Domain Decomposition for real time Simulation of needle insertion

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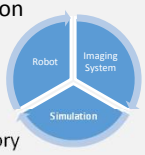
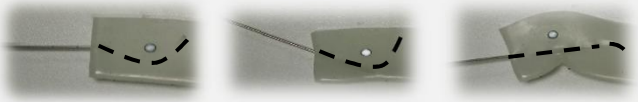
Context

- Real time simulation for needle trajectory optimization during robotic insertions in deformable objects:
  1. Planned trajectory
  2. Deformation due to insertion
  3. Camera capture the surface of the gel
  4. FE Simulation to compute the deformed trajectory





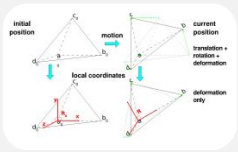
Simulation

Deformable Model

Corotational Model [1]:

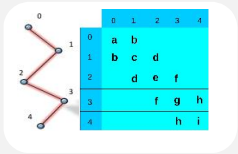
- Linear relation between stress tensor σ and deformation tensor ϵ (Hooke's law)
- Geometrical non linarites filtering:

$$\boldsymbol{\sigma = RCR^T\epsilon}$$



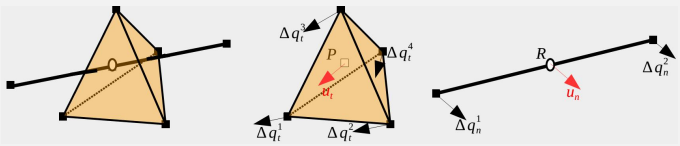
Beam Model :

- FEM corotational beam model
- Elasticity and bending stiffness
- Topology is a sequence of segments
- Stiffness matrix is a block-tri-diagonal



Interaction Model:

- Constraints are imposed using Lagrangian Multipliers (LM) [2]
- Re-meshing is avoided



- Principle of virtual works: the displacement of a virtual point inside a tetrahedral element is given as a linear relation **J** of the degrees of freedom **q**:

$$\boldsymbol{u_t = J_t \Delta q_t} \qquad \boldsymbol{u_n = J_n \Delta q_n}$$

where n denotes the needle and t the deformable object.

Time integration

The dynamic equation of simulated bodies is given by:

$$\boldsymbol{M\ddot{q} = P - F(q,\dot{q}) + R(q,\lambda)}$$

where $\boldsymbol{M}$ is the inertia matrix, $\boldsymbol{F}$ the internal forces, $\boldsymbol{P}$ the external forces and $\boldsymbol{R}$ the constraint forces given by LM $\boldsymbol{\lambda}$.

Implicit time integration is used with backward Euler scheme:

- Stability, accuracy, interaction between models
- First order Taylor expression

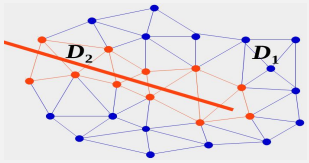
$$\begin{cases} \boldsymbol{A_t x_t} & \boldsymbol{= b_t + H_t^T \lambda} \\ \boldsymbol{A_n x_n} & \boldsymbol{= b_n + H_n^T \lambda} \\ \boldsymbol{\delta = H_n^T x_n + H_t^T x_t} \end{cases} \rightarrow \begin{cases} \boldsymbol{x_t} & \boldsymbol{= A_t^{-1}(b_t + H_t^T \lambda)} \\ \boldsymbol{x_n} & \boldsymbol{= A_n^{-1}(b_n + H_n^T \lambda)} \\ \boldsymbol{\delta = \underbrace{\left[H_n A_n^{-1} H_n^T + H_t A_t^{-1} H_t^T\right]}_{W} \lambda + \delta_0} \end{cases}$$

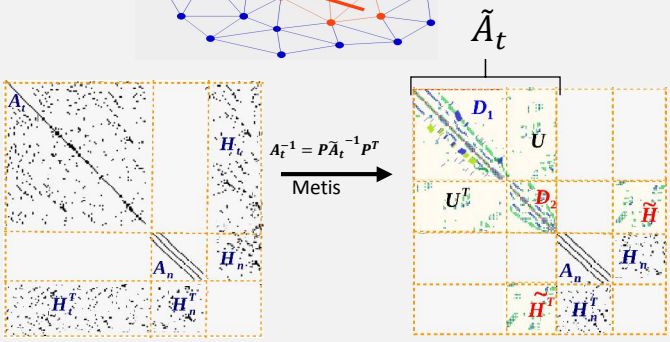
where $\boldsymbol{x}$= $\Delta\boldsymbol{\dot{q}}$, $\boldsymbol{H}$ is the derivative of the constraints, **A the** implicit fem matrix, $\boldsymbol{\delta}$ the constraint violation and $\boldsymbol{W}$ is known as the *Delasus* operator

Contribution: Domain Decomposition to compute W in Real Time

Domain Decomposition

- $\boldsymbol{A_n}$ is a bloc-tri-diagonal matrix that can be inverted in real time using Thomas algorithm.[2]
- $\boldsymbol{A_t}$ is a large matrix
  - The trajectory of the needle is known before the simulation
  - The LM are applied on few degrees of freedom





- Using Sherman Morrison formula

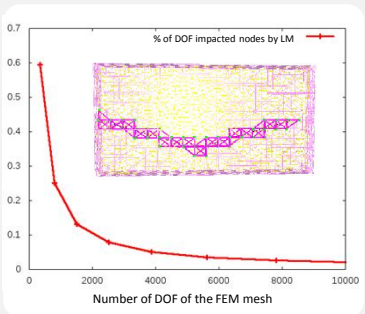
$$\boldsymbol{H_t^T A_t^{-1} H_t = H_t^T P\left(D_2 - U^T D_1^{-1} U\right)^{-1} P^T H_t}$$

- $\boldsymbol{U^T D_1^{-1} U}$ is precomputed (far from the trajectory)
- $\boldsymbol{D_2 - U^T D_1^{-1} U}$ is inverted at each time step using blas library

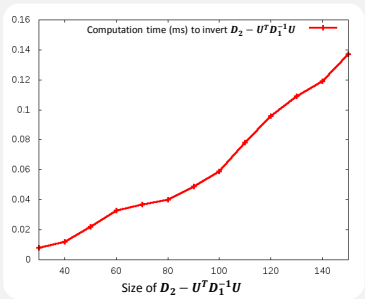
Result

- Few DOF are impacted by the LM along the planed trajectory:

87 DOF for 1512 nodes meshes
105 DOF for 3872 nodes meshes
130 DOF for 7800 nodes meshes



- $\boldsymbol{D_2 - U^T D_1^{-1} U}$ can be inverted in real-time using blas library for any mesh resolution



[1] Michael Hauth and Wolfgang Strasser "*Corotational Simulation of deformable Model*" Sand 14, D-72076 Tübingen, Germany.

[2] C. Duriez , C. Guebert, M. Marchal and S. Cotin ,L. Grisoni "*Interactive Simulation of Flexible Needle Insertions Based on Constraint Models. In Medical Image Computing and Computer-Assisted Intervention*" In Medical Image Computing and Computer-Assisted Intervention.