Fault Isolation and Quantification from Gaussian Residuals with Application to Structural Damage Quantification

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Abstract: Fault detection for structural health monitoring has been a topic of much research during the last decade. Localization and quantification of damages, which are linked to fault isolation, have proven to be more challenging, and at the same time of higher practical impact. While damage detection can be essentially handled as a data-driven approach, localization and quantification require a strong connection between data analysis and physical models. This paper builds upon a hypothesis test that checks if the mean of a Gaussian residual vector – whose parameterization is linked to possible damage locations – has become non-zero in the faulty state. It is shown how the damage location and extent can be inferred and robust numerical schemes for their estimation are derived based on QR decompositions and minmax approaches. Finally, the relevance of the approach is assessed in numerical simulations of two structures.

Keywords: Fault isolation, residual evaluation, statistical tests, damage quantification, structural health monitoring.

1. INTRODUCTION

Damage in structures impairs their operation if not detected early enough, in which case the operational state can be improved and the remaining life of the structure lengthened. For this reason, the question of vibration-based damage diagnosis, i.e. damage detection, localization and quantification, has been studied from many different angles in the last decade [Fan and Qiao, 2011].

For example, purely data-driven empirical approaches based e.g. on mode shapes curvatures or the wavelet transform have been considered for certain classes of structures such as beams. On the other side, model-based approaches rely on an analytical structural model, on the premise that damage causes changes in the parameters of such a finite element model (FEM), involving the mass, damping and stiffness matrices of the structure. Based on a well-fitted reference model, the problem of damage localization and quantification may be resolved through sensitivity analysis and model updating with measurements from the faulty state [Teughels et al., 2002, Viet Hà and Golinval, 2010]. However, FEM updating is often an ill-posed problem, since usually the dimension of the FEM is much larger than the number of identified parameters from measurements.

Alternative methods for damage localization with a theoretical background avoid the model updating step, while linking FEM information to data from both safe and damaged structures in statistical tests. For example, changes in the structural flexibility are interrogated [Marin et al., 2012] or the FEM parameters of a Gaussian residual vector are tested for a change based on the local approach [Benveniste et al., 1987, Basseville et al., 2000, 2004, Döhler and Mevel, 2013, Döhler et al., 2014b], on which this paper is based. While many fault detection and isolation (FDI) methods concern additive faults, e.g. [Dong and Verhaegen, 2009, Dong et al., 2012], our approach assumes a general description of the fault affecting the state space matrices.

In this work, FDI is based on a Gaussian residual vector with zero mean in the reference state and non-zero mean in the faulty state. The fault isolation problem consists in deciding which components in a related parameterization have changed by hypothesis testing [Basseville and Nikiforov, 1993, Basseville, 1997]. For FEM parameterizations, this is related to damage localization in [Basseville et al., 2004, Balmès et al., 2008]. It was demonstrated in [Döhler et al., 2014b] that the minmax test has more appropriate properties and is better suited for the damage localization problem when the parameters are not fully independent and multiple parameters are changing. Minmax approaches have also been applied to damping monitoring [Zouari et al., 2009] or damage detection with temperature rejection [Balmès et al., 2009]. From a numerical point of view, QR approaches have been proved to be a tool to enforce stability for such algorithms [Zhang and Basseville, 2003, Döhler et al., 2014b]. While this framework allows fault isolation for Gaussian residual vectors, it has never been attempted to quantify the absolute change in the faulty parameter components in this context. In this paper, we extend the local approach to damage quantification.

This paper is organized as follows. In Sections 2 and 3, structural modelization and numerical schemes for hypothesis testing are recalled. In Section 4, fault quantification is addressed and applied to structural damage diagnosis in

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Section 5. Numerical applications of the proposed scheme are reported in Section 6.

2. MOTIVATION: STRUCTURAL DAMAGE DIAGNOSIS BASED ON LOCAL APPROACH

The behavior of linear time-invariant dynamical structures subject to unknown ambient excitation can be described by system

$$\mathcal{M}\ddot{\mathcal{X}}(t) + \mathcal{C}\dot{\mathcal{X}}(t) + \mathcal{K}\mathcal{X}(t) = v(t) \tag{1}$$

where t denotes continuous time; $\mathcal{M}, \mathcal{C}, \mathcal{K} \in \mathbb{R}^{m\times m}$ are mass, damping, and stiffness matrices, respectively; the (high dimensional) state vector $\mathcal{X}(t) \in \mathbb{R}^m$ is the displacement vector of the m degrees of freedom of the structure; and $v(t)$ is the external unmeasured force (noise).

Observing system (1) at r sensor positions (e.g. displacement, velocity or acceleration sensors) at discrete time instants $t = k\tau$ (with sampling rate $1/\tau$), it can be transformed to an equivalent discrete-time state space system [Juang, 1994]

$$\begin{cases} x_{k+1} = Ax_k + v_k \\ \quad y_k = Cx_k + w_k \end{cases} \tag{2}$$

with the states $x_k \in \mathbb{R}^n$, the measured outputs $y_k \in \mathbb{R}^r$, the state transition matrix

$$A = \exp\left(\begin{bmatrix} 0 & I \\ -\mathcal{M}^{-1}\mathcal{K} & -\mathcal{M}^{-1}\mathcal{C} \end{bmatrix}\tau\right) \in \mathbb{R}^{n\times n}$$

and the observation matrix

$$C = \begin{bmatrix} L_d - L_a\mathcal{M}^{-1}\mathcal{K} & L_v - L_a\mathcal{M}^{-1}\mathcal{C} \end{bmatrix} \in \mathbb{R}^{r\times n},$$

where $n = 2m$ is the model orderand $L_d, L_v, L_c \in \{0,1\}^{r\times m}$ are selection matrices indicating the positions of displacement, velocity or acceleration sensors. The state noise v_k and output noise w_k are unmeasured and assumed to be Gaussian, zero-mean, white.

Our system parameter θ of interest is linked to the structural properties of system (1), and can be e.g. mass parameters of the elements in a finite element model (FEM), i.e. $\mathcal{M} = \mathcal{M}(\theta)$, or parameters corresponding to element stiffness such as Young's modulus of the individual structural elements, i.e. $\mathcal{K} = \mathcal{K}(\theta)$. In particular, $A = A(\theta)$ in the corresponding system (2). Structural damage leads to changes in θ, and hence the considered faults in system (2) are changes in θ. They are not additive. It is assumed that structural damage is local and thus affects only a small part of the parameters in θ. After damage has been localized, i.e. it has been decided which parameters in θ are faulty ("fault isolation") based on measurements $\{y_k\}_{k=1,\dots,N}$ of the faulty system, we propose a strategy to determine the change δ in the faulty parameters in order to perform damage quantification.

To achieve this task, our work is based on the asymptotic local approach for change detection [Benveniste et al., 1987]. It assumes the close hypotheses

$$\begin{aligned} &\mathbf{H}_0: \ \theta = \theta_0 && \text{(reference system)}, \\ &\mathbf{H}_1: \ \theta = \theta_0 + \delta/\sqrt{N} && \text{(faulty system)}, \end{aligned} \tag{3}$$

where vector θ_0 denotes the system parameter in the reference state and vector δ is unknown but fixed. With this statistical framework, very small changes in the system parameter θ can be detected if the number of measurements N is large enough. For FDI, a subspace-based residual vector ζ_N has been proposed in [Basseville et al., 2000] and is detailed in Section 5. Using the structural parameterization θ as explained above, it yields the central limit theorem (CLT)

$$\zeta_N \xrightarrow{d} \begin{cases} \mathcal{N}(0,\Sigma) & \text{under } \mathbf{H}_0 \\ \mathcal{N}(\mathcal{J}\delta,\Sigma) & \text{under } \mathbf{H}_1 \end{cases} \tag{4}$$

for $N \to \infty$, where $\mathcal{J}$ and Σ are the asymptotic sensitivity and covariance, respectively. In this framework, we recall suitable statistical tests for fault isolation in the following section.

3. BASIC FAULT ISOLATION TESTS

3.1 Definitions

Let a parameter vector $\theta \in \mathbb{R}^l$ and a Gaussian residual vector $\zeta = \zeta(\theta) \in \mathbb{R}^h$ be given, yielding (4). Its sensitivity matrix $\mathcal{J} \in \mathbb{R}^{h\times l}$ is assumed to have full column rank and its covariance $\Sigma \in \mathbb{R}^{h\times h}$ is positive definite. Many methods have been proposed in the literature for fault detection (decide if there is a change in θ) and isolation (decide which elements of vector θ have changed) by the means of such a Gaussian residual vector ζ [Basseville and Nikiforov, 1993, Basseville, 1997].

For both problems of fault isolation and quantification, different partitions of the vector δ into two subvectors are considered. Without loss of generality, let this partition be

$$\delta = \begin{bmatrix} \delta_a \\ \delta_b \end{bmatrix}.$$

For fault isolation, a decision between $\delta_a = 0$ and $\delta_a \neq 0$ is made for each partition. Let $F = \mathcal{J}^T\Sigma^{-1}\mathcal{J}$ be the Fisher information matrix of the parameter θ contained in vector ζ, and let $\mathcal{J}$ and F be partitioned accordingly as

$$\mathcal{J} = [\mathcal{J}_a \ \ \mathcal{J}_b], F = \begin{bmatrix} F_{aa} & F_{ab} \\ F_{ba} & F_{bb} \end{bmatrix} = \begin{bmatrix} \mathcal{J}_a^T\Sigma^{-1}\mathcal{J}_a & \mathcal{J}_a^T\Sigma^{-1}\mathcal{J}_b \\ \mathcal{J}_b^T\Sigma^{-1}\mathcal{J}_a & \mathcal{J}_b^T\Sigma^{-1}\mathcal{J}_b \end{bmatrix}.$$

In the next sections, two isolation tests are recalled and extended to the quantification of the faults, i.e. the estimation of δ_a in Section 4.

3.2 Sensitivity Test

The simplest possibility for testing $\delta_a = 0$ (no change in parameter subset 'a') against $\delta_a \neq 0$ is to assume $\delta_b = 0$ and thus $\zeta \sim \mathcal{N}(\mathcal{J}_a\delta_a, \Sigma)$. The corresponding Generalized Likelihood Ratio (GLR) test writes as [Basseville, 1997]

$$t_{\text{sens}} = \zeta^T\Sigma^{-1}\mathcal{J}_a\left(\mathcal{J}_a^T\Sigma^{-1}\mathcal{J}_a\right)^{-1}\mathcal{J}_a^T\Sigma^{-1}\zeta, \tag{5}$$

which is called sensitivity test. The test variable t_{sens} is χ^2-distributed with $\dim(\theta_a)$ degrees of freedom and non-centrality parameter $\delta_a^T F_{aa}\,\delta_a$, if $\delta_b = 0$ is actually true. For a decision, the test variable is compared to a threshold.

Remark 1. If the assumption $\delta_b = 0$ does not hold, the non-centrality parameter of t_{sens} follows as

$$\delta_a^T F_{aa}\,\delta_a + 2\delta_a^T F_{ab}\delta_b + \delta_b^T F_{ba}F_{aa}^{-1}F_{ab}\delta_b. \tag{6}$$

3.3 Minmax Test

Instead of assuming $\delta_b = 0$, the variable δ_b is replaced by its least favorable value for a decision about δ_a, as follows. Define the partial residuals as

$$\zeta_a \stackrel{\text{def}}{=} \mathcal{J}_a^T \Sigma^{-1} \zeta, \tag{7a}$$

$$\zeta_b \stackrel{\text{def}}{=} \mathcal{J}_b^T \Sigma^{-1} \zeta, \tag{7b}$$

the robust residual as [Basseville, 1997]

$$\zeta_a^* \stackrel{\text{def}}{=} \zeta_a - F_{ab} F_{bb}^{-1} \zeta_b \tag{8}$$

and

$$F_a^* \stackrel{\text{def}}{=} F_{aa} - F_{ab} F_{bb}^{-1} F_{ba}. \tag{9}$$

Then, the mean of the robust residual ζ_a^* is sensitive to changes δ_a, but blind to δ_b, and it holds

$$\zeta_a^* \sim \mathcal{N}\left(F_a^* \, \delta_a, \; F_a^*\right). \tag{10}$$

The corresponding GLR test for $\delta_a = 0$ (no change in parameter subset 'a') against $\delta_a \neq 0$ writes as

$$t_{\text{mm}} = \zeta_a^{*T} F_a^{*-1} \zeta_a^*, \tag{11}$$

which is called minmax test. The test variable t_{mm} is χ^2-distributed with $\dim(\theta_a)$ degrees of freedom and non-centrality parameter $\delta_a^T F_a^* \, \delta_a$, independently of δ_b. For a decision, the test variable is compared to a threshold. Note that the invertibility of all matrices in the computation is guaranteed, since $\mathcal{J}$ is assumed to have full column rank and Σ is positive definite.

3.4 Numerically Efficient Computation

Both the sensitivity and the minmax test require a number of matrix inversions, which may be numerically critical due to possible ill-conditioning of the covariance matrix Σ. In [Zhang and Basseville, 2003, Döhler et al., 2014b] numerically more robust computations of both tests were suggested, which are recalled in the following. They make use of the decomposition

$$\Sigma^{-1} = (\Sigma^{-1/2})^T \Sigma^{-1/2}, \tag{12}$$

which can be obtained e.g. from an SVD of $\Sigma = U \Delta U^T$ as $\Sigma^{-1/2} = \Delta^{-1/2} U^T$, as well as QR decompositions. Note that an efficient computation of $\Sigma^{-1/2}$ is detailed in [Döhler and Mevel, 2011, Döhler et al., 2014a].

Sensitivity Test Using the thin QR decomposition [Golub and Van Loan, 1996]

$$\Sigma^{-1/2} \mathcal{J}_a = QR, \tag{13}$$

the sensitivity test in (5) writes as [Zhang and Basseville, 2003]

$$t_{\text{sens}} = \alpha^T \alpha, \quad \text{where} \quad \alpha = Q^T \Sigma^{-1/2} \zeta. \tag{14}$$

Here, the number of numerically critical matrix inversions and multiplications is limited to a minimum compared to a direct computation in (5).

Minmax Test Let the thin QR decomposition of

$$\Sigma^{-1/2} \left[\mathcal{J}_b \;\, \mathcal{J}_a\right] = \left[Q_b \;\, Q_a\right] \begin{bmatrix} R_{bb} & R_{ba} \\ 0 & R_{aa} \end{bmatrix} \tag{15}$$

be given and partitioned accordingly. Then, the minmax test in (11) writes as [Döhler et al., 2014b]

$$t_{\text{mm}} = \beta^T \beta, \quad \text{where} \quad \beta = Q_a^T \Sigma^{-1/2} \zeta. \tag{16}$$

4. FAULT QUANTIFICATION

In this section, estimates of the fault δ_a are derived based on the properties of the sensitivity and minmax tests, respectively. Special care is also taken of a numerically sensible computation of these estimates.

4.1 Sensitivity Approach

Theorem 2. Define

$$\widehat{\delta}_a^{\text{sens}} \stackrel{\text{def}}{=} (\mathcal{J}_a^T \Sigma^{-1} \mathcal{J}_a)^{-1} \mathcal{J}_a^T \Sigma^{-1} \zeta. \tag{17}$$

Then, under the assumption $\delta_b = 0$,

$$\widehat{\delta}_a^{\text{sens}} \;\sim\; \mathcal{N}\left(\delta_a, F_{aa}^{-1}\right).$$

Proof. Since $\zeta \sim \mathcal{N}\left(\mathcal{J} \, \delta, \Sigma\right)$, it follows under the assumption $\delta_b = 0$ (see Section 3.2) that

$$\zeta \sim \mathcal{N}\left(\mathcal{J}_a \, \delta_a, \Sigma\right).$$

It follows easily

$$\mathcal{J}_a^T \Sigma^{-1} \zeta \sim \mathcal{N}(\mathcal{J}_a^T \Sigma^{-1} \mathcal{J}_a \, \delta_a, \mathcal{J}_a^T \Sigma^{-1} \mathcal{J}_a)$$

and, since $F_{aa} = \mathcal{J}_a^T \Sigma^{-1} \mathcal{J}_a$, the assertion follows. □

Since the sensitivity approach requires the assumption $\delta_b = 0$, which is in reality never guaranteed, the potential error can be analyzed in the following corollary.

Corollary 3. For δ_b arbitrary, it holds

$$\widehat{\delta}_a^{\text{sens}} \;\sim\; \mathcal{N}\left(\delta_a + F_{aa}^{-1} F_{ab} \, \delta_b, F_{aa}^{-1}\right). \tag{18}$$

Proof. Since $\zeta \sim \mathcal{N}\left(\mathcal{J}_a \, \delta_a + \mathcal{J}_b \, \delta_b, \Sigma\right)$, the assertion follows from the definition of $\widehat{\delta}_a^{\text{sens}}$ in (17). □

A numerically sensible computation of $\widehat{\delta}_a^{\text{sens}}$ can be achieved as follows.

Corollary 4. With the QR decomposition (13), namely $\Sigma^{-1/2} \mathcal{J}_a = QR$, it holds

$$\widehat{\delta}_a^{\text{sens}} = R^{-1} Q^T \Sigma^{-1/2} \zeta,$$

coinciding with $R^{-1} \alpha$ in the test computation in (14).

Proof. With decomposition (12), $\widehat{\delta}_a^{\text{sens}}$ in (17) can be written as

$$\widehat{\delta}_a^{\text{sens}} = \left((\Sigma^{-1/2} \mathcal{J}_a)^T \Sigma^{-1/2} \mathcal{J}_a\right)^{-1} (\Sigma^{-1/2} \mathcal{J}_a)^T \Sigma^{-1/2} \zeta.$$

Plugging in the QR decomposition $\Sigma^{-1/2} \mathcal{J}_a = QR$, where $Q^T Q = I$ and R is invertible, yields

$$\widehat{\delta}_a^{\text{sens}} = \left(R^T Q^T Q R\right)^{-1} R^T Q^T \Sigma^{-1/2} \zeta,$$

leading to the assertion. □

4.2 Minmax Approach

Theorem 5. Let the variables for the minmax test be given in (7a)–(9). Define

$$\widehat{\delta}_a^{\text{mm}} \stackrel{\text{def}}{=} (F_a^*)^{-1} \zeta_a^*. \tag{19}$$

Then,

$$\widehat{\delta}_a^{\text{mm}} \;\sim\; \mathcal{N}(\delta_a, (F_a^*)^{-1}).$$

Proof. The assertion follows immediately from property (10). □

Similar to the minmax test (11), the computation of $\widehat{\delta}_a^{\text{mm}}$ in (19) is a numerical challenge since the computation of ζ_a^* and F_a^* requires a number of matrix operations (inversions, subtractions) that may be critical especially if Σ is badly conditioned. In (15)–(16) a solution for the minmax test computation was recalled from [Döhler et al., 2014b] that is based on the QR decomposition, requiring a minimum of the former numerically critical operations. Based on these results, a simple computation for $\widehat{\delta}_a^{\mathrm{mm}}$ is based on the same QR decomposition as follows.

Corollary 6. With the QR decomposition (15), it holds

$$\widehat{\delta}_a^{\mathrm{mm}} = R_{aa}^{-1} Q_a^T \Sigma^{-1/2} \zeta,$$

coinciding with $R_{aa}^{-1}\beta$ in the test computation in (16).

Proof. In the proof of [Döhler et al., 2014b, Thm. 1] it was shown that QR decomposition (15) yields

$$\zeta_a^* = R_{aa}^T Q_a^T \Sigma^{-1/2} \zeta, \quad F_a^* = R_{aa}^T R_{aa}.$$

Substituting these results in (19) leads immediately to the assertion. □

5. STRUCTURAL DAMAGE LOCALIZATION AND QUANTIFICATION

We now return to our application on structural damage localization and quantification, as laid out in Section 2. Based on the asymptotic local approach for change detection [Benveniste et al., 1987], we assume the close hypotheses (3), where θ is a parameter vector containing local structural parameters, as for example the stiffness corresponding to each structural element.

In this framework, a subspace-based residual vector has been defined in [Basseville et al., 2000] based on outputs $\{y_k\}_{k=1,\ldots,N}$ of system (2) as

$$\zeta_N = \sqrt{N}\,\mathrm{vec}(S^T \widehat{\mathcal{H}}_{p+1,q}), \tag{20}$$

where $\widehat{\mathcal{H}}_{p+1,q}$ is an estimate of the Hankel matrix

$$\mathcal{H}_{p+1,q} \stackrel{\mathrm{def}}{=} \begin{bmatrix} R_1 & R_2 & \ldots & R_q \\ R_2 & R_3 & \ldots & R_{q+1} \\ \vdots & \vdots & \ddots & \vdots \\ R_{p+1} & R_{p+2} & \ldots & R_{p+q} \end{bmatrix}$$

containing the output correlations $R_i = \mathbf{E}(y_k y_{k-i}^T)$, and S is the left null space of $\mathcal{H}_{p+1,q}$ from the reference system.

The residual vector ζ_N in (20) satisfies the Central Limit Theorem (4) [Basseville et al., 2000]. The estimation of its asymptotic sensitivity $\mathcal{J}$ is described in detail in [Basseville et al., 2004, Balmès et al., 2008] for a chosen structural parameterization θ, where the residual is first derived wrt. the modal parameters (frequencies, mode shapes) obtained from the eigenvalues and eigenvectors of system (2), which are then derived wrt. the structural parameters using FEM (1). The estimation of Σ is described in [Döhler and Mevel, 2011, Döhler et al., 2014a].

Based on these considerations, both the sensitivity and the minmax tests from Section 3 can be applied to residual ζ_N (asymptotically, N large enough) for each of the structural parameters in $\theta = \left[\theta^1 \ \ldots \ \theta^l\right]$ to perform damage localization. A parameter whose test value exceeds a threshold is regarded as faulty and corresponds to a damage location in the structure. Finally, the quantification of the damage extent can be performed with the respective estimators derived in Section 4. Note that estimates $\widehat{\delta}$ are obtained. The parameter change that we are interested in for damage quantification, however, is the absolute change $\theta - \theta_0$ of the structural parameters, which can be obtained from

$$\widehat{\theta} - \theta_0 = \widehat{\delta}/\sqrt{N}$$

due to the local hypothesis in (4).

6. APPLICATION

The damage localization and quantification techniques have been applied to two simulated structures. The first example is a simple mass-spring chain, the second a truss structure. In both cases, output-only datasets with displacement samples were generated at the sensor coordinates in reference and damaged states from white noise excitation. Measurement noise was added with a magnitude of 5% of each generated output signal. The damping was always defined such that all modes have a damping ratio of 2%. All parameters of the tests (S, $\mathcal{J}$, Σ) were estimated based on the dataset in the reference state and the information from the respective FEM.

In the damaged state, both the sensitivity and the minmax tests from Section 3 were applied to the datasets for each structural element, and the damage extent was estimated for the damaged element(s) based on the respective tests as shown in Sections 4–5. In each quantification example, 100 realizations of the simulated time series were used. The quantification results are shown for several damage cases and extents, each as the mean from the 100 estimates together with their standard deviation.

6.1 Mass-spring chain

A mass-spring chain with eight elements is considered (Fig. 1) with masses $m_1 = m_3 = m_5 = m_7 = 1, m_2 = m_4 = m_6 = m_8 = 2$ and stiffnesses $k_1 = k_3 = k_5 = k_7 = 200, k_2 = k_4 = k_6 = k_8 = 100$. Output time series of length $N = 100{,}000$ are simulated at time step $\tau = 0.05$ s at four elements. $\mathcal{J}$ is computed on all eight modes of the system. We consider three damage cases, where damage is simulated by a stiffness reduction in one or two springs.

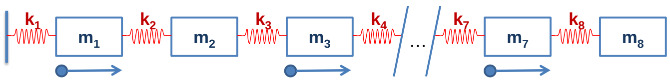

Fig. 1. Mass-spring chain with four sensors.

Damage localization

Damage in element 4, Fig. 2 While the minmax tests behave perfectly, the sensitivity test for element 3 also reacts due to the violation of $\delta_b = 0$. The strong reaction only for element 3 among the undamaged elements becomes clear when evaluating the non-centrality parameter of the test in (6), in particular the value of $F_{ba}F_{aa}^{-1}F_{ab}$ where 'a' corresponds to any undamaged element and 'b' corresponds to the damaged element 4: this value is at 22.3 for element 3 while being lower than 0.4 for all other undamaged elements.

Damage in elements 2 and 4, Fig. 3 The minmax tests behave perfectly again, while the damage in spring 2 cannot be detected with the sensitivity tests as the test value of element 3 is significant. This is analogous to the previous case, where it was explained why the test for element 3 reacts when element 4 is damaged (see Fig. 2). The minmax test is robust to changes in the non-tested parameters θ_b by design.

Damage in elements 3 and 4, Fig. 4 The sensitivity and minmax tests behave very well in this case. Note that the

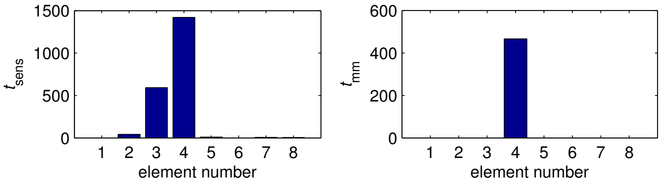

Fig. 2. Sensitivity tests (left) and minmax tests (right) for 10% damage in element 4.

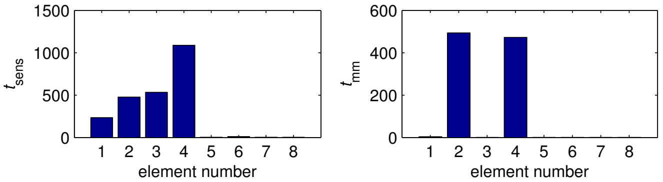

Fig. 3. Sensitivity (left) and minmax tests (right) for 5% damage in element 2 and 10% damage in element 4.

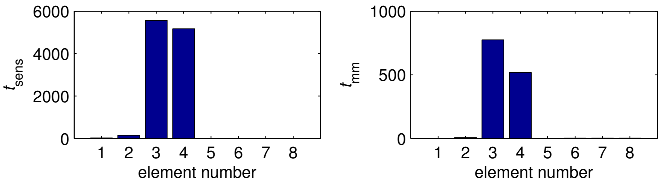

Fig. 4. Sensitivity (left) and minmax tests (right) for 5% damage in element 3 and 10% damage in element 4.

test values for element 3 is in both tests larger than for element 4, while element 4 is more damaged. Indeed, the test values serve only for the decision if the respective element is damaged or not by comparing it to a threshold. They are not directly linked to the damage extent, which is estimated separately in the following as shown in Section 4.

Damage quantification

Damage in element 4, Fig. 5 (top) The damage extents are well estimated for both the sensitivity and the minmax approaches, only large extents are slightly overestimated.

Damage in elements 2 and 4, Fig. 5 (left) The stiffness reduction in element 2 is half as large as in element 4 for each damage. Results from the minmax approach are satisfying, where again only large damage extents are slightly overestimated. The damage quantification from the sensitivity approach is biased (here underestimated), since the assumption $\delta_b = 0$ is violated. As shown in Corollary 3, the theoretic bias for δ_a is $F_{aa}^{-1} F_{ab}\, \delta_b$ in (18), and $F_{aa} > 0$ by definition. Indeed, $\widehat{F}_{ab} = -0.4 < 0$ between the parameters corresponding to damaged elements 2 and 4 in this case, which explains the underestimation.

Damage in elements 3 and 4, Fig. 5 (right) The results for the minmax approach are similar as in the previous two-damage case, whereas the damage extent is strongly overestimated now when using the sensitivity approach. This is in line with $\widehat{F}_{ab} = 2.09 > 0$ for the parameters corresponding to elements 3 and 4 in this case.

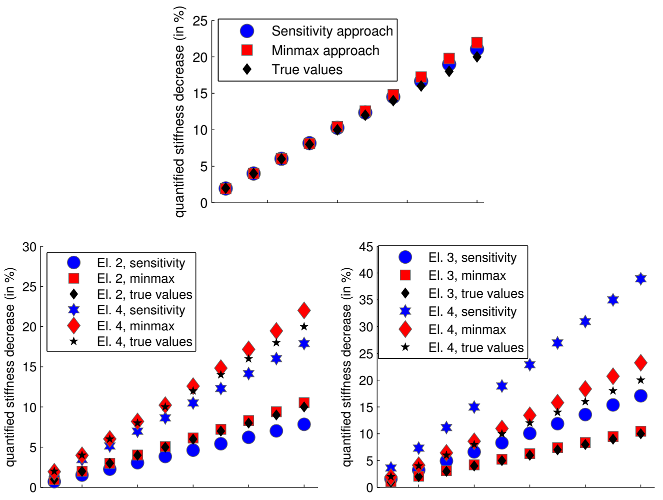

Fig. 5. Quantification of different damage extents in elements 4 only (top), 2 and 4 (left), 3 and 4 (right).

6.2 Truss

A truss structure with 25 elements of equal stiffness properties, having six sensors, has been considered as a more complex example (Fig. 6). Output time series of length $N = 50{,}000$ are simulated at time step $\tau = 0.05$ s. At this sampling frequency, we are in the more realistic case of modal truncation, since only ten modes are present in the data that can be taken into account for the computation of $\mathcal{J}$, compared to altogether 25 modes of the analytical model. In general, only a small number of modes can be obtained from measurements of system (2) compared to the number of modes present in the FEM (1). Furthermore, S is estimated on the simulated data in this example and not on the analytical modes of the model. Damage is simulated by decreasing the element stiffness.

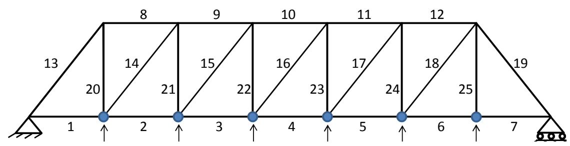

Fig. 6. Truss structure with six sensors.

Two scenarios are considered, one with damage in element 16, and another one with damage in both elements 16 and 23, where the stiffness reduction in element 16 is half as large as in element 23 for each considered damage extent. We show only quantification results for these cases and skip localization results for brevity. Still, damage was localized correctly in both cases with the sensitivity and the minmax tests, where the test values for the undamaged elements are much lower in the minmax tests, as expected.

Damage in element 16, Fig. 7 (left) The estimates from both the sensitivity and the minmax approach are close to the true values. The estimation errors are however larger for large damage extents, maybe due to the nature of the local approach: the relation between ζ_N and δ is non-linear and the first-order approximation involving $\mathcal{J}$ (computed at $\delta = 0$) may thus be less accurate for large changes δ.

Damage in elements 16 and 23, Fig. 7 (right) The minmax approach yields quite accurate estimates, while the sensitivity approach overestimates the damage extent significantly, similar to the example in Fig. 5 (right). This overestimation can be explained analogously by $\widehat{F}_{ab} = 0.2 > 0$ between damaged elements 16 and 23.

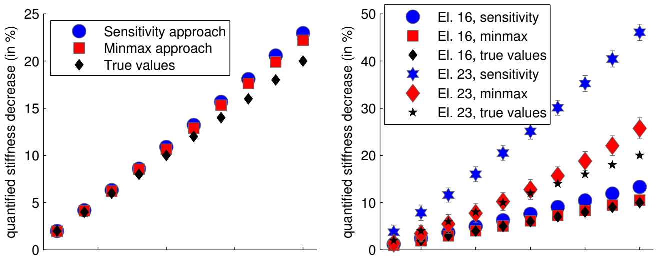

Fig. 7. Quantification of different damage extents in element 16 (left) and both elements 16 and 23 (right).

While this example does not satisfy the theoretical assumptions perfectly anymore (modal truncation), it still gives reasonable results. This shows that the developed damage quantification approach is promising, and further investigation is required for application on real structures.

7. CONCLUSIONS

In this paper, the quantification of parameter changes has been investigated based on a Gaussian residual vector and hypothesis testing. It has been shown that the local hypothesis modeling for a parameter change in θ as $\delta/\sqrt{N}$ is not only a convenient mathematical tool for establishing the CLT for the Gaussian residual, but also has a sensible and meaningful interpretation leading to an effective quantification of the parameter change. Based on sensitivity and minmax tests, estimators for the parameter change have been derived together with QR based robust numerical schemes for their computation. An application was shown for vibration-based structural damage localization and quantification in a simulation study, where changes in local parameters of a structure indicate damage. It has been noticed that the local approach is more precise if parameter changes are small, as expected. With more than one parameter changing, i.e. more than one damage, the minmax test proved to be more effective than the sensitivity test for both damage localization and quantification, where it avoids false alarms at undamaged elements for localization and it avoids bias in the damage quantification. Further work should link damage quantification and compressed sensing to handle FEM dimensions that largely exceed the modal parameter dimension, which is related to the number of sensors and identified modes.

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