

## Domain Decomposition for real time Simulation of needle insertion

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# Domain Decomposition for real time Simulation of needle insertion

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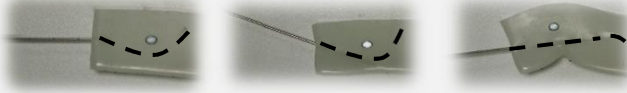
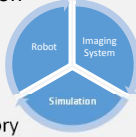
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## Context

- Real time simulation for needle trajectory optimization during robotic insertions in deformable objects:

- Planned trajectory
- Deformation due to insertion
- Camera capture the surface of the gel
- FE Simulation to compute the deformed trajectory

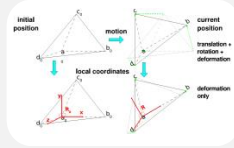


## Simulation

### Deformable Model

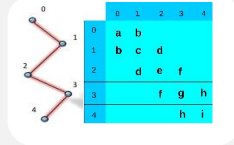
#### Corotational Model [1]:

- Linear relation between stress tensor  $\sigma$  and deformation tensor  $\epsilon$  (Hooke's law)
- Geometrical non linearities filtering:  
$$\sigma = RCR^T \epsilon$$



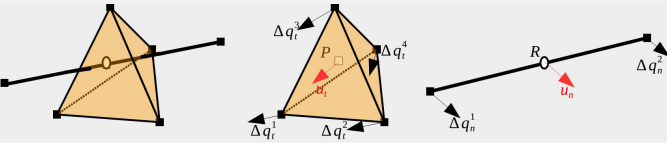
#### Beam Model:

- FEM corotational beam model
- Elasticity and bending stiffness
- Topology is a sequence of segments
- Stiffness matrix is a block-tri-diagonal



### Interaction Model:

- Constraints are imposed using Lagrangian Multipliers (LM) [2]
- Re-meshing is avoided



- Principle of virtual works: the displacement of a virtual point inside a tetrahedral element is given as a linear relation  $J$  of the degrees of freedom  $q$ :

$$u_t = J_t \Delta q_t \quad u_n = J_n \Delta q_n$$

where  $n$  denotes the needle and  $t$  the deformable object.

### Time integration

The dynamic equation of simulated bodies is given by:

$$M\ddot{q} = P - F(q, \dot{q}) + R(q, \lambda)$$

where  $M$  is the inertia matrix,  $F$  the internal forces,  $P$  the external forces and  $R$  the constraint forces given by LM  $\lambda$ .

Implicit time integration is used with backward Euler scheme:

- Stability, accuracy, interaction between models
- First order Taylor expression

$$\begin{cases} A_t x_t = b_t + H_t^T \lambda \\ A_n x_n = b_n + H_n^T \lambda \\ \delta = H_n^T x_n + H_t^T x_t \end{cases} \rightarrow \begin{cases} x_t = A_t^{-1}(b_t + H_t^T \lambda) \\ x_n = A_n^{-1}(b_n + H_n^T \lambda) \\ \delta = \underbrace{[H_n A_n^{-1} H_n^T + H_t A_t^{-1} H_t^T]}_W \lambda + \delta_0 \end{cases}$$

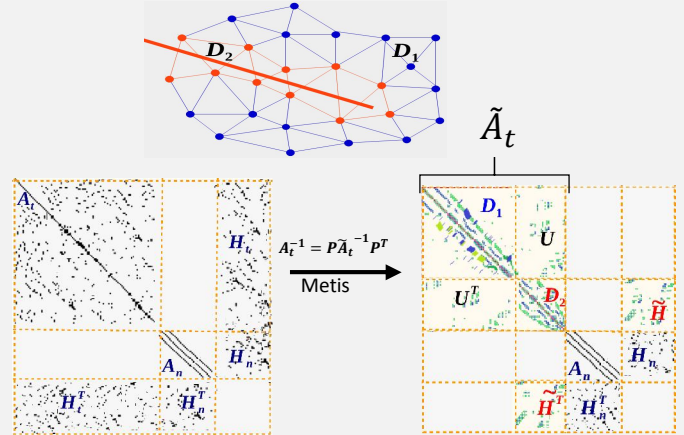
where  $x = \Delta \dot{q}$ ,  $H$  is the derivative of the constraints,  $A$  the implicit fem matrix,  $\delta$  the constraint violation and  $W$  is known as the Delassus operator

**Contribution:** Domain Decomposition to compute  $W$  in Real Time

## Domain Decomposition

- $A_n$  is a bloc-tri-diagonal matrix that can be inverted in real time using Thomas algorithm.[2]

- $A_t$  is a large matrix
  - The trajectory of the needle is known before the simulation
  - The LM are applied on few degrees of freedom



- Using Sherman Morrison formula

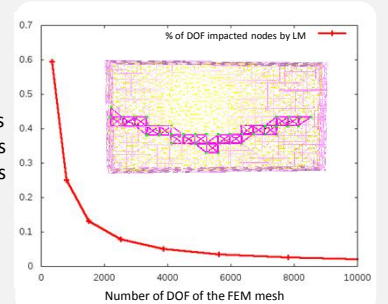
$$H_t^T A_t^{-1} H_t = H_t^T P (D_2 - U^T D_1^{-1} U)^{-1} P^T H_t$$

- $U^T D_1^{-1} U$  is precomputed (far from the trajectory)
- $D_2 - U^T D_1^{-1} U$  is inverted at each time step using blas library

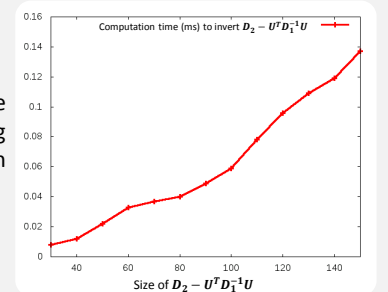
## Result

- Few DOF are impacted by the LM along the planned trajectory:

87 DOF for 1512 nodes meshes  
105 DOF for 3872 nodes meshes  
130 DOF for 7800 nodes meshes



- $D_2 - U^T D_1^{-1} U$  can be inverted in real-time using blas library for any mesh resolution



[1] Michael Hauth and Wolfgang Strasser "Corotational Simulation of deformable Model" Sand 14, D-72076 Tübingen, Germany.

[2] C. Duriez, C. Guebert, M. Marchal and S. Cotin, L. Grisoni "Interactive Simulation of Flexible Needle Insertions Based on Constraint Models. In Medical Image Computing and Computer-Assisted Intervention" In Medical Image Computing and Computer-Assisted Intervention.