



**HAL**  
open science

## Computing Periodic Triangulations

Mikhail Bogdanov, Manuel Caroli, Monique Teillaud

► **To cite this version:**

Mikhail Bogdanov, Manuel Caroli, Monique Teillaud. Computing Periodic Triangulations. Shape up - Exercises in Materials Geometry and Topology, Sep 2015, Berlin, Germany. pp.60-61. hal-01224549

**HAL Id: hal-01224549**

**<https://inria.hal.science/hal-01224549>**

Submitted on 5 Nov 2015

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# Computing Periodic Triangulations

Mikhail Bogdanov and Manuel Caroli

*This work was done while the first two authors were PhD students at INRIA Sophia Antipolis - Méditerranée*

Monique Teillaud\*  
INRIA Nancy - Grand Est  
CNRS, LORIA and  
Université de Lorraine

This work was initially motivated by the need for periodic Delaunay triangulations in  $\mathbb{R}^3$  in numerous domains, including astronomy, material engineering, biomedical computing, fluid dynamics, etc (see [3] for references). Periodic triangulations in the hyperbolic plane  $\mathbb{H}^2$  also arise in very diverse fields, such as physics, solid modeling, and neuromathematics (see [1]).

Periodic triangulations can be seen as triangulations of a manifold that is the quotient of the space ( $\mathbb{R}^3$  or  $\mathbb{H}^2$  in the abovementioned two cases) under the action of some discrete group of isometries (some crystallographic group for the Euclidean case, some Fuchsian group for hyperbolic case).

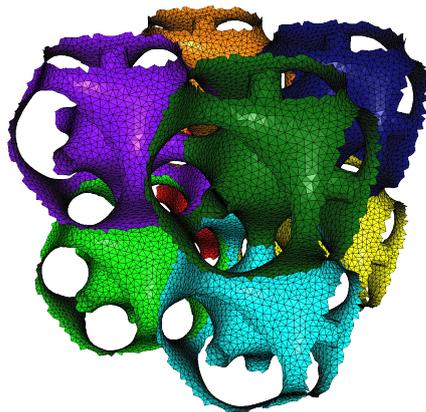


FIG. 1. Mesh of a 3D triply-periodic volume (*work with Aymeric Pellé*).

Delaunay triangulations in the Euclidean space  $\mathbb{R}^d$  have been extensively studied. They are an essential tool for a large number of fields. A classic incremental algorithm consists in inserting points one after another; for each new point, the region of cells in conflict is computed, then it is partitioned by new cells having the new point as vertex. This algorithm yields a very efficient implementation, publicly available in the CGAL library ([www.cgal.org](http://www.cgal.org)). The Delau-

nay complex in  $\mathbb{H}^d$  can be derived from the Delaunay triangulation in  $\mathbb{R}^d$  [2].

Much less is known for the case of quotient manifolds. We studied cycles in the graph of edges of the projection of the Delaunay triangulation in  $\mathbb{R}^3$  or  $\mathbb{H}^2$  onto the studied quotient. We studied covering spaces that allow to avoid cycles of length 1 and 2, thus guaranteeing that the projection of the Delaunay triangulation onto the covering space is a simplicial complex, which allows us to extend the algorithm described above [1, 3].

While the general scheme is similar for Euclidean and hyperbolic manifolds, the case of hyperbolic manifolds is more tricky, if only because hyperbolic translations do not commute. We could consider the case of general closed Euclidean  $d$ D manifolds, while our study in the hyperbolic case is currently limited to the Bolza surface. In the specific case of the 3D flat torus, the work led to an implementation in CGAL, and a collaboration with astrophysicists (see e.g., <http://www.computational-geometry.org/SoCG-videos/socg12video/>). Further work includes the computation of 3D Euclidean periodic meshes (Fig. 1) and plane hyperbolic meshes (Fig.2).

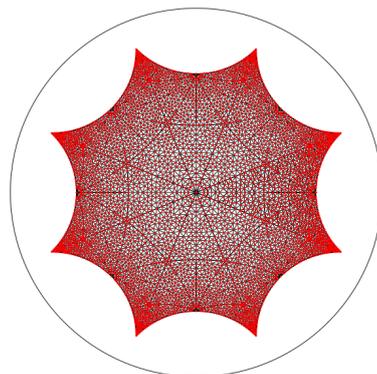


FIG. 2. Periodic mesh in the hyperbolic plane (*work with Mathieu Schmitt*).

More references can be found at <http://www.loria.fr/~teillaud/other-geometries/>

- [1] M. Bogdanov and M. Teillaud. Delaunay triangulations and cycles on closed hyperbolic surface. Research Report INRIA No 8434 (2013). <http://hal.inria.fr/hal-00921157>
- [2] M. Bogdanov, O. Devillers, and M. Teillaud. Hyperbolic Delaunay complexes and Voronoi diagrams made practical. Proc. SoCG, pp. 67-76, 2013. <http://hal.inria.fr/hal-00833760>
- [3] M. Caroli and M. Teillaud. Computing 3D Periodic Triangulations. Proc. ESA. LNCS 5757, pp. 59–70 (2009). doi: 10.1007/978-3-642-04128-0\_6. - See also SoCG'08 video at <http://www.computational-geometry.org/SoCG-videos/socg08video/>

\* email: [Monique.Teillaud@inria.fr](mailto:Monique.Teillaud@inria.fr)