Leader-follower fixed-time consensus for multi–agent systems with unknown nonlinear inherent dynamics

Michael Defoort, Andrey Polyakov, Guillaume Demesure, Mohamed Djemai

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Michael Defoort[a,*], Andrey Polyakov[b], Guillaume Demesure[a], Mohamed Djemai[a,c]

[a] *LAMIH, CNRS UMR 8201, University of Valenciennes and Hainaut-Cambresis, Le Mont Houy, 59313 Valenciennes Cedex 9, France*
[b] *NON-A team, Inria Lille-Nord Europe, 59300 Lille, France*
[c] *Univ. Pierre et Marie CURIE, CNRS, UMR-7222, ISIR, 4 Place Jussieu, F-75005 Paris, France*

Abstract

This paper focuses on the design of fixed-time consensus for first order multi-agent systems with unknown inherent nonlinear dynamics. A distributed control protocol, based on local information, is proposed to ensure the convergence of the tracking errors in finite time. Some conditions are derived to select the controller gains in order to obtain a prescribed convergence time regardless of the initial conditions. Simulations are performed to validate the theoretical results.

Keywords: Multi-agent system, Consensus, Fixed-time convergence, Distributed tracking

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1. Introduction

Multi-agent systems (MAS) have been studied with great attention during the last few decades because multiple agents may perform a task more efficiently than a single one, reduce sensibility to possible agent fault and give high flexibility during the mission execution. Many recent works deal with cooperative scheme for MAS due to its broad range of applications in various areas, e.g. flocking [1], rendezvous [2], formation control [3, 4], containment control [5] and sensor network [6].

Among them, the consensus problem, which objective is to design distributed control policies that enable agents to reach an agreement regarding a certain quantity of interest by relying on neighbors' information [7], has received considerable attention. They can be categorized into two separated directions depending on whether there is a leader or not. Many consensus schemes have been developed recently [7, 8, 9, 10]. However, most of the existing consensus protocols have been derived when there is no leader or when the leader is static. Nevertheless, in many missions, a dynamic leader is required.

Many applications may require a dynamic leader, which could be virtual for its followers. Its behavior is independent of the other agents. The problem of leader-follower consensus is also referred to the cooperative or distributed consensus tracking [11]. The coordination of a group of mobile autonomous agents following a leader has been firstly motivated in [12]. A multi-agent consensus problem with an active leader and a variable topology has been proposed in [13]. In [14], it has been highlighted the strong links between existing leader-follower formation control approaches and consensus-based strategies. The leader-following consensus problem for multi-agent systems with Lipschitz nonlinear dynamics is discussed in [15]. Recently, a distributed output regulation approach has been proposed for the leader-follower consensus problem in [16].

In the study of consensus problem, the convergence rate has been an important topic. Indeed, this significant performance index is of high interest to study the effectiveness of a consensus protocol in the context of MAS. Most of the existing consensus algorithms focus on asymptotic convergence, where the settling time is infinite. However, many

☆This work has been supported by International Campus on Safety and Intermodality in Transportation, the European Community, the Regional Delegation for research and Technology, the Nord-Pas-de-Calais Region, the Ministry of Higher Education and Research and the National Center for Scientific Research under the FUI EQUITAS and the ARCIR SUCRe projects.

*Corresponding author. Email address: michael.defoort@univ-valenciennes.fr

applications require a high speed convergence generally characterized by a finite-time control strategy [17]. Moreover, finite-time control allows some advantageous properties such as good disturbance rejection and good robustness against uncertainties. A robust finite-time consensus tracking problem based on terminal sliding-mode control has been studied in [18], with input disturbances rejection. The finite-time consensus problem for multi-agent systems has been studied for single integrator [11, 19], double integrator [20] and inherent non-linear dynamics [21, 22].

Using finite time controller, an estimation of the settling time could be very useful when switching topology or networks of clusters are considered. It is worthy of noting that, for the above-mentioned works, the explicit expressions for the bound of the settling time depend on the initial states of agents. Therefore, the knowledge of these initial conditions usually prevent us from the estimation of the settling time using distributed architectures. A new approach, called fixed-time stability has been recently proposed to define algorithms which guarantee that the settling time is upper bounded regardless to the initial conditions [23]. This concept is promising since one can design a controller such that some control performances are obtained in a given time and independently of initial conditions. In the case of leaderless strategies, the fixed-time average consensus problem has been solved for single integrator linear dynamics without disturbance in [25] and with bounded matched disturbances in [26] even in the case of switching communication topologies [27]. To the best of our knowledge, although the finite-time leader-follower consensus problem has been well studied, the estimation of the convergence time off-line has not yet been proposed.

Motivated by the above works, in this paper, the fixed-time consensus tracking problem for MAS with inherent nonlinear dynamics is considered. The contribution of this paper is twofold. First, it is proposed a generalization of the finite-time leader-follower consensus protocol dealing with MAS with nonlinear inherent dynamics. Then, an explicit estimation of the settling time is provided regardless of the initial conditions.

This paper is organized as follows. Sections 2 is devoted to the formulation of the leader-follower consensus problem. Section 3 presents the distributed consensus protocol, based on local information, to ensure the convergence of the tracking errors in finite time. Some conditions are derived to select the controller gains in order to obtain a prescribed convergence time regardless of the initial conditions. In Section 4, an illustrative example is discussed.

Notations: We denote the transpose of a matrix M by M^T and $eig(M)$ its eigenvalues. $\lambda_{min}(M)$ (resp. $\lambda_{max}(M)$) are the smallest and the largest eigenvalues of M.
For a square symmetric matrix $P \in \mathbb{R}^{n\times n}$, $P \succ \mathbf{0}$ (resp. $P \prec \mathbf{0}$) indicates that P is positive (resp. negative) definite.

Let $\mathbf{1_N} = (1,1,\ldots,1)^T \in \mathbb{R}^n$ and $\mathrm{diag}(a_1, a_2 \ldots a_{n-1}, a_n)$ the diagonal matrix $\begin{pmatrix} a_1 & 0 & \ldots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \ldots & 0 & a_n \end{pmatrix} \in \mathbb{R}^{n\times n}$. For a given vector $x \in \mathbb{R}^n$, $|x|$ (resp. $||x||$) denotes the 1-norm (resp. 2-norm) of x.
For $x = (x_1, \ldots, x_n)^T \in \mathbb{R}^n$ and $b \geq 0$, let us define $\lfloor x \rceil^b = \left(\mathrm{sign}(x_1)\,|x_1|^b, \ldots, \mathrm{sign}(x_n)\,|x_n|^b\right)^T$.

2. Formulation of the leader-follower consensus problem

Consider a multi-agent system consisting of a virtual leader and N followers. The dynamics of each follower and of the leader agent are given by

$$\begin{array}{rcll} \dot{x}_i & = & f(t, x_i) + u_i, & i \in \{1, \cdots, N\} \\ \dot{x}_0 & = & f(t, x_0) + u_0 & \end{array} \tag{1}$$

where $x_0 \subset \mathbb{R}^m$ (resp. $u_0 \in \mathbb{R}^m$) is the state (resp. control input) of the virtual leader, $x_i \in \mathbb{R}^m$ is the state of agent i and $u_i \in \mathbb{R}^m$ is the control input of agent i. Moreover, $f : \mathbb{R}^+ \times D \to \mathbb{R}^m$ is an uncertain nonlinear dynamics of agent i, continuous in t, $D \subset \mathbb{R}^m$ a domain containing the origin. Furthermore, there exist positive constants l_1, l_2 and l_3 such that $\forall x_1, x_2 \in D$, $\forall t \geq 0$

$$||f(t,x_1) - f(t,x_2)||^2 \leq l_1 + l_2||x_1 - x_2||^2 + l_3||x_1 - x_2||^4 \tag{2}$$

Remark 1. *The presented class of systems include Lipschitz ($l_1 = l_3 = 0$) and quasi-Lipschitz ($l_3 = 0$) as well as some essentially nonlinear (e.g polynomial) models ($l_3 \neq 0$). The polynomial term may appear, for example, in mechanical models with a viscous friction force proportional to square of velocity. Indeed, if x_i is the velocity of some fast mobile vehicle, then $\sqrt{l_3}$ is proportional to a coefficient of viscous friction.*

It is worthy of noting that condition (2) is less restrictive than most of the existing works dealing on consensus for MAS with nonlinear inherent dynamics [21]. Since $f(t,x_i)$ is uncertain, it is not possible to cancel it with the control, namely (1) is not feedback equivalent to an m–dimensional integrator.
It is also assumed that the control u_0 of the leader is bounded by a known positive constant l, i.e.

$$\|u_0\| \leq l \tag{3}$$

To solve the coordination problem and model exchanged information between agents, graph theory is briefly recalled hereafter. The communication topology among all followers is fixed and is represented by an undirected graph $\mathscr{G}$ which consists of a nonempty set of nodes $\mathscr{V} = \{1,2,\cdots,N\}$ and a set of edges $\mathscr{E} \subset \mathscr{V} \times \mathscr{V}$. Here, each node in $\mathscr{V}$ corresponds to an agent i, and each edge $(i,j) \in \mathscr{E}$ in the undirected graph corresponds to an information link between agent i and agent j. The topology of graph $\mathscr{G}$ is represented by the weighted adjacency matrix $A = (a_{ij}) \in \mathbb{R}^{N\times N}$ given by $a_{ij} > 0$ if $(j,i) \in \mathscr{E}$ and $a_{ij} = 0$, otherwise. The Laplacian matrix of $\mathscr{G}$ is defined as $L = (l_{ij}) \in \mathbb{R}^{N\times N}$ with $l_{ii} = \sum_{j=1}^{N} a_{ij}$ and $l_{ij} = -a_{ij}$ for $i \neq j$. For an undirected graph, L is symmetric positive semi-definite. Graph $\mathscr{G}$ is connected if and only if its Laplacian matrix has a simple zero eigenvalue with associated eigenvector $\mathbf{1_N}$. The graph $\bar{\mathscr{G}}$ is fixed and describes the communication topology of all followers and the virtual leader. The topology of $\bar{\mathscr{G}}$ is described by the weighted matrix $H = L + D \in \mathbb{R}^{N\times N}$ where $D = diag(a_{10},\ldots,a_{N0})$ with $a_{i0} > 0$ if the leader state is available to follower i and with $a_{i0} = 0$ otherwise.

The objective of this paper is to design a distributed control protocol u_i, based on available local information, such that each follower tracks the virtual leader in a prescribed time T_{max}, i.e.

$$x_i(t) = x_0(t), \qquad \forall t \geq T_{max}, \qquad \forall i \in \{1,\cdots,N\} \tag{4}$$

Remark 2. *For the sake of simplicity, in the following, it is assumed that $m = 1$. The analysis is valid for any arbitrary dimension m with the difference being that the expression should be written in terms of the Kronecker product.*

3. Fixed-time consensus tracking protocol

In this section, a nonlinear finite time distributed control protocol will be designed such that each follower tracks the virtual leader in a prescribed time T_{max}. Before that let us introduce the concept of fixed-time stability and a useful lemma.

3.1. Preliminaries on fixed-time stability

Let us consider the following system

$$\begin{cases} \dot{x}(t) & \in & F(t,x(t)) \\ x(0) & = & x_0 \end{cases} \tag{5}$$

where $x \in \mathbb{R}^n$ is the state, $F : \mathbb{R}^+ \times \mathbb{R}^n \to \mathbb{R}^n$ be an upper semicontinuous convex-valued mapping, such that the set $F(t,x)$ is non-empty for any $(t,x) \in \mathbb{R}^+ \times \mathbb{R}^n$ and $F(t,0) = 0$ for $t > 0$. The solutions of (5) are understood in the Filippov sense [28].

Definition 1. *[29] The origin of system (5) is a globally finite time equilibrium if there is a function $T : \mathbb{R}^n \mapsto \mathbb{R}^+$ such that for all $x_0 \in \mathbb{R}^n$, the solution $x(t,x_0)$ of system (5) is defined and $x(t,x_0) \in \mathbb{R}^n$ for $t \in [0,T(x_0))$ and* $\lim_{t\to T(x_0)} x(t,x_0) = 0$. *$T(x_0)$ is called the settling time function.*

Definition 2. *[23] The origin of system (5) is a globally fixed-time equilibrium if it is globally finite-time stable and the settling time function $T(x_0)$ is bounded by some positive number $T_{max} > 0$, i.e. $T(x_0) \leq T_{max}$ for $\forall x_0 \in \mathbb{R}^n$.*

The fixed-time stabilization at the origin can be demonstrated on the simplest scalar control system $\dot{x}(t) = u(t)$, where $x \in \mathbb{R}$ is the state and $u \in \mathbb{R}$ is the control input. If the so-called negative relay feedback $u = -\operatorname{sign}(x)$ is applied, then the closed-loop system is finite-time stable with the settling-time $T(x_0) = |x_0|$. It is worthy of noting that the settling time depends on the initial conditions and is unbounded. The fixed-time control algorithm [23] has the form $u = -(|x|^2 + 1)\operatorname{sign}(x)$. It guarantees finite convergence time independently of the initial conditions, namely, $T(x_0) \leq \frac{\pi}{2}$, where $\pi = 3.1415926...$ is the well-know irregular number.

More strong result is provided by the following lemma that refines the results of [23] and [24].

Lemma 1. *[25] Assume that there exists a continuously differentiable positive definite and radially unbounded function $V : \mathbb{R}^n \to \mathbb{R}^+$ such that*

$$\sup_{t>0, y\in F(t,x)} \frac{\partial V(x)}{\partial x} y \leq -aV^p(x) - bV^q(x) \quad for \quad x \neq 0$$

where $a, b > 0, p = 1 - \frac{1}{\mu}, q = 1 + \frac{1}{\mu}, \mu \geq 1$.

Then, the origin of the differential inclusion (5) is globally fixed-time stable with the settling time estimate

$$T(x_0) \leq T_{\max} = \frac{\pi\mu}{2\sqrt{ab}}.$$

In the next subsection, this lemma will be used to design a distributed nonlinear fixed-time control protocol and to analyze its robustness with respect to uncertainties and disturbances.

3.2. Fixed-time consensus protocol design

Based on Lemma 1, a distributed control protocol is derived to guarantee that each follower tracks the leader in a prescribed time T_{max} regardless the initial condition in the following Theorem.

Theorem 1. *Suppose that graph $\mathscr{G}$ is connected and at least one $a_{i0} > 0$. The control protocol*

$$u_i = \alpha \left\lfloor \sum_{j=0}^{N} a_{ij}(x_j - x_i) \right\rceil^2 + \beta \sum_{j=0}^{N} a_{ij}(x_j - x_i) + \gamma \text{sign}\left(\sum_{j=0}^{N} a_{ij}(x_j - x_i) \right) \tag{6}$$

where the gains are defined as

$$\begin{aligned} \alpha &= \frac{\sqrt{l_3}\,(\lambda_{\max}(H))^{3/2}}{\lambda_{min}^{7/2}(H)} + \frac{\varepsilon\sqrt{N}}{(2\lambda_{min}(H))^{\frac{3}{2}}} \\ \beta &= \frac{\sqrt{l_2}}{\lambda_{min}(H)} \\ \gamma &= l + \sqrt{l_1 N} + \frac{\varepsilon}{\lambda_{min}(H)} \end{aligned} \tag{7}$$

with $\varepsilon > 0$, achieves the convergence of the tracking errors $\tilde{x}_i = x_i - x_0$, $\forall i \in \{1, \cdots, N\}$ to zero in a finite time which is bounded by

$$T_{max} = \frac{\pi}{\varepsilon} \tag{8}$$

Proof. Let us define the tracking errors as $\tilde{x}_i = x_i - x_0$, $\forall i \in \{1, \cdots, N\}$. Equations (1), (6) yield

$$\dot{\tilde{x}}_i = \alpha \left\lfloor \sum_{j=0}^{N} a_{ij}(x_j - x_i) \right\rceil^2 + \beta \sum_{j=0}^{N} a_{ij}(x_j - x_i) + \gamma \text{sign}\left(\sum_{j=0}^{N} a_{ij}(x_j - x_i) \right) + f(t, x_i) - f(t, x_0) - u_0 \tag{9}$$

In a compact form, system (9) becomes

$$\dot{\tilde{x}} = -\alpha \lfloor H\tilde{x} \rceil^2 - \beta H\tilde{x} - \gamma \text{sign}(H\tilde{x}) + \tilde{f} - \mathbf{1_N} u_0$$

with

$$\begin{aligned} \tilde{x} &= (\tilde{x}_1, \ldots, \tilde{x}_N)^T \\ \tilde{f} &= (f(t, x_1) - f(t, x_0), \ldots, f(t, x_N) - f(t, x_0))^T \end{aligned}$$

Considering the candidate Lyapunov function

$$V = \frac{1}{2}\tilde{x}^T H \tilde{x}$$

$H \succ \mathbf{0}$ since graph $\mathscr{G}$ is connected and there is at least one a_{i0} positive [11]. Its time derivative is given by

$$\begin{array}{rcl} \dot{V} & = & \tilde{x}^T H \dot{\tilde{x}} \\ & = & -\alpha \tilde{x}^T H \lfloor H\tilde{x} \rceil^2 - \beta \|H\tilde{x}\|_2^2 - \gamma \|H\tilde{x}\|_1 - \tilde{x}^T H \mathbf{1_N} u_0 + \tilde{x}^T H \tilde{f} \end{array} \tag{10}$$

The later considerations uses the classical result about equivalence of the norms $\|\cdot\|_q$

$$\|y\|_q = \left(\sum_{i=1}^{N} |y_i|^q \right)^{\frac{1}{q}}, \quad y = (y_1, ..., y_N)^T \in \mathbb{R}^N, \quad q > 0$$

in a finite dimensional linear space. Namely, for $p > r > 0$ we always have

$$\|y\|_p \leq \|y\|_r \leq N^{\frac{1}{r} - \frac{1}{p}} \|y\|_p$$

The first term of (10) can be estimated as follows

$$\begin{array}{rcl} -\alpha \tilde{x}^T H \lfloor H\tilde{x} \rceil^2 & = & -\alpha \sum_{i=1}^{N} \left| \sum_{j=0}^{N} a_{ij}(x_j - x_i) \right|^3 = -\alpha \|H\tilde{x}\|_3^3 \\ & \leq & -\alpha N^{-\frac{1}{2}} \|H\tilde{x}\|_2^3 = -\alpha N^{-\frac{1}{2}} \left(\tilde{x}^T H^T H \tilde{x} \right)^{\frac{3}{2}} \\ & \leq & -\alpha N^{-\frac{1}{2}} \left(2\lambda_{min}(H) \right)^{\frac{3}{2}} V^{\frac{3}{2}} \end{array}$$

The last term of (10) admits the following representation

$$\begin{aligned} \tilde{x}^T H \tilde{f} &\leq \|H\tilde{x}\|_2 \|\tilde{f}\|_2 \leq \|H\tilde{x}\|_2 \left(l_1 N + l_2 \|\tilde{x}\|_2^2 + l_3 \|\tilde{x}\|_4^4 \right)^{1/2} \\ &\leq \|H\tilde{x}\|_2 \left(\sqrt{l_1 N} + \sqrt{l_2} \|\tilde{x}\|_2 + \sqrt{l_3} \|\tilde{x}\|_4^2 \right) \\ &\leq \sqrt{l_1 N} \|H\tilde{x}\|_1 + \frac{\sqrt{l_2}}{\lambda_{min}(H)} \|H\tilde{x}\|_2^2 + \frac{\sqrt{l_3} N^{-\frac{1}{2}} \left(2\lambda_{\max}(H) \right)^{3/2}}{\lambda_{min}^2(H)} V^{3/2} \end{aligned}$$

It follows that

$$\dot{V} \leq \left(-\alpha N^{-\frac{1}{2}} \left(2\lambda_{min}(H) \right)^{\frac{3}{2}} + \frac{\sqrt{l_3} N^{-\frac{1}{2}} \left(2\lambda_{\max}(H) \right)^{3/2}}{\lambda_{min}^2(H)} \right) V^{\frac{3}{2}} - \left(\beta - \frac{\sqrt{l_2}}{\lambda_{min}(H)} \right) \|H\tilde{x}\|_2^2 - \gamma \|H\tilde{x}\|_1 + (l + \sqrt{l_1 N}) \|H\tilde{x}\|_1 \tag{11}$$

Let the gains be designed by (7). Therefore, one gets

$$\dot{V} \leq -\varepsilon V^q - \varepsilon V^p$$

with

$$\begin{array}{rcl} \mu & = & 2 \\ p & = & 1 - \frac{1}{\mu} \\ q & = & 1 + \frac{1}{\mu} \end{array}$$

Hence, Lemma 1 guarantees that the origin of the closed-loop system (9) is globally fixed-time stable with the settling time bounded by $T_{max} = \frac{\pi}{\varepsilon}$.

■

Remark 3. *It is worthy of noting that the settling time does not depend on the initial conditions. If a prescribed convergence time is required, one can easily tune the controller gains according to* (7) *to solve the leader-follower consensus problem. Such a requirement occurs when considering switching topologies. An extension of these results to the case of switching network is possible. Indeed, in this case, the sum of time intervals over which the graph is strongly connected must be larger that the settling time to guarantee that each follower tracks the virtual leader.*

4. Numerical simulations

Let us consider the multi-agent system (1) with $N = 6$ followers and one virtual leader. The communication topology, given in Fig. 1 is undirected and fixed. It is connected and the corresponding matrix H is given by

$$H = \begin{bmatrix} 2 & -1 & 0 & 0 & 0 & 0 \\ -1 & 3 & -1 & 0 & 0 & 0 \\ 0 & -1 & 4 & -1 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & -1 & -1 & 3 & -1 \\ 0 & 0 & 0 & 0 & -1 & 2 \end{bmatrix}$$

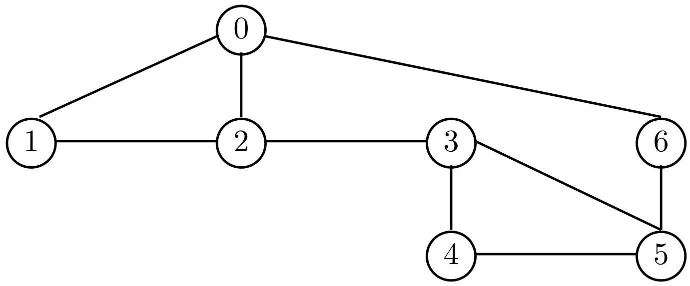


Figure 1: Communication topology.

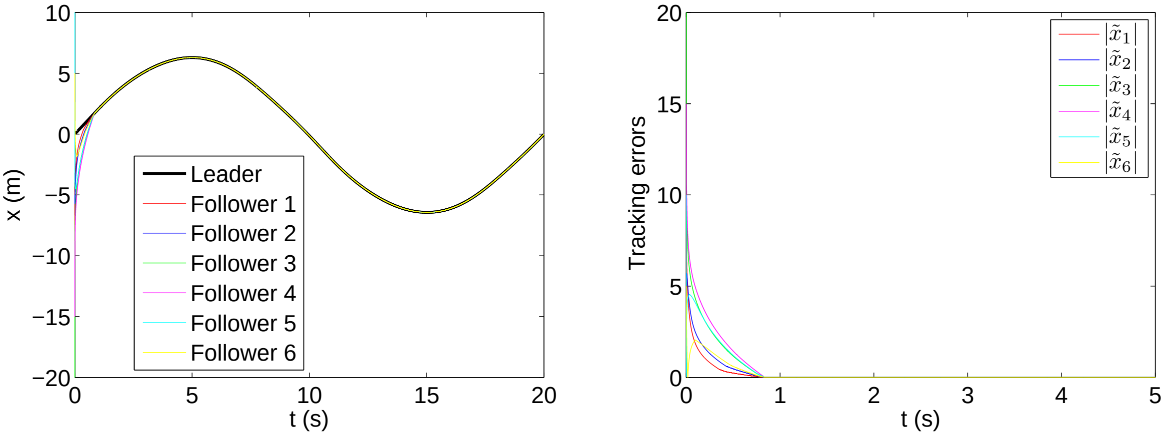
Hereafter, two scenarios are discussed.

Scenario 1: We consider the case $m = 1$. The nonlinear dynamics, assumed uncertain, are $f(t,x_i) = 0.1\sin(x_i)$, $\forall i = \{0,\ldots,N\}$. The Lipschitz condition is verified. Indeed, in this case, $l_1 = l_3 = 0$ and $l_2 = 0.1$. The control input of the virtual leader is chosen as $u_0 = 2\cos(0.1\pi t)$. It is bounded by $l = 2$. From Theorem 1, the control protocol (6) achieves the convergence of the tracking errors $\tilde{x}_i = x_i - x_0$, $\forall i \in \{1,\cdots,N\}$ to zero in finite time. The control parameters are chosen as $\alpha = 5.38$, $\beta = 1.07$, $\gamma = 5.38$. The estimated upper bounds of the settling time for the proposed distributed control scheme (6) is $T_{max} = 1.5s$.

The six followers start from the following initial conditions: $x_1(0) = -10$, $x_2(0) = -5$, $x_3(0) = -20$, $x_4(0) = -15$, $x_5(0) = 10$ and $x_6(0) = 5$. The virtual leader starts from $x_0(0) = 0$. Fig. 2(a) depicts the trajectories of the virtual leader and the followers when the control protocol (6) is used. Fig. 2(b) shows that the consensus is achieved after $0.8s$. It confirms the theoretical results obtained from Theorem 1.

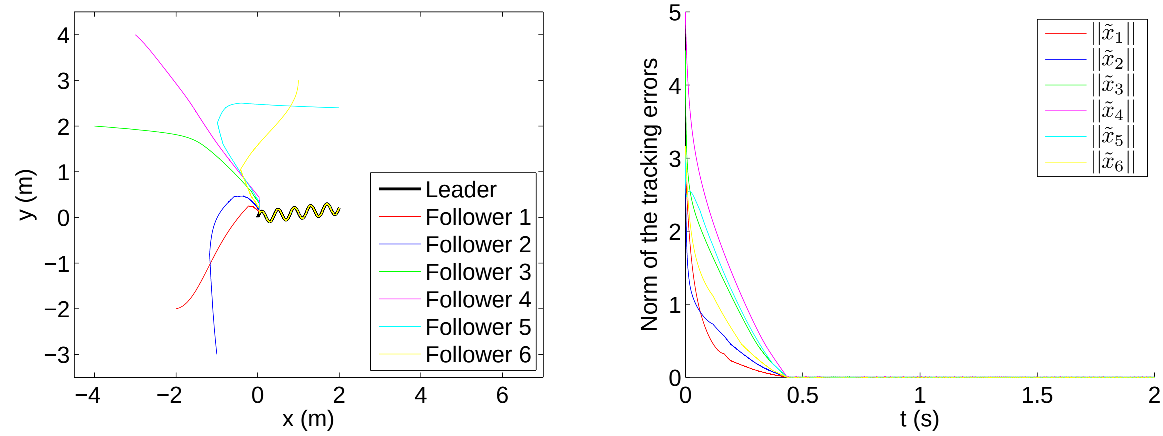
Scenario 2: Now, we consider the case $m = 2$. The state of the virtual leader and the followers x_i are written as $x_0 = \left(\eta_0^x, \eta_0^y\right)^T \in D$ and $x_i = \left(\eta_i^x, \eta_i^y\right)^T \in D$ where $D = [-10,10] \times [-10,10]$. The nonlinear dynamics, assumed uncertain, are $f(t,x_i) = 0.01\left(0.01(\eta_i^x)^2, 0.01\,\mathrm{sign}\left((\eta_i^y)\right)\right)^T$. This function does not satisfy the Lipschitz condition. Nevertheless, in this case, $l_1 = 0.02$ and $l_3 = 0.1$ on D. The control input of the virtual leader is chosen as $u_0 = (0.1, 2\cos(\pi/2t))$. It is bounded by $l = 0.64$. From Theorem 1, the control protocol (6) achieves the convergence of the tracking errors $\tilde{x}_i = x_i - x_0$, $\forall i \in \{1,\cdots,N\}$ to zero in finite time. The control parameters are chosen as $\alpha = 7.11$, $\beta = 0$, $\gamma = 4.36$. The estimated upper bounds of the settling time for the proposed distributed control scheme (6) is $T_{max} = 1.5s$.

The six followers start from the following initial conditions: $x_1(0) = (-2,-2)^T$, $x_2(0) = (-1,-3)^T$, $x_3(0) = (-4,2)^T$, $x_4(0) = (-3,4)^T$, $x_5(0) = (2,2.4)^T$ and $x_6(0) = (1,3)^T$. The virtual leader starts from $x_0(0) = (0,0)^T$. Fig. 3(a) depicts the trajectories of the virtual leader and the followers when the control protocol (6) is used. Fig. 3(b) shows that the consensus is achieved after $0.5s$. It confirms the theoretical results obtained from Theorem 1.



(a) (b)

Figure 2: Leader-follower consensus for multi-agent systems with unknown Lipschitz nonlinearities (a) Trajectories of each agent. (b) Tracking errors.



(a) (b)

Figure 3: Leader-follower consensus for multi-agent systems with unknown non Lipschitz nonlinearities (a) Trajectories of each agent. (b) Norm of the tracking errors.

5. Conclusion

In this paper, the fixed-time consensus tracking problem for MAS with inherent nonlinear dynamics is considered. The contribution of this paper is twofold. First, it is proposed a generalization of the finite-time leader-follower consensus protocol dealing with MAS with nonlinear inherent dynamics. Then, an explicit estimation of the settling time is provided regardless of the initial conditions. Some conditions are derived to select the controller gains in order to obtain a prescribed convergence time regardless of the initial conditions.

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