ϵ-Invariant Output Stabilization: Homogeneous Approach and Dead Zone Compensation

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***Abstract*— This work addresses the stabilization of dynamical systems in presence of uncertain bounded perturbations using ϵ-invariance theory. Under some assumptions, the problem is reduced to the stabilization of a chain of integrators subject to a perturbation and is treated in two steps. The evaluation of the disturbance and its compensation. Homogeneous observer and control [5], [19] are the tools utilized to achieve a global asymptotic stability and robustness. The result is formally proven and, to validate the theory, it is applied to the control of the telescopic link of a hydraulic actuated industrial crane used in forestry. Experimental results and a comparison with a standard PI controller are presented.**

I. Introduction

The problem of uniform stabilization of dynamical systems in the presence of uncertain bounded inputs has a rather long history [24]. By uniformity in this context we understand invariance (exact or approximate) of the closed-loop system with respect to disturbing inputs (disturbance rejection or cancellation are another names of that problem). Initiated by a French engineer Jean-Victor Poncelet [20], these ideas received a large attention in Soviet Union following the theory developed by Georgy Vladimirovich Shipanov [22], which is called the theory of ε-invariance (it was supposed to provide invariance up to $\varepsilon > 0$ deviations caused by disturbances of a given class). Next, many different solutions for ε-invariant stabilization have been proposed: time delay control [25], active disturbance rejection [11], universal integral controls [14], [9], various sliding-mode control algorithms [15], [7] converging in a finite time, model-free control [8] (just to mention a few, there are also many other adaptive/fuzzy/neural control solutions). The statement of ε-invariant control design problem can be given following a recent development [8] (model-free control). Consider a SISO uncertain nonlinear system, whose model is given in the implicit form (it is not resolved with respect to the highest derivative):

$$f[y(t), \dot{y}(t), \dots, y^{(n)}(t), u(t), d(t)] = 0,\ t \geq 0,$$

where $y(t) \in \mathbb{R}$ is the measured output, $u(t) \in \mathbb{R}$ is the control input, $d(t) \in \mathbb{R}^m$ is the vector of uncertain parameters/signals, $n \geq 1$ is the system dimension, which may be unknown, $f : \mathbb{R}^{n+m+1} \to \mathbb{R}$ is an unknown nonlinear function ensuring existence of the system solutions at least locally. Fixing $k \geq 1$, a local model can be extracted:

$$y^{(k)}(t) = u(t) + F(t),$$

where $F(t) \in \mathbb{R}$ is a new unknown input including $y, y^{(1)}, \dots, y^{(n)}$, u and d. This model may have sense only locally, but under assumption that the dynamics of $y^{(k+1)}, \dots, y^{(n)}$ are stable (*i.e.* the system is minimum phase with relative degree k [14], [8]) the original stabilization problem for uncertain nonlinear system can be reduced to uniform (ε-invariant) stabilization of a chain of k integrators subjected by unknown matched input F (frequently assumed bounded along with its derivatives). There are many solutions to this problem, which are based on the idea that if it is possible to estimate $y^{(k)}(t)$ then $F(t) = y^{(k)}(t) - u(t)$ can be evaluated and compensated by the control. The difference is mainly in the tools used for estimation of $y^{(k)}(t)$ (high-gain observers in [14], [9], sliding-mode differentiators in [15], [7] or algebraic ones in [8]). Time delay is frequently introduced to break the algebraic loop [25], [3], [10], [13], which appears when using the estimate $y^{(k)}(t) - u(t)$ in the control $u(t)$ itself. Another difference between [7], [8], [9], [14], [15] consists in the type of feedback used for the system stabilization. Theoretically sliding-mode controls provide a finite-time exact cancellation of matched disturbances [15], [7], which is better than ε-invariance provided by linear feedbacks from [8], [9], [14], [25]. But in practice the sliding-mode controls suffer from chattering that returns them back to ε-invariance setting. A related difference is robustness with respect to different nonlinearities of $y, y^{(1)}, \dots, y^{(n)}$ hidden in F (for example, linear feedback treats only Lipschitz or linear perturbations). In order to improve robustness and to avoid chattering, an intermediate solution could be represented by homogeneous high-gain controls [5] and observers [19]. Due to homogeneity, local asymptotic stability of this systems implies global one, and robustness with respect to disturbances is inherited next [4]. Adjusting nonlinear gains in control and estimation algorithms from [5], [19] it is possible to get a needed degree of robustness with respect to F. A development of ε-invariant output control based on [5], [19] is presented in this work and thus applied to control a specific link of an actuated industrial crane used in forestry. The aim is to achieve precision and smooth extension and retraction in a way that standard methods cannot guarantee

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[12]. In the treated example the telescopic link of the crane must track a reference trajectory using the proposed approach to compensate incertitudes due to a dead zone of the control input modeled as in [23] along with other perturbations.

II. Preliminaries

Through the paper the following notations is used:

- $\mathbb{R}_+ = \{x \in \mathbb{R} : x \geq 0\}$, where $\mathbb{R}$ is the set of real number.
- $|\cdot|$ denotes the absolute value in $\mathbb{R}$, $\|\cdot\|$ denotes the Euclidean norm on $\mathbb{R}^n$.
- For a (Lebesgue) measurable function $d : \mathbb{R}_+ \to \mathbb{R}^m$ define the norm $||d||_{[t_0,t_1)} = \text{ess}\ \sup_{t\in[t_0,t_1)}\|d(t)\|$, then $||d||_\infty = ||d||_{[0,+\infty)}$ and the set of $d(t)$ with the property $||d||_\infty < +\infty$ we further denote as $\mathcal{L}_\infty$ (the set of essentially bounded measurable functions).
- A continuous function $\alpha : \mathbb{R}_+ \to \mathbb{R}_+$ belongs to the class $\mathcal{K}$ if $\alpha(0) = 0$ and the function is strictly increasing. The function $\alpha : \mathbb{R}_+ \to \mathbb{R}_+$ belongs to the class $\mathcal{K}_\infty$ if $\alpha \in \mathcal{K}$ and it is unbounded. A continuous function $\beta : \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}_+$ belongs to the class $\mathcal{KL}$ if $\beta(\cdot, t) \in \mathcal{K}_\infty$ for each fixed $t \in \mathbb{R}_+$ and $\lim_{t\to+\infty} \beta(s, t) = 0$ for each fixed $s \in \mathbb{R}_+$.
- $\lceil\cdot\rfloor^\alpha$ denotes the following operation $|\cdot|^\alpha\text{sign}(\cdot)$.
- The notation $DV(x)f(x)$ stands for the directional derivative of a continuously differentiable function V with respect to the vector field f evaluated at point x.

Following [6], consider a nonlinear system

$$\dot{x}(t) = f[x(t), d(t)], \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state, $d(t) \in \mathbb{R}^m$ is the external input, $d \in \mathcal{L}_\infty$, and $f : \mathbb{R}^{n+m} \to \mathbb{R}^n$ is a locally Lipschitz (or Hölder) continuous function, $f(0, 0) = 0$. For an initial condition $x_0 \in \mathbb{R}^n$ and input $d \in \mathcal{L}_\infty$, define the corresponding solutions by $x(t, x_0, d)$ for any $t \geq 0$ for which the solution exists.

Definition 1. The system (1) is called *input-to-state practically stable (ISpS)*, if for any input $d \in \mathcal{L}_\infty$ and any $x_0 \in \mathbb{R}^n$ there are some functions $\beta \in \mathcal{KL}$, $\gamma \in \mathcal{K}$ and $c \geq 0$ such that

$$\|x(t, x_0, d)\| \leq \beta(\|x_0\|, t) + \gamma(||d||_{[0,t)}) + c \quad \forall t \geq 0.$$

The function γ is called *nonlinear asymptotic gain*. The system is called *ISS* if $c = 0$.

Definition 2. A smooth function $V : \mathbb{R}^n \to \mathbb{R}_+$ is called *ISpS Lyapunov function* for the system (1) if for all $x \in \mathbb{R}^n$, $d \in \mathbb{R}^m$ and some $r \geq 0$, $\alpha_1, \alpha_2, \alpha_3 \in \mathcal{K}_\infty$ and $\theta \in \mathcal{K}$:

$$\alpha_1(\|x\|) \leq V(x) \leq \alpha_2(\|x\|),$$
$$DV(x)f(x, d) \leq r + \theta(\|d\|) - \alpha_3(\|x\|).$$

Such a function V is called *ISS Lyapunov function* if $r = 0$.

Note that an ISS Lyapunov function can also satisfy the following equivalent condition for some $\chi \in \mathcal{K}$:

$$\|x\| > \chi(\|d\|) \Rightarrow DV(x)f(x, d) \leq -\alpha_3(\|x\|).$$

Theorem 1. *[6] The system (1) is ISS (ISpS) iff it admits an ISS (ISpS) Lyapunov function.*

A. Weighted homogeneity

Following [2], for fixed strictly positive numbers r_i, $i = 1, \ldots, n$ called weights and $\lambda > 0$, one can define:

- the *vector of weights* $\mathbf{r} = (r_1, \ldots, r_n)^T$, $r_{\max} = \max_{1\leq j\leq n} r_j$ and $r_{\min} = \min_{1\leq j\leq n} r_j$;
- the *dilation matrix* function $\Lambda_r(\lambda) = diag\{\lambda^{r_i}\}_{i=1}^n$, note that $\forall x \in \mathbb{R}^n$ and $\forall\lambda > 0$ we have $\Lambda_r(\lambda)x = (\lambda^{r_1}x_1, \ldots, \lambda^{r_i}x_i, \ldots, \lambda^{r_n}x_n)^T$.

Definition 3. A function $g : \mathbb{R}^n \to \mathbb{R}$ is $\mathbf{r}$–*homogeneous with degree* $\mu \in \mathbb{R}$ if $\forall x \in \mathbb{R}^n$ and $\forall\lambda > 0$ we have:

$$\lambda^{-\mu} g(\Lambda_r(\lambda)x) = g(x).$$

A vector field $f : \mathbb{R}^n \to \mathbb{R}^n$ is $\mathbf{r}$–*homogeneous with degree* $\nu \in \mathbb{R}$, with $\nu \geq -r_{\min}$ if $\forall x \in \mathbb{R}^n$ and $\forall\lambda > 0$ we have:

$$\lambda^{-\nu}\Lambda_r^{-1}(\lambda)f(\Lambda_r(\lambda)x) = f(x),$$

which is equivalent for i-th component of f being a $\mathbf{r}$–homogeneous function of degree $r_i + \nu$.

The system (1) with $d = 0$ is $\mathbf{r}$–homogeneous of degree ν if the vector field f is $\mathbf{r}$–homogeneous of degree ν.

Theorem 2. *[21] For the system (1) with $d = 0$ and $\mathbf{r}$–homogeneous and continuous function f the following properties are equivalent:*

- *the system (1) is (locally) asymptotically stable*;
- *there exists a continuously differentiable $\mathbf{r}$–homogeneous Lyapunov function $V : \mathbb{R}^n \to \mathbb{R}_+$ such that*

$$\alpha_1(\|x\|) \leq V(x) \leq \alpha_2(\|x\|),\ DV(x)f(x, 0) \leq -\alpha(\|x\|),$$
$$\lambda^{-\mu}V(\Lambda_r(\lambda)x) = V(x),\ \mu > r_{\max},$$

$\forall x \in \mathbb{R}^n$ and $\forall\lambda > 0$, for some $\alpha_1, \alpha_2 \in \mathcal{K}_\infty$ and $\alpha \in \mathcal{K}$.

Define

$$\tilde{f}(x, d) = [f(x, d)^T\ 0_m]^T \in \mathbb{R}^{n+m},$$

it is an extended auxiliary vector field for the system (1), where 0_m is the zero vector of dimension m.

Theorem 3. *[4] Let the vector field $\tilde{f}$ be homogeneous with the weights $\mathbf{r} = [r_1, \ldots, r_n] > 0$, $\tilde{\mathbf{r}} = [\tilde{r}_1, \ldots, \tilde{r}_m] > 0$ with a degree $\nu \geq -r_{\min}$, i.e. $f(\Lambda_r(\lambda)x, \Lambda_{\tilde{r}}(\lambda)d) = \lambda^\nu\Lambda_r(\lambda)f(x, d)$ for all $x \in \mathbb{R}^n$, $d \in \mathbb{R}^m$ and all $\lambda > 0$. Assume that the system (1) is globally asymptotically stable for $d = 0$, then the system (1) is ISS.*

Therefore, for homogeneous system (1) its ISS property follows asymptotic stability for $d = 0$ (as for linear systems [6]). The nonlinear asymptotic gain function has been also estimated in [4].

B. Homogeneous stabilizing control

Consider a nonlinear system

$$\begin{aligned}\dot{\xi}_i &= \xi_{i+1},\ i=1,\dots,n-1, \\ \dot{\xi}_n &= -\sum_{i=1}^{n} a_i \lceil \xi_i \rfloor^{\alpha_i},\end{aligned} \quad (2)$$

where $\xi = [\xi_1, \dots, \xi_n] \in \mathbb{R}^n$ is the state vector, α_i and a_i are real parameters. For $r_i = 1 + (i-1)\nu$ and $\alpha_i = \frac{1+n\nu}{1+(i-1)\nu}$, $i = 1, \dots, n$, where $\nu > -\frac{1}{n-1}$, the system (2) is $\mathbf{r}$–homogeneous of degree ν.

Theorem 4. *[5] Let $a_1, \dots, a_n$ form a Hurwitz polynomial, then there exists $0 < \varrho < \frac{1}{n-1}$ such that for any $\nu \in (-\frac{1}{n-1} + \varrho, 0)$ the system (2) with $\alpha_i = \frac{1+n\nu}{1+(i-1)\nu}$, $i = 1, \dots, n$ is globally finite-time stable.*

C. Homogeneous observer

Consider a nonlinear system

$$\begin{aligned}\dot{\xi}_i &= \xi_{i+1} - \lambda_i \lceil \xi_1 \rfloor^{\beta_i},\ i=1,\dots,n-1, \\ \dot{\xi}_n &= -\lambda_n \lceil \xi_1 \rfloor^{\beta_n},\end{aligned} \quad (3)$$

where $\xi = [\xi_1, \dots, \xi_n] \in \mathbb{R}^n$ is the state vector, λ_i and β_i are real parameters. For $r_i = 1 + (i-1)\mu$ and $\beta_i = 1 + i\mu$ for all $i = 1, \dots, n$, where $\mu > -\frac{1}{n}$, the system (3) is $\mathbf{r}$–homogeneous of degree μ.

Theorem 5. *[19] Let $\lambda_1, \dots, \lambda_n$ form a Hurwitz polynomial, then there exists $0 < \varrho < \frac{1}{n}$ such that for any $\mu \in (-\frac{1}{n}+\varrho, 0)$ the system (3) with $\beta_i = 1 + i\mu$, $i = 1, \dots, n$ is globally finite-time stable.*

III. Problem statement

The following state-space representation will be considered in this work:

$$\begin{aligned}\dot{x}_i(t) &= x_{i+1}(t),\ i=1,\dots,k-1, \\ \dot{x}_k(t) &= u(t) + F(t),\ y(t) = x_1(t),\end{aligned} \quad (4)$$

where $x(t) = [x_1(t), \dots, x_k(t)]^T \in \mathbb{R}^k$ is the state space vector of the system (4) at time instant $t \geq 0$; $u(t) \in \mathbb{R}$ and $F(t) \in \mathbb{R}$ are the control and disturbance inputs, respectively; $y(t) \in \mathbb{R}$ is the measured output. Since (4) is linear, then the measurement noise can be transferred to the input and included in F. The following restrictions are introduced for (4).

Assumption 1. *Let $F \in \mathcal{L}_\infty$ and $\dot{F} \in \mathcal{L}_\infty$, in addition a constant $f > 0$ is given such that*

$$||F||_\infty \leq f,\ ||\dot{F}||_\infty \leq f.$$

It is required to design a dynamical output feedback u such that for the given $\varepsilon > 0$ and all initial conditions $x_0 \in \mathbb{R}^k$,

$$\lim_{t\to+\infty} \|x(t)\| \leq \varepsilon$$

for all F satisfying Assumption 1. The conditions of that assumption can also be relaxed supposing that F is a nonlinear function of x and asking for a semi-global ε-invariance.

IV. Control design

First, the vector x has to be estimated. Due to the system structure this problem is equivalent to the estimation of the derivatives $y^{(1)}(t), \dots, y^{(k-1)}(t)$ for the output $y(t)$, for this purpose the following linear filter can be designed

$$\begin{aligned}\dot{z}_i &= z_{i+1} + l_i(y - z_1),\ i=1,\dots,k-1, \\ \dot{z}_k &= l_k(y - z_1) + u,\end{aligned} \quad (5)$$

where $z = [z_1, \dots, z_k]^T \in \mathbb{R}^k$ and high-gain tuning parameters $l_i > 0$ for $i = 1, \dots, k$ form a Hurwitz polynomial (more precise restrictions on l_i will be given later). Denote $\hat{y}^{(i)}$ as an estimate of $y^{(i)} = x_{i+1}$, then we can select $\hat{y}^{(i)} = z_{i+1}$ for $i = 0, \dots, k-1$ and the filter estimation error $e = x - z$ has dynamics:

$$\begin{aligned}\dot{e}_i &= e_{i+1} - l_i e_1,\ i=1,\dots,k-1, \\ \dot{e}_k &= -l_k e_1 + F.\end{aligned}$$

From the last equation the following estimate

$$\hat{F} = \hat{\dot{e}}_k + l_k e_1$$

of F can be calculated, where $\hat{\dot{e}}_k$ is an estimate of $\dot{e}_k$. In order to calculate $\hat{\dot{e}}_k$ a second filter/differentiator should be designed that has to converge faster than exponentially (the rate of decay in the linear one (5)). For this purpose a homogeneous high-gain differentiator can be used:

$$\begin{aligned}\dot{\zeta}_i &= \zeta_{i+1} - l_i e_1 + \lambda_i \lceil e_1 - \zeta_1 \rfloor^{\beta_i},\ i=1,\dots,k, \\ \dot{\zeta}_{k+1} &= \lambda_{k+1} \lceil e_1 - \zeta_1 \rfloor^{\beta_{k+1}} + l_k l_1,\end{aligned} \quad (6)$$

where $\zeta = [\zeta_1, \dots, \zeta_k]^T \in \mathbb{R}^{k+1}$ and the tuning parameters $\beta_i > 0$ and $\lambda_i > 0$ for $i = 1, \dots, k+1$ will be derived later. Denote $\bar{e} = [e^T\ e_{k+1}]^T$, where

$$\begin{aligned}e_{k+1} &= -l_k e_1 + F, \\ \dot{e}_{k+1} &= -l_k \dot{e}_1 + \dot{F} = -l_k(e_2 - l_1 e_1) + \dot{F},\end{aligned}$$

then we can select $\hat{\dot{e}}_k = \zeta_{k+1}$ and the estimation error $\epsilon = \bar{e} - \zeta$ has dynamics:

$$\begin{aligned}\dot{\epsilon}_i &= \epsilon_{i+1} - \lambda_i \lceil \epsilon_1 \rfloor^{\beta_i},\ i=1,\dots,k, \\ \dot{\epsilon}_{k+1} &= -\lambda_{k+1} \lceil \epsilon_1 \rfloor^{\beta_{k+1}} - l_k e_2 + \dot{F},\end{aligned}$$

which is $\mathbf{r}$–homogeneous of order $\mu > -\frac{1}{k+1}$ for $r_i = 1 + (i-1)\mu$ and $\beta_i = 1 + i\mu$ for all $i = 1, \dots, k+1$ (if $\mu = -\frac{1}{k+1}$ then $\beta_{k+1} = 0$ and (6) reduces to a high-order sliding mode observer, while for all $\mu > -\frac{1}{k+1}$ the filter (6) stays continuous), i.e. a perturbed version of (3). Therefore,

$$\hat{F} = \zeta_{k+1} + l_k e_1. \quad (7)$$

Second, the control u can be introduced

$$u = -\sum_{i=0}^{k-1} a_{i+1} \left\lceil \hat{y}^{(i)} \right\rfloor^{\alpha_{i+1}} - \hat{F} = -\sum_{i=1}^{k} a_i \lceil z_i \rfloor^{\alpha_i} - \hat{F}, \quad (8)$$

where the coefficients a_i, $i = 1, \dots, k$ form a Hurwitz polynomial and $\alpha_i > 0$, $i = 1, \dots, k$ will be defined in the next section.

To conclude, the proposed model-free invariance control algorithm includes two filters (5) and (6) (one for differentiation and another for decoupling the control u and the estimate of F appearing into the same equation), the unknown input F estimate (7) and the stabilizing control (8).

Therefore, the following result can been proven[1].

Theorem 6. *Let Assumption 1 be satisfied. Then for any given $\epsilon > 0$ there exist $l \in \mathbb{R}^k$, $\lambda \in \mathbb{R}^{k+1}$, $a \in \mathbb{R}^k$, $\mu \in (-\frac{1}{k+1}, 0)$, $\nu \in (-\frac{1}{k-1}, 0)$, $\beta_i = 1 + i\mu$ for $i = 1, \dots, k+1$, $\alpha_i = \frac{1+k\nu}{1+(i-1)\nu}$ for $i = 1, \dots, k$, such that in the system (4) with the output regulator (5)–(8) for all initial conditions*

$$x \in \mathcal{L}_\infty^k, z \in \mathcal{L}_\infty^k, \zeta \in \mathcal{L}_\infty^{k+1},$$

and

$$\lim_{t \to +\infty} \|x(t)\| \leq \varepsilon.$$

Moreover, the system (4), (5)–(8) is ISS with respect to the input $(F, \dot{F})$.

Remark 1. Since dynamics of all variables, $x(t)$, $\epsilon(t)$ and $e(t)$, are homogeneous, their asymptotic gains can be evaluated as it is proposed in [3] and using the parameters γ_l, γ_λ and γ_a. Finally, for given f the value of $\varepsilon(t)$ can be estimated.

Since (6) and (8) contain nonlinear gains, then the asymptotic gain of (4), (5)–(8) (see [4] for an algorithm of its estimation) close to the origin is better than in a pure linear system (*i.e.* replacing (6) and (8) by linear filter and feedback, respectively).

V. Dead zone compensation

We are going to present the compensation of an input nonlinearity, known as a dead zone, which is depicted in Fig. 1. This dead zone model is a static representation of diverse physical phenomena with negligible fast dynamics, see [23]. One well-known example is the model of an industrial electro-hydraulic valve in which the spool occludes the orifice with some overlap. In this case, system (4) should be rewritten as below:

$$\begin{aligned} \dot{x}_i(t) &= x_{i+1}(t),\ i = 1, \dots, k-1, \\ \dot{x}_k(t) &= D(u(t)) + F(t),\ y(t) = x_1(t), \end{aligned} \tag{9}$$

where the dead zone input is represented by $D(u(t))$ and it has the following structure:

$$D(u(t)) = \begin{cases} m_r(u - b_r) & \text{if} \quad u \geq b_r, \\ 0 & \text{if} \quad -b_l \leq u \leq b_r, \\ m_l(u - b_l) & \text{if} \quad u \leq -b_l. \end{cases} \tag{10}$$

where $m_i = m_0 + \Delta m_i$ and $b_i = b_0 + \Delta b_i$, with $i = l, r$; the subscript l stands for "left" and r for "right", m_0 and b_0 are the nominal values while Δm_i and Δb_i are uncertain terms.

[1]The Proof is omitted due to space limitation.

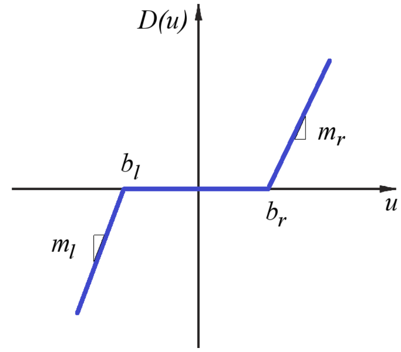

Fig. 1. Dead zone input nonlinearity.

Let $u_0(t)$ be the control signal from a model-free invariance control design. Then, our approach is based on the design of a nominal dead zone inverse, $DI(u_0(t))$, where the remaining uncertain terms Δm_i and Δb_i will be canceled by the invariance control algorithm. For this aim the nominal parameters m_0 and b_0 are assumed to be known and they are used for the construction of a static nominal dead zone inverse:

$$u(t) = DI(u_0(t)) = m_0^{-1}\,(u_0(t) + m_0\, b_0\ \ \text{sign}(u_0)). \tag{11}$$

Substituting (11) in (9) we obtain the following dynamics:

$$\begin{aligned} \dot{x}_i(t) &= x_{i+1}(t),\ i = 1, \dots, k-1, \\ \dot{x}_k(t) &= u_0(t) + F_0(t),\ y(t) = x_1(t), \end{aligned} \tag{12}$$

where $u_0(t)$ is the final control input and $F_0(t)$ represents a perturbation containing new terms related with the uncertain parameters of a dead zone. Note that the inverse of a dead zone is a relay-type discontinuity that can be canceled if the inverse is exact, see [23]. Besides, with a nominal dead zone inverse the structure of system (4) is recovered and the uncertain terms are to be compensated by a model free invariance controller.

VI. Hydraulic Actuator Case Study

The experimental setup is the telescopic link of a laboratory prototype of a typical industrial hydraulic forestry crane. Such industrial equipment is widely used in forestry and is a subject of many researches aimed at automation of these systems, see [18].

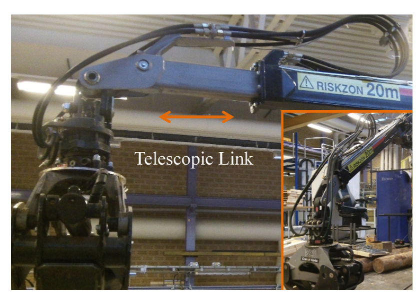

Fig. 2. Industrial hydraulic forestry crane.

The telescopic link of the crane, see Fig. 2, consists of a double-acting single-side hydraulic cylinder and a solid load, which is attached to a piston of the cylinder. This link can be described as a restricted 1-DOF mechanical system actuated by a hydraulic force:

$$m\ddot{x} = f_h - f_{grav} - f_{fric}, \tag{13}$$

where m is the mass, f_h is the force generated by the hydraulics, f_{grav} is the gravity and f_{fric} is the friction force. The force generated by the hydraulics is controlled via a current signal to the valve of the cylinder and it is given by

$$f_h = \begin{cases} p_a\,A_a - p_b\,A_b, & \text{if variables are in } R, \\ f_{uncertain}, & \text{otherwise,} \end{cases} \tag{14}$$

where the validity region

$$R = \{x \in (0, \bar{x}_p), \quad |\dot{x}| < \bar{x}_v, \quad p_a, p_b \in (p_t, p_s)\}, \tag{15}$$

is defined by certain constant bounds; p_s is the pump pressure, and p_t is the return (exit) pressure; the piston areas A_a and A_b are known geometric parameters; p_a and p_b are the measured pressures in chambers A and B of the cylinder. The friction and gravity terms are considered as unknown perturbations. Note that the position of the link, x, is limited by geometrical constrains, the velocity is limited by the maximum achievable flow from a pump, pressures are limited through a set of service anti-cavitation and pressure-relief valves, which, in particular, ensure $p_t \leq p_i \leq p_s$, $i = a, b$. These devices play a fault-preventing role and do not influence a normal operation. Moreover, the initial conditions are within the region R and $f_{uncertain}$ prevent from leaving the region. The dynamics of the pressures can be modeled, see [16, Sec. 3.8], by:

$$\begin{array}{ll} \dot{p}_a = \frac{\beta}{V_a(x)}\,(-\dot{x}\,A_a + q_a), & \text{if variables are in } R, \\ \dot{p}_b = \frac{\beta}{V_b(x)}\,(\dot{x}\,A_b - q_b), & \text{if variables are in } R, \\ \dot{p}_i = p_{uncertain}, \quad i = a, b & \text{otherwise,} \end{array} \tag{16}$$

and initial conditions $p_{a,b}(0) \in (p_t, p_s)$; $V_a(x) = V_{a0} + x\,A_a$ and $V_b(x) = V_{b0} - x\,A_b$ are volumes of the chambers A and B at the given piston position x, V_{a0} and V_{b0} are known geometric constants, β is a known bulk modulus, q_a and q_b are flows to the chamber A and from the chamber B. The flow q_a is positive when the oil goes into chamber A, and the flow q_b is positive when the oil goes out of chamber B. Following [16], [17], the nonlinear equations describing the fluid flow distribution in the valve can be written, in their simplest forms, as: $q_a = c_a\,S_a(x_s)\,\sqrt{p_s - p_a}$ and $q_b = c_b\,S_b(x_s)\,\sqrt{p_b - p_t}$ for $x_s \geq 0$; $q_a = -c_a\,S_a(x_s)\,\sqrt{p_a - p_t}$ and $q_b = -c_b\,S_b(x_s)\,\sqrt{p_s - p_b}$ for $x_s < 0$. Here c_a and c_b are constant coefficients which depend on physical values (fluid density, discharge coefficient and other), $S_a(x_s)$ and $S_b(x_s)$ are (non-negative) areas of orifices for the ports A and B, x_s is a displacement of a spool inside a valve, this spool is actuated by an electromagnetic actuator where an input (current) signal u is applied, $u \in [-u^-, u^+]$. Assuming that the valve is symmetric, i.e. $\forall x_s : S_a(x_s) = S_b(x_s)$, we introduce the signed area function $S(x_s)$ given by $S(x_s) = S_a(x_s)\,\mathrm{sign}(x_s) = S_b(x_s)\,\mathrm{sign}(x_s)$. The absolute value of this function is equal to the area of the orifices and the sign indicates directions of the flows. Then, the flow equations can be rewritten as: $q_a = c_a\varphi_a S(x_s)$ and $q_b = c_b\varphi_b S(x_s)$, with: $\varphi_a = \sqrt{p_s - p_a}\frac{(\mathrm{sign}(x_s)+1)}{2} - \sqrt{p_a - p_t}\frac{(\mathrm{sign}(x_s)-1)}{2}$ and $\varphi_b = \sqrt{p_b - p_t}\frac{(\mathrm{sign}(x_s)+1)}{2} - \sqrt{p_s - p_b}\frac{(\mathrm{sign}(x_s)-1)}{2}$. Note that in industrial hydraulic systems a nonzero pressure difference through the valve is ensured by a set of service valves, *i.e.* $\varphi_a \geq 0$ and $\varphi_b \geq 0$. Taking into account a high-response servo valve and assuming that the spool displacement is proportional to the input signal, the signed area can be modeled as defined by the input signal u through a nonlinear static relation $S(x_s) = D(u)$. The shape of the function D strongly depends on a type of the valve; for industrial heavy-duty systems a dead-zone due to a leakage-preventing closed-center spool and a saturation due to limiting screws are common, see [1] and references therein. Taking the derivative of (14) and substituting (16), one obtains:

$$\dot{f}_h = -\varphi_0\dot{x} + \varphi_1 D(u), \tag{17}$$

where $\varphi_0 = \beta(\frac{A_a^2}{V_a(x)} + \frac{A_b^2}{V_b(x)})$ and $\varphi_1 = \beta(\frac{c_a A_a \varphi_a}{V_a(x)} + \frac{c_b A_b \varphi_b}{V_b(x)})$ with $0 < \underline{\varphi_i} \leq \varphi_i \leq \overline{\varphi_i}$ for $i = 0, 1$. Besides, the nonlinear function $D(u)$ can be represented by (10). Since $\dot{f}_h$ is bounded, sytem (17) can be rewritten as:

$$\dot{x} = \varphi D(u) - \varphi_0^{-1}\dot{f}_h, \tag{18}$$

where $\varphi = \frac{\varphi_1}{\varphi_0}$ and $\varphi = \bar{\varphi} + \Delta\bar{\varphi}$. Given an appropriate desired trajectory for (15), x_{ref}, with Lipschitz continuous second derivative, $\ddot{x}_{ref}$, the objective of this section is to design a control law for u to achieve the tracking of the cylinder position x.

A. Experiments

The experiments are carried out using a real-time platform dSpace 1401 with sample time of 1ms using forward Euler integration method. The position of the telescopic link is measured with a wire-actuated encoder which provides 2381 counts for the range from 0 to 1.55m with a quantization interval of $Q = 0.651$mm. The desired trajectory is selected as a sinusoidal signal: $x_d = 0.8 + 0.4\sin(\omega t)$. In the experiments the values used for the dead zone compensation (11) are $m_0 = 1$ and $b_0 = 0.3$; being such values the nominal ones, the controller should be able to compensate the gap between nominal and real values, namely compensate the remaining uncertain terms Δm_i and Δb_i along with other perturbations. Firstly, it is necessary to tune the a_1, λ_1 and l coefficients, then to examine the performances of the controller varying the parameter α_1 which determines the non-linearity of the controller, lastly on has to evaluate the effect of the $\hat{F}_0$ estimate. The error in the tracking is smaller decreasing α_1 and even smaller adding the $\hat{F}_0$ compensation, which proves clearly the efficiency of the method. It is worth to remark thought, that a high frequency oscillatory behavior is present in the error (and control input) for $\alpha_1 < 1$ which is not desirable in the application considered. An α_1 equal or close to one had to be chosen to avoid it. The closest α_1 to 1 the faster the oscillations, which are not propagated from the control input to the telescopic link guaranteeing a

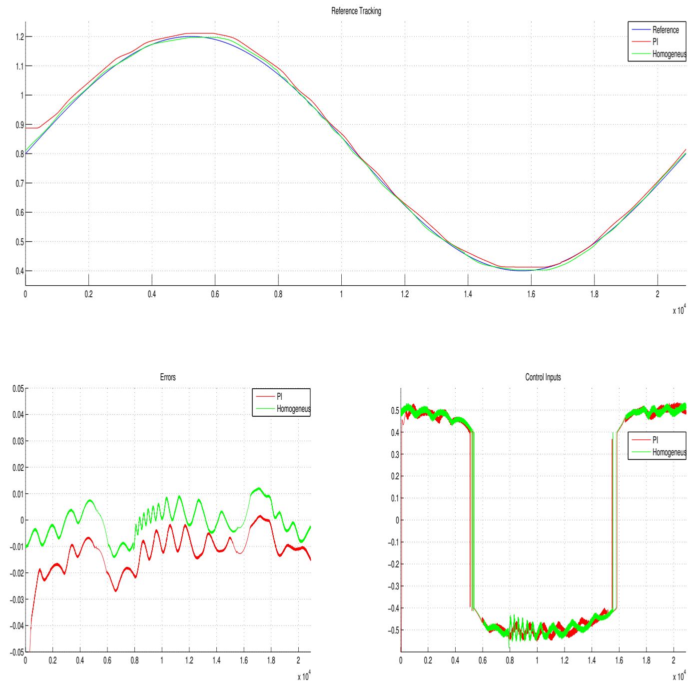

Fig. 3. Comparison between the Homogeneous controller and the PI approach. (x-axis in samples, y-axis in meter)

smooth extension and retraction. To lay stress on the good performances of the method, Fig. 3 shows the behavior in comparison with a standard PI approach often used to regulate such a system. It can be seen that, although the integral action applied, the link could not follow the reference as well as the homogeneous control which does not have what appears to be a static error with respect to the reference; that it could be caused by the nominal values used for the dead zone compensation. As stated above the homogeneous controller is able to handle the error between nominal and real values, that's not achieved with the PI control despite the tuning to have the best behavior possible.

VII. Conclusion

This paper presented the synthesis of a control method for system which can be described as in (4). A linear filter allows the decoupling of the control variables and the disturbances, which are afterward estimated starting from the estimation error using a homogeneous high gain observer. The estimation of perturbations is then included in the controller to compensate them. The approach is fully proven theoretically then applied on the telescopic link of a hydraulic actuated robotic crane used in forestry. To deal with such a platform an easy but effective dead zone compensation technique has been implemented; such a compensation relies just on nominal parameters. The proposed controller behaves as expected, compensating perturbations and dead zone incertitudes, and outperforming the standard PI control as it is shown with experiments.

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