

Feature Based Hole Filling Algorithm on Triangular Mesh

Bin Xu, Zhongke Li, Ying Tan

► **To cite this version:**

Bin Xu, Zhongke Li, Ying Tan. Feature Based Hole Filling Algorithm on Triangular Mesh. Dao-liang Li; Yingyi Chen. 7th International Conference on Computer and Computing Technologies in Agriculture (CCTA), Sep 2013, Beijing, China. Springer, IFIP Advances in Information and Communication Technology, AICT-419 (Part I), pp.90-101, 2014, Computer and Computing Technologies in Agriculture VII. <10.1007/978-3-642-54344-9_11>. <hal-01220900>

HAL Id: hal-01220900

<https://hal.inria.fr/hal-01220900>

Submitted on 27 Oct 2015

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



Feature Based Hole Filling Algorithm on Triangular Mesh

Bin xu¹, Zhongke li², Ying³ tan

1. Teaching and Research Section of computer, The Second Artillery Engineering
University, Xi'an, 710025, China Email: xubinliuxia@sina.com

Abstract: Aiming at the problem of triangular mesh hole repairing, a new geometric feature based hole filling algorithm was presented. The holes boundary were extracted and pretreated, and the advancing front mesh technique is used to cover the hole with newly created triangles; The Euclidean-coordinate and Laplacian of mesh points near the hole boundary were chosen as training sample for support vector machines and deduced Laplacian of filling points; calculated coordinate of filling point through position equations based on deduced Laplacian of filling points to fill the hole precisely. Examples proved that this algorithm furnished geometric details to missing area of triangular mesh surface very good.

Keywords: Triangular mesh model; hole repairing; Laplacian; least-squares support vector machines; position equation

1 Introduction

Polygonal representations of 3D objects, and particular triangular meshes, have become prevalent in numerous application domains. As 3D optical scanners become widespread, triangle meshes can be easier created and widely applied in the fields of CAD and reverse engineering. Even with high-fidelity scanners, the data obtained is often incomplete. The existence of holes makes it difficult for many operations based on meshes, such as model rebuilding, rapid prototyping and finite element analysis. Therefore, certain repairs must be done before taking incomplete mesh models into actual applications, and hole-filling is the most important one among them. In many computer-aided engineering applications, detailed geometric features are very important, so how to make patched surface to match the missing geometry well is a core problem. Surround this center various mesh hole-filling approaches have been proposed in recent years.

Davis [1] apply a volumetric diffusion process to extend a signed distance function through this volumetric representation until its zero set bridges whatever holes may be present. Guo[2] employed space carving and iso-surface extraction to fill holes. Ju [3] constructed an inside or outside volume using an octree grid and re-constructed the surface by contouring. Joshua and Szymon [4] used a min-cut algorithm to split space

into inside and outside portions, and patched the holes simultaneously in a globally sensitive manner. Holes with regular boundary over a relatively planar region can be easily patched via planar triangulation, which has been described in detail by a number of textbooks and papers [5, 6, 7]. However, filling a complex hole over an irregular region is much more difficult. To solve this problem, Carr [8] used radial basis function to construct an implicit surface to cover the hole. This method works well for convex surfaces and can handle irregular holes. But difficulties arise when the underlying surface is too complex to be described by a single-value function. Liepa [9] presented an umbrella operator to fair the triangulation over the hole to estimate the underlying geometry. However, the $O(n^3)$ performance of the triangulation method limits this method from being used widely. Jun [10] proposed a hole-filling method based on a piecewise scheme. His method divides a complex hole into several simple holes and all sub-holes are sequentially filled with planar triangulation; sub-division and refinement are then employed to smooth the new triangles. Chen [11] proposed a hole-filling method which can fill the hole and recover its sharp feature involved in the hole area. With this method, holes are filled using a radial basis function; a feature enhancement process based on Bayesian classification [12] and sharpness dependent filter [13] is then applied if there exists any sharp feature on the hole boundary. Some hole-filling algorithms for parametric surfaces have been presented [14, 15, 16]. Since the boundaries of the holes handled are usually made up of a B-spline curve, conserving continuity is more important for these holefilling algorithms. Ideally, hole-filling algorithm should possess the following properties: (1) able to cover an arbitrary hole for any model (robustness), (2) capable of filling large holes in a reasonable amount of time (efficiency), (3) enable the patched surface to match the missing geometry well (precision). Unfortunately, due to the complexity and diversity of the holes, no existing hole-filling methods satisfy all the above desirable properties. In this thesis, we present a novel hole-filling algorithm for mesh models. The advancing front mesh technique is employed first to generate a new triangular mesh to cover the existing hole. Then we utilize the Poisson equation to optimize the new mesh. The algorithm is intended to be simple, fast and robust. The main advantage of algorithm in this thesis is can preserve geometric features to certain extent. Moreover, the hole-filling mesh models are of excellent quality for engineering.

2 Main Mathematical Tool

There are three important mathematical tools in this thesis, one is least-squares support vector machines, another is Laplacian on triangular mesh surface and the last one is discrete Poisson equation. As mentioned above, keeping geometric features is very important to hole filling algorithm in many applications. Therefore, how to enable the patched surface to match the missing geometry well (precision) is a core problem in this thesis.

2.1 Lapacian on triangular mesh surface

Lapacian is the most popular mathematic tool to describe geomatic features on triangular mesh surface. A local film of one vertex on triangular mesh model surface comprised by vertex V_i and vertices on one edge. Fig1 is local film of vertex V_i .

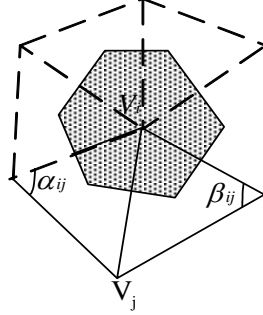


Fig.1:Local film of V_i

Eq (1) is Lapacian of vertex V_i , w_{ij} is weight coefficient. Eq (2) and Eq (3) is calculating formulas for weight coefficient.

$$\delta_i = \sum_{v_j \in N(i)} w_{ij}(v_j - v_i) \quad (1)$$

$$w_{ij} = \frac{\omega_{ij}}{\sum_{v_j \in N(i)} \omega_{ij}} \quad (2)$$

$$\omega_{ij} = \cot \alpha_{ij} + \cot \beta_{ij} \quad (3)$$

δ_i is cotangent lapacian of V_i calculating by Eq (1) 、Eq (2) and Eq (3) .

$$\bar{k}_i n_i = \delta_{ci} = \frac{1}{4A(v_i)} \sum_{v_j \in N(i)} (\cot \alpha_{ij} + \cot \beta_{ij})(v_i - v_j) \quad (4)$$

As Eq (4) , cotangent lapacian linear approximate mean curvature in value and approximate principal curvature in direction , therefore cotangent lapacian is perfect mathematic tools to describe geometrical details of triangular mesh. $A(v_i)$ in Eq (4) is Voronoi graph (Shaded area in Fig1) .

$$L = \begin{bmatrix} L_{11} & \cdots & L_{1n} \\ \vdots & L_{ij} & \vdots \\ L_{n1} & \cdots & L_{nn} \end{bmatrix} \quad (5)$$

$$L_{ij} = \begin{cases} -1 & i = j \\ w_{ij} & v_j \in N(i) \\ 0 & \text{otherwise} \end{cases} \quad (6)$$

$$\Delta_d = LV_d = [\delta_{1d}, \delta_{2d}, \delta_{3d}, \dots, \delta_{nd}]^T, \quad d \in \{x, y, z\} \quad (7)$$

$$V_d = [v_{d1}, v_{d2}, v_{d3}, \dots, v_{dn}]^T, \quad d \in \{x, y, z\} \quad (8)$$

For the purpose to calculate lapacian on vertex , lapacian matrix (as Eq(5)) has been introduced in this thesis. As Eq (5) , L is lapacian matrix of order $n \times n$, n is the number of vertexes in computational domain.

Form of L_{ij} in Eq (5) such as Eq (6) , and V_d is component of the vertex coordinates in a coordinate axis , Δ_d is component of the lapacian in a coordinate axis.

2.2 least-squares support vector machines

least-squares support vector machines is introduced by Suykens first , in this method the least square linear system be used as loss function , thus outstanding advantages of this method is high speed and robustness. The application of least-squares support vector in this thesis is a typical regression problem, regression purpose is optimize fuction $f(x)$. The training sample set in this thesis is components of spatial coordinate around the hole area mesh vertices in the same coordinate axis.

$$S = \{(V_{id}, \delta_{id}), V_{id} \in R, \delta_{id} \in R\}_{i=1}^l \quad (9)$$

As in Eq (9) , $d \in \{x, y, z\}$, l is the of samples in training sample set. Based on the structural risk minimization principle , and considering the complexity and the fitting error fuction , the regression problem can be described as the following optimization problem (as Eq (10) and Eq (11)) .

$$\delta_{id} = w^T \varphi(V_{id}) + b + e_i \quad (10)$$

$$\min Q(w, b, e) = \frac{1}{2} \|w\|^2 + \frac{\gamma}{2} \sum_{i=1}^l e_i^2 \quad (11)$$

As in Eq (11) , w is weighting vector, e_i is slack variable , γ is regularization parameter , deviation variable , in the purpose to solve above optimizatoin problem , Lagrange fuction need to be established. (As Eq (12))

$$L(w, b, e, a) = Q(w, b, e) - \sum_{i=1}^l a_i [w^T \phi(x_i) + b + e_i - y_i] \quad (12)$$

$$\begin{cases} \frac{\partial L}{\partial w} = 0 \Rightarrow w - \sum_{i=1}^l a_i \phi(x_i) = 0 \\ \frac{\partial L}{\partial b} = 0 \Rightarrow \sum_{i=1}^l a_i = 0 \\ \frac{\partial L}{\partial e_i} = 0 \Rightarrow C e_i - a_i = 0 \\ \frac{\partial L}{\partial a_i} = 0 \Rightarrow w^T \phi(x_i) + b + e_i - y_i = 0 \end{cases} \quad (13)$$

Write Eq (13) into matrix form , and expunction w and e can get Eq (12) .

$$\begin{bmatrix} 0 & \bar{I}^T \\ \bar{I} & \Omega + \gamma^{-1}I \end{bmatrix} \begin{bmatrix} b \\ a \end{bmatrix} = \begin{bmatrix} 0 \\ y \end{bmatrix} \quad (14)$$

In Eq (14) $\Omega_{ij} = K(x_i, x_j)$, and Solve formula(12) to get regression function as Eq (15) .

$$f(x) = \sum_{i=1}^l a_i K(x, x_i) + b \quad (15)$$

2.3 Discrete possion equation

In our implementation, we choose the Poisson equation with Dirichlet boundary conditions to refine the patch mesh. The Poisson equation with the Dirichlet boundary is formulated as $\Delta f = \text{div} h$, $f|_{\partial\Omega} = f^*|_{\partial\Omega}$, where f is unknown scalar fuction h is the guidance vector filed, $\text{div} h$ is the divergence of h , and f^* is a known scalar function providing the boundary condition. It can be verified that the Poisson equation is the equivalent to the minimization problem as Eq (16) .

$$\min_f \int |\nabla f - h|^2, \text{with } \mathcal{F}|_{\partial\Omega} = f^*|_{\partial\Omega} \quad (16)$$

The discrete Poisson equation is actually a sparse linear system $Ax = b$, where the unknown vector x represents special coordinates of all vertices on the reconstructed patch mesh, the coefficient matrix A is determined by Eq (5) , and the vector b is a known vector field obtained from the collection of divergence values at all boundary vertices formulated by Eq (1) , which is taken as the boundary condition. The Poisson equation implies that in order to reconstruct the patch mesh we need a guidance vector field defined on the triangles of the patch mesh.

3 Hole-Filling Algorithm

Algorithm in this thesis can be divided into three following parts. (1) The hole boundary pre-treatment: Include a hole boundary identification and boundary edge pre-treatment. (2) Initial patch mesh generation: Include initialize the front using the boundary vertices of the hole; Calculate the angle between two adjacent boundary edges; Starting create new triangles on the plane determined by two adjacent edge until the whole hole has been patched by all newly created triangles. (3) Filling patch refinement based on Poisson equation: Include gather train sample and regression function's solving of least-squares support vector machine; re-positioned vertices of filling patch by solving poisson equation in order to make the patch mesh connect the boundary vertices smoothly and approximate the missing geometry more accurately is employed to refine the patch mesh.

3.1 Hole boundary pretreatment

There is two steps in this section : hole boundary identification and boundary edge pretreatment. Since a vertex-based topological structure is used in this work, all boundary vertices can be easily identified by checking the numbers of their 1-ring triangles and 1-ring edges, in this thesis octree be used in this to increase identification speed. The concrete step of boundary identification as follows:

Step1: Compute center of each triangle patch and contaction between the center and its vertices;

Step2: Generate octree based on triangle patch centers;

Step3: Searching octree, and detecting topology connection between its edges, if one edge belong to one triangle patch, the edge is boundary edge;

Step4: Distinguishing different boundary loop by vertex geometric connectivity, showing different holes.

The second step in this section is boundray edge pretreatment. Because data information of hole area in not complete, shape of reconstructed triangle patch is not always good, some patch is long and narrow triangle. In propose to reduce influence of bad shape triangle patch, edge pre-treatment must do before filling the mesh hole. The concrete step as follows:

$v_i (i = 1, 2, \dots, n)$ is n vertices on hole boundary, \mathcal{E} is edges on hole boundary.

Step1: Compute average length of edges on hole boundary L_{ave} .

Step2 : Traversal all edge on hole boundary, if length of one edge ($\mathcal{E}_{ij} = (v_i, v_j)$) $L_{\mathcal{E}_{ij}} > k \cdot L_{ave}$, compute middle point v_{new} of \mathcal{E}_{ij} .

Step3: Take v_{new} as new boundary point, and delete \mathcal{E}_{ij} and triangle patch $v_i v_j$, add edge $v_i v_{new}$ 、 $v_{new} v_j$ and triangle patch $v_i v_{new}$ 、 $v_{new} v_j$, as shown in Fig2.

At last iterate step3, until boundary edge didn't change.

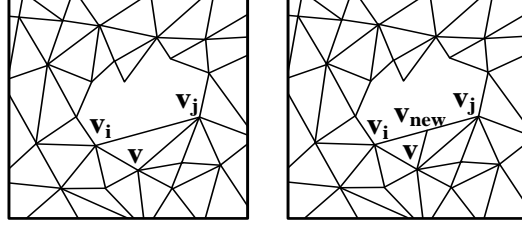


Fig.2:Hole boundary Pretreatment

3.2 Initial patch mesh generation

In this thesis, we adopt the advancing front mesh (AFM) technique to generate an initial patch mesh over the hole. The method consists of the following six steps:

Step 1: Initialize the front using the boundary vertices of the hole.

Step 2: Calculate the angle θ_i between two adjacent boundary edges (e_i and e_{i+1}) at each vertex V_i on the front.

Step 3: Starting from the vertex V_i with the smallest angle θ_i , create new triangles on the plane determined by e_i and e_{i+1} with the three rules shown in Fig. 3.

Step 4: Compute the distance between each newly created vertex and every related boundary vertex; if the distance between them is less than the given threshold, they are merged.

Step 5: Update the front.

Step 6: Repeat Steps 2 through 5 until the whole region has been patched by all newly created triangles.

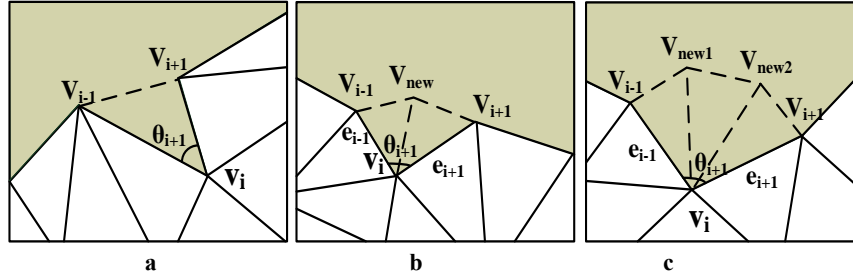


Fig.3: Rules for creating triangles:

a $\theta_i \leq 75^\circ$; b $75 < \theta_i \leq 135^\circ$; c $\theta_i > 135^\circ$

3.3 Establish the training sample set

Up to the present all mesh hole filling algorithm deduce geometric characteristic of filling patch depend on geometric characteristic on initial mesh surface surrounding the hole. In this thesis we take lapacian and euclidean coordinate on vertices surrounding the hole boundary as training sample set to least-squares support vector machines.

In case of V_e is vertex set on hole boundray B , $SV_j(j=0,1,\dots,k)$ is j layer of vertex around hole boundary, in practice there is four step in the training sample collection method.

Step1: Take vertex v_i on hole boundary, search vertex in same traingle patch with v_i , if that vertex does not in V_e , put it in SV_1 .

Step2: Traversing every vertex in arrays V_e , repet step1, than gathered vertices on the first layer from hole boundary and put these vertices in SV_1 .

Step3: To vertices in $SV_j(j=1,2,\dots,k-1)$ search vertex in same traingle patch with them, than put vertices not in SV_j and SV_{j-1} into SV_{j+1} .

Step4: Iterate step3, until the setted up layer number of $SV_j(j=0,1,\dots,k)$ attained. Under this circumstances all vertices and its lapacian consist training samples set, the set of sample points as Fig4.

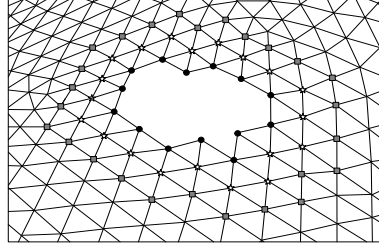


Fig.4: Training sample set for least-squares support vector machines

3.4 Filling patch refinement based on Poisson equation

According to method mentioned in section 2.2, Put training sample set in least-squares support vector machines, get three regression fuction: $f_x(x)$, $f_y(x)$, $f_z(x)$. By three regression fuction we can get new lapacian field that the poisson equation requires. Once a discrete lapacian fields is given, we consider the new lapacian field as the lapacian field in the Poisson equation, a piecewise continuous scalar function, a connected mesh, can be reconstructed, which computes euclidean coordinate of vertices on filling patch in the least squares sense.

In this thesis, process of computing new coordinates of vertices on filling patch is solution of $n \times n$ linear equstions as (18).

$$V'_d = [v'_{d1}, v'_{d2}, v'_{d3}, \dots, v'_{dn}]^T, \quad d \in \{x, y, z\} \quad (17)$$

$$L_{vn} V'_{nd} = \Delta'_{dn} \quad (18)$$

$$\Delta'_{dn} = L_{vn} V_d = [f_d(v'_{1d}), f_d(v'_{2d}), \dots, f_d(v'_{nd})]^T, \quad d \in \{x, y, z\} \quad (19)$$

L_{vn} in (18) is contangent lapacian matrix established by method mentioned in section 2.1 of filling patch, Δ'_{dn} in (19) is one component of lapacian of vertices on reconstructed filling patch. $f_d(v'_{nd})$ in (19) is one regression fuction.

4 Implementation and Analysis

The proposed hole-filling algorithm has been implemented with VisualC++2010 and OpenGL. All experimental results in this paper were obtained on a 2.0 GHz Pentium IV personal computer with 1024MB memory. Many examples have been used to test the robustness, efficiency and accuracy of the method.

Figure5a shows head model, there are a large number of geometric details on this mesh model. Figure5b shows head model with hole in the part of hair. In this section the hole-filling result on head model by direct filling algorithm、algorithm in reference[17]and algorithm in this thesis. The reason of taking algorithm in reference[17]compare to algorithm in this thesis is the algorithm in reference[17]is based on solution of poisson equation same to algorithm in this thesis, the difference between them is least-squares support vector machine has been used in algorithm in this thesis to estimate geometric details of filling patch, so the comparison of two algorithms show the advantage of algorithm in this thesis at keeping geometric style between filling patch and other part of mesh model.

Figure6 depicts filling result by direct filling algorithm, obviously very rough. Figure7 depicts filling result by filling algorithm in reference[17], as shown in figure7, filling result is very good, mesh quality of filling patch is well two, the only disfigurement is filling patch and other part of model is not coincident in geometric style at all. Figure8 depicts filling result by algorithm in this thesis, from the picture we can see that mesh quality is as good as in Figure7, and keeping geometric style between filling patch and other part of mesh very well.

Table 1 is error evaluation for algorithm in reference[17]and in thesis. Table 2 is running time of two algorithms states above.

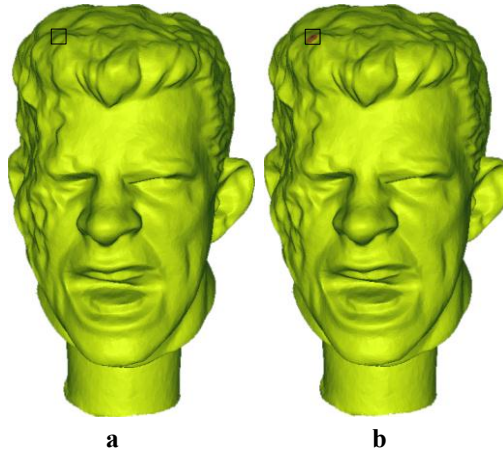


Fig.5:Preliminary head model and head model with hole

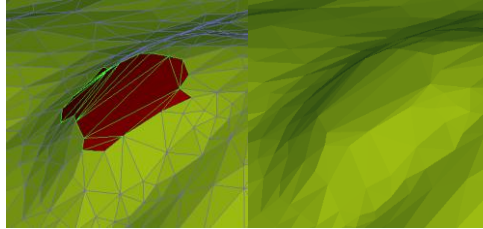


Fig.6:Filling result by direct filling alorithm

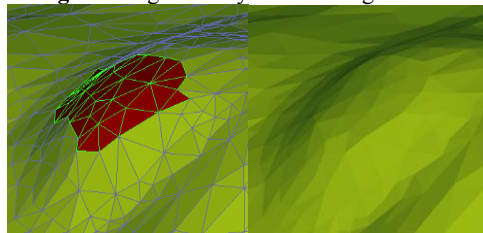


Fig.7: Filling result by filling alorithm in reference 17

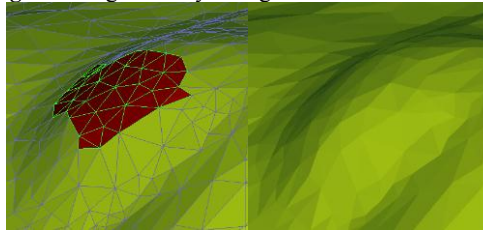


Fig.8: Filling result by filling alorithm in this thesis

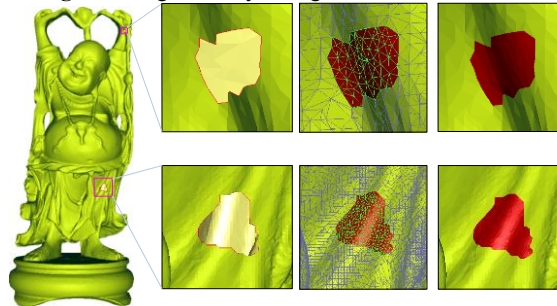


Fig.9: Filling result on happyvrip model by filling alorithm in this thesis

Table1.error evaluation

Algorithm	Vertices num on filling patch	Triangle num on filling patch	Average distance	Average error
In reference 17	28	69	0.05 (mm)	0.0015
In this thesis	31	75	0.042 (mm)	0.00142

Table2.computing time

Algorithm	Vertices num on filling patch	Triangle num on filling patch	Time of Initial patch mesh generation	Time of solving possion equation
In reference 17	28	69	10.6ms	15.7ms
In this thesis	31	75	10.4ms	15.9ms

In this thesis the distances between the vertices on the patch mesh and the original analytic surface are used to evaluate the precision of our algorithm. The quotient of

the average distance and the square root of analytic surface area is considered as the error of our algorithm. From table 1 and table 2 we can see that by algorithm in this thesis we can get higher precision while spend similar time.

Fig9 depicts filling result by algorithm in this thesis on big hole, from the picture we can see that the algorithm work very well in keeping geometric style between filling patch and other part when the hole need to fill is compared bigger.

5 Conclusion

The innovation point of this thesis is taking laplacian of vertices on filling patch as output of regression function and reconstruction filling patch by solving poisson equation. The advantage of this algorithm is use laplacian to depict geometric details of mesh surface model, therefore making filling mesh patch and original mesh surface insculated more naturally.

Limitation of algorithm in this thesis is filling result may lose some geometric details when the hole is too big, this limitation is existing in all popular mesh surface filling algorithms. The method dealing with this problem is taking whole geometric characteristic of mesh model as reference while filling the mesh surface hole, this is also research area of this thesis.

References

1. J. Davis, S.R. Marschner, M. Garr and M. Levoy, "Filling holes in complex surfaces using volumetric diffusion" In: First International Symposium on 3D Data Processing Visualization and Transmission, Padova, pp. 428-438, 2002.
2. T.-Q. Guo, J.-J. Li, J.-G. Weng and Y.-T. Zhuang, "Filling holes in complex surfaces using oriented voxel diffusion" In: 2006 International Conference on Machine Learning and Cybernetics, IEEE, Los Alamitos, pp. 4370-4375, 2006.
3. Ju, T: Robust repair of polygonal models. In: Processing of SIGGRAPH, Los Angeles, CA, USA, 8-12 August 2004 pp. 888-895. ACM Press, New York (2004).
4. Joshua, P, Szymon, R.: Atomic volumes for mesh completion. In: Proceedings Eurographics Symposium on Geometry Processing, Dublin, Ireland. 29 August-2 September 2005, pp. 33-41. Blackwell Press (2005).
5. A. Brunton, S. Wuhler, C. Shu, P. Bose and E.D. Demaine, "Filling holes in triangular meshes by curve unfolding" In: IEEE International Conference on Shape Modeling and Applications, pp. 66-72, 2009.
6. B.J. Li, X.K. Zhang P. Zhou and P. Hu, "Mesh parameterization based on one-step inverse forming, " *Comput. Aided Des.* 42(7), pp. 633-640, 2010.
7. S. Gao, W. Zhao, H. Lin, F. Yang and X. Chen, "Feature suppression based cad mesh model simplification" *Computer-Aided Design*, 42(12), pp. 1178-1188, 2010.
8. Carr.J. Beatson, R., Cherrie, J., Mitchell, T., Fright, W., McCallum, B: Reconstruction and representation of 3D objects with radial basis functions. In: Processing of SIGGRAPH, Los Angeles, CA, 12-17 August 2001, pp. 67-76. ACM Press, New York (2001).

9. Liepa, P: Filling hole in meshes. In: Proceedings Eurographics Symposium on Geometric Processing, Granada, Spain, 1–6 September 2003, pp. 200–207. Blackwell Press (2003).
10. Jun, Y.: A piecewise hole-filling algorithm in reverse engineering. *Comput. Aided Des.* 22(8), 263–270 (2005).
11. Chen, C.Y., Cheng, K.Y., Liao, H.Y.M.: A sharpness dependent approach to 3D polygon mesh hole filling. In: Proceedings of Eurographic, Trinity College, Dublin, Ireland, 29 August–2 September 2005, pp. 13–16. Blackwell Press (2005).
12. Chen, C.Y., Cheng, K.Y.: A sharpness dependent filter for mesh smoothing. *Comput. Aided Geom. Des.* 22(5), 376–391 (2005).
13. Chen, C.Y., Cheng, K.Y., Liao, H.Y.M.: Fairing of polygon meshes via Bayesian discriminate analysis. The 12th International Conference in Central Europe on Computer Graphics, Visualization and Computer Vision, Plzen-Bory, Czech Republic, pp. 175–182. UNION Agency-Science Press (2004).
14. R. Lou, J.-P. Pernot, A. Mikchevitch and P. V éron, “ Merging enriched finite element triangle meshes for fast prototyping of alternate solutions in the context of industrial maintenance,” *Comput. Aided Des.* 42(8), pp. 670-681, 2010.
15. T. Ju, “Fixing geometric errors on polygonal models: a survey,” *Journal of Computer Science and Technology*, 24(1), pp. 19-29, 2009.
16. Z. Li, D.S. Meek and D.J. Walton, “Polynomial blending in a mesh hole-filling application,” *Computer-Aided Design*, 42, pp. 340-349, 2010.
17. W. Zhao, S.M. Gao and H.W. Lin, “A robust hole-filling algorithm for triangular mesh,” *The Visual Computer: International Journal of Computer Graphics*, 23(12), pp.987-997, 2007.