Impedance Transmission Conditions for the Electric Potential across a Highly Conductive Casing

Aralar Erdozain, Victor Péron

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A. Erdozain[1,*], **V. Péron**[1,2]
[1]INRIA Bordeaux Sud-Ouest, Team magique 3D
[2]Université de Pau et des Pays de l'Adour
*Email: aralar.erdozain@inria.fr

Abstract

We present Impedance Transmission Conditions (ITCs) for the electric potential in the framework of borehole through-casing resistivity measurements. Such ITCs substitute the part of the domain corresponding to a highly conductive casing. The naturally small thickness of the casing makes it ideal for exhibiting ITCs. We numerically observe the delivered order of accuracy.

Keywords: Impedance Conditions, Electric Potential, Borehole, Casing, Resistivity.

1 Introduction

Borehole resistivity measurements are commonly used when trying to obtain a better characterization of the earth's subsurface. Often a metallic casing is employed to surround the well, which allows to protect the well and avoid possible collapses. The use of such casing highly complicates the analysis due to large contrast between the conductivities of the casing and the rock formations.

This work is motivated by realistic configurations [2] where the conductivity of the casing is $\sigma_{\rm c} \approx \varepsilon^{-3}$ when ε denotes the thickness of the casing. In this framework, our aim is to derive ITCs for the electromagnetic field across such a casing. As a first approach we derive ITCs for the electric potential.

We refer to [1] where the authors derive ITCs for eddy current models with a conductivity parameter of a thin sheet of the form $\sigma \approx \varepsilon^{-2}$.

We first introduce the mathematical model. Then, we explicit two asymptotic models of order two and four. Finally, we numerically analyse the performance and order of accuracy of the ITCs.

2 The Mathematical Model

We consider a transmission problem for the static case of the electric potential, the governing equations read as follows

$$\left\{\begin{array}{rlll} \sigma_{\rm i}\Delta u_{\rm i} & = f_{\rm i} & \text{in} & \Omega_{\rm i}^\varepsilon \\ \sigma_{\rm e}\Delta u_{\rm e} & = f_{\rm e} & \text{in} & \Omega_{\rm e}^\varepsilon \\ \sigma_{\rm c}\Delta u_{\rm c} & = 0 & \text{in} & \Omega_{\rm c}^\varepsilon \\ u_{\rm i} & = u_{\rm c} & \text{on} & \Gamma_{\rm i}^\varepsilon \\ u_{\rm c} & = u_{\rm e} & \text{on} & \Gamma_{\rm e}^\varepsilon \\ \sigma_{\rm i}\partial_n u_{\rm i} & = \sigma_{\rm c}\partial_n u_{\rm c} & \text{on} & \Gamma_{\rm i}^\varepsilon \\ \sigma_{\rm c}\partial_n u_{\rm c} & = \sigma_{\rm e}\partial_n u_{\rm e} & \text{on} & \Gamma_{\rm e}^\varepsilon \\ u & = 0 & \text{on} & \partial\Omega \end{array}\right. \quad (1)$$

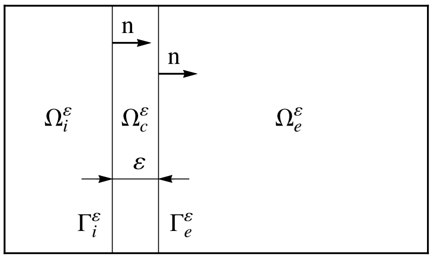

Figure 1: Domain of interest.

The considered domain $\Omega \subset \mathbb{R}^2$ is formed by three subdomains $\Omega_{\rm i}^\varepsilon$, $\Omega_{\rm c}^\varepsilon$, $\Omega_{\rm e}^\varepsilon$, which are characterized by different conductivities $\sigma_{\rm i}$, $\sigma_{\rm e}$, $\sigma_{\rm c}$. These subdomains have rectangular shape and $\Omega_{\rm c}^\varepsilon$ is a thin layer of uniform thickness ε, as shown in Figure 1. In (1), $f_{\rm i}$, $f_{\rm e}$, $\sigma_{\rm i}$, $\sigma_{\rm e}$ and $\sigma_{\rm c}$ are known data and u corresponds to the unknown, whose restrictions to the different subdomains are denoted as $u|_{\Omega_{\rm i}^\varepsilon} = u_{\rm i}$, $u|_{\Omega_{\rm e}^\varepsilon} = u_{\rm e}$, $u|_{\Omega_{\rm c}^\varepsilon} = u_{\rm c}$.

In this framework, we address the issue of ITCs for u (as $\varepsilon \to 0$) when the conductivity of the casing is $\sigma_{\rm c} = \alpha\varepsilon^{-3}$ ($\alpha \in \mathbb{R}$).

3 Main Results

For developing ITCs, we perform a formal expansion of the solution u in power series of ε. This leads to a collection of problems which can be solved successively. Then truncating the series we build asymptotic models by obtaining ITCs between $\Gamma_{\rm i}^\varepsilon$ and $\Gamma_{\rm e}^\varepsilon$.

Definition 1 *Let u be the solution of problem (1). We say that $u^{[k]}$ satisfies an asymptotic model of order $k+1$ when (for ε small enough)*

$$||u - u^{[k]}||_{L^2(\Omega_i^\varepsilon \cup \Omega_e^\varepsilon)} \leq C\varepsilon^{k+1}, \qquad C \in \mathbb{R}.$$

We derive two asymptotic models of order two and four.

Order 2 model

$$\begin{cases} \sigma_{\rm i}\Delta u_{\rm i}^{[1]} & = f_{\rm i} \quad \text{in} \quad \Omega_{\rm i}^\varepsilon \\ u_{\rm i}^{[1]} & = 0 \quad \text{on} \quad \partial\Omega_{\rm i}^\varepsilon \end{cases}$$

$$\begin{cases} \sigma_{\rm e}\Delta u_{\rm e}^{[1]} & = f_{\rm e} \quad \text{in} \quad \Omega_{\rm e}^\varepsilon \\ u_{\rm e}^{[1]} & = 0 \quad \text{on} \quad \partial\Omega_{\rm e}^\varepsilon \end{cases}$$

Order 4 model

$$\begin{cases} \sigma_{\rm i}\Delta u_{\rm i}^{[3]} & = f_{\rm i} & \text{in} & \Omega_{\rm i}^\varepsilon \\ \sigma_{\rm e}\Delta u_{\rm e}^{[3]} & = f_{\rm e} & \text{in} & \Omega_{\rm e}^\varepsilon \\ \left[u^{[3]}\right] & = 0 & & \\ \left[\sigma\partial_n u^{[3]}\right] & = -\frac{\alpha}{\varepsilon^2}\Delta_\Gamma \left\{u^{[3]}\right\} & & \\ u_{\rm i}^{[3]} & = 0 & \text{on} & \partial\Omega_{\rm i}^\varepsilon \backslash \Gamma_{\rm i}^\varepsilon \\ u_{\rm e}^{[3]} & = 0 & \text{on} & \partial\Omega_{\rm e}^\varepsilon \backslash \Gamma_{\rm e}^\varepsilon \end{cases}$$

Here, the jump and mean value of a function u across the domain $\Omega_{\rm c}^\varepsilon$ are

$$\begin{aligned} [u] &:= u_{\rm e}|_{\Gamma_{\rm e}^\varepsilon} - u_{\rm i}|_{\Gamma_{\rm i}^\varepsilon} \\ \{u\} &:= \frac{1}{2}\left(u_{\rm e}|_{\Gamma_{\rm e}^\varepsilon} + u_{\rm i}|_{\Gamma_{\rm i}^\varepsilon}\right). \end{aligned}$$

The validation of these models consists in proving estimates for $u - u^{[k]}$ (see Definition 1). Hereafter, we present numerical validations for each asymptotic model.

4 Numerical Results

We developed a finite element code to solve problem (1) and the two asymptotic models. We consider $f_{\rm i} = 1$, $f_{\rm e} = 1$ as right hand sides and we select $\sigma_{\rm i} = 3$, $\sigma_{\rm e} = 5$, $\sigma_{\rm c} = \varepsilon^{-3}$ as the different conductivities.

We compute the L^2 error between the solution of problem (1) and the solution of each asymptotic model for different values of ε by using triangular elements and degree five Lagrange polynomials. We observe these results in Figure 2 and the corresponding slopes of the graphics in Table 1. Each numerical convergence rate converges to the formal order of accuracy.

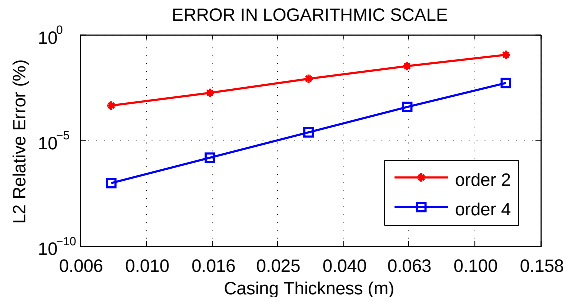

Figure 2: L^2 error of the order 2 and order 4 model for different values of ε

Casing Thickness ε	0.0117	0.0234	0.0469	0.0938
Order 2 Slopes	1.9944	1.9848	1.9537	1.8456
Order 4 Slopes	3.9906	3.9699	3.8962	3.6430

Table 1: Slopes corresponding to the curves of Figure 2

5 Perspectives

Among the future perspectives, we would like to derive asymptotic models for different configurations including the non-static case of the electric potential and electromagnetic field. We plan to perform mathematical proofs to validate these asymptotic models.

References

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