

## Domain Decomposition for real time Simulation of needle insertion

Yinoussa Adagolodjo, Hadrien Courtecuisse, Raffaella Trivisonne, Laurent Goffin, Stephane Pierre-Alain Bordas, Michel De Mathelin

► **To cite this version:**

Yinoussa Adagolodjo, Hadrien Courtecuisse, Raffaella Trivisonne, Laurent Goffin, Stephane Pierre-Alain Bordas, et al.. Domain Decomposition for real time Simulation of needle insertion. DD23, Jul 2015, Jeju, South Korea. <hal-01229514>

**HAL Id: hal-01229514**

**<https://hal.inria.fr/hal-01229514>**

Submitted on 30 Aug 2016

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# Domain Decomposition for real time Simulation of needle insertion

Yinoussa Adagolodjo<sup>1</sup>

Hadrien Courtecuisse<sup>1</sup>

Raffaella Trivisonne<sup>1</sup>

<sup>1</sup>University of strasbourg

Laurent Goffin<sup>1</sup>

Stéphane Bordas<sup>2</sup>

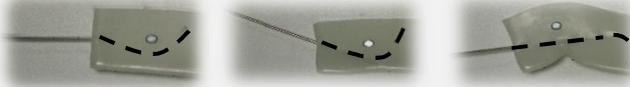
Michel De Mathelin<sup>1</sup>

<sup>2</sup>University of Luxembourg

## Context

- Real time simulation for needle trajectory optimization during robotic insertions in deformable objects:

- Planned trajectory
- Deformation due to insertion
- Camera capture the surface of the gel
- FE Simulation to compute the deformed trajectory

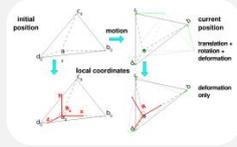


## Simulation

### Deformable Model

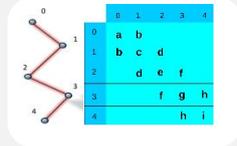
#### Corotational Model [1]:

- Linear relation between stress tensor  $\sigma$  and deformation tensor  $\epsilon$  (Hooke's law)
- Geometrical non linearities filtering:  
$$\sigma = RCR^T \epsilon$$



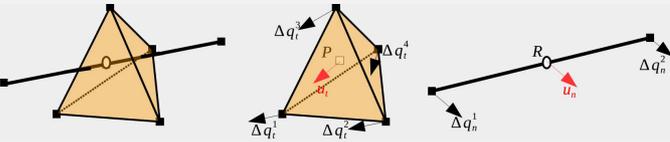
#### Beam Model :

- FEM corotational beam model
- Elasticity and bending stiffness
- Topology is a sequence of segments
- Stiffness matrix is a block-tri-diagonal



### Interaction Model:

- Constraints are imposed using Lagrangian Multipliers (LM) [2]
- Re-meshing is avoided



- Principle of virtual works: the displacement of a virtual point inside a tetrahedral element is given as a linear relation  $J$  of the degrees of freedom  $q$ :

$$u_t = J_t \Delta q_t \quad u_n = J_n \Delta q_n$$

where  $n$  denotes the needle and  $t$  the deformable object.

### Time integration

The dynamic equation of simulated bodies is given by:

$$M\ddot{q} = P - F(q, \dot{q}) + R(q, \lambda)$$

where  $M$  is the inertia matrix,  $F$  the internal forces,  $P$  the external forces and  $R$  the constraint forces given by LM  $\lambda$ .

Implicit time integration is used with backward Euler scheme:

- Stability, accuracy, interaction between models
- First order Taylor expression

$$\begin{cases} A_t x_t = b_t + H_t^T \lambda \\ A_n x_n = b_n + H_n^T \lambda \\ \delta = H_n^T x_n + H_t^T x_t \end{cases} \rightarrow \begin{cases} x_t = A_t^{-1}(b_t + H_t^T \lambda) \\ x_n = A_n^{-1}(b_n + H_n^T \lambda) \\ \delta = \underbrace{[H_n A_n^{-1} H_n^T + H_t A_t^{-1} H_t^T]}_W \lambda + \delta_0 \end{cases}$$

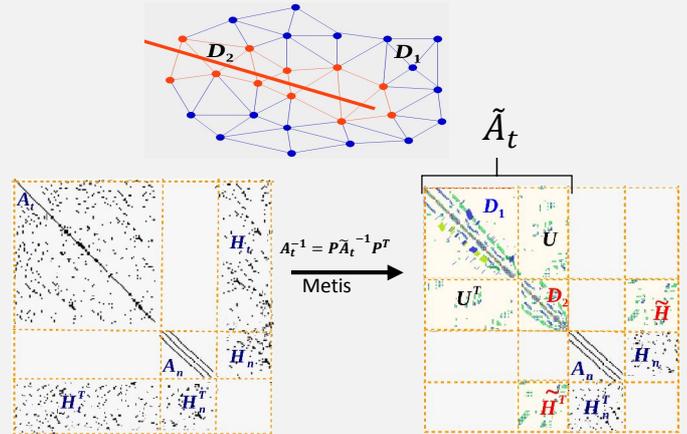
where  $x = \Delta \dot{q}$ ,  $H$  is the derivative of the constraints,  $A$  the implicit fem matrix,  $\delta$  the constraint violation and  $W$  is known as the Delaunay operator

**Contribution:** Domain Decomposition to compute  $W$  in Real Time

## Domain Decomposition

- $A_n$  is a bloc-tri-diagonal matrix that can be inverted in real time using Thomas algorithm.[2]

- $A_t$  is a large matrix
  - The trajectory of the needle is known before the simulation
  - The LM are applied on few degrees of freedom



- Using Sherman Morrison formula

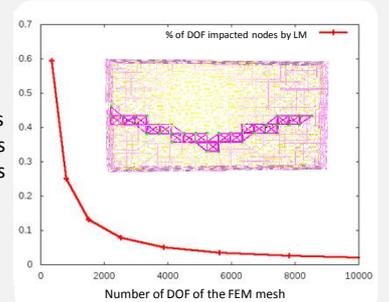
$$H_t^T A_t^{-1} H_t = H_t^T P (D_2 - U^T D_1^{-1} U)^{-1} P^T H_t$$

- $U^T D_1^{-1} U$  is precomputed (far from the trajectory)
- $D_2 - U^T D_1^{-1} U$  is inverted at each time step using blas library

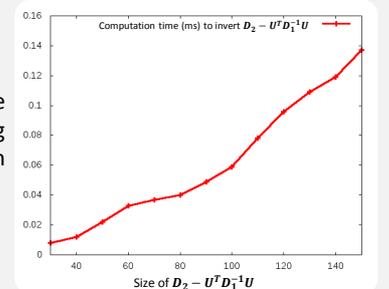
## Result

- Few DOF are impacted by the LM along the planned trajectory:

87 DOF for 1512 nodes meshes  
105 DOF for 3872 nodes meshes  
130 DOF for 7800 nodes meshes



- $D_2 - U^T D_1^{-1} U$  can be inverted in real-time using blas library for any mesh resolution



[1] Michael Hauth and Wolfgang Strasser "Corotational Simulation of deformable Model" Sand 14, D-72076 Tübingen, Germany.

[2] C. Duriez, C. Guebert, M. Marchal and S. Cotin, L. Grisoni "Interactive Simulation of Flexible Needle Insertions Based on Constraint Models. In Medical Image Computing and Computer-Assisted Intervention" In Medical Image Computing and Computer-Assisted Intervention.