Stochastic Hybrid System Actuator Fault Diagnosis by Adaptive Estimation

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Abstract: Based on the interacting multiple model (IMM) estimator for hybrid system state estimation and on the adaptive Kalman filter for time varying system joint state-parameter estimation, a new algorithm, the *adaptive IMM estimator*, is proposed in this paper for actuator fault diagnosis in stochastic hybrid systems. The working modes of the considered hybrid systems are described by stochastic state-space models, and the mode transitions are characterized by a Markov model. Actuator faults are modeled as parameter changes, and the related fault diagnosis problem is solved by the proposed adaptive IMM estimator through joint state-parameter estimation. Numerical examples are presented to illustrate the performance of the proposed method.

Keywords: Fault diagnosis, hybrid system, random uncertainty, actuator fault, joint state-parameter estimation.

1. INTRODUCTION

In order to improve the safety, the reliability and the performance of complex industrial systems, the topic of fault diagnosis is attracting more and more researchers during the last decades (Basseville and Nikiforov (1993); Gertler (1998); Chen and Patton (1999); Ding (2008); Isermann (2006); Korbicz (2004); Blanke et al. (2003); Simani et al. (2003)). Most of these studies are model-based, assuming that the considered systems are described by differential equations or by their discrete time counterpart. However, complex industrial systems may have behaviors that cannot be described by a single set of differential equations, because of working mode changes. Typically, each of the working modes is modeled by a different set of differential equations. Such systems are known as *hybrid systems*. It is thus necessary to develop methods for hybrid system fault diagnosis in order to cope with complex industrial systems.

In general a hybrid system involves both continuous dynamics and discrete events that may be combined in a more or less complex manner (Blom and Lygeros (2006); Lunze and Lamnabhi-Lagarrigue (2009)). In this paper it is assumed that each considered hybrid system has a finite number of working modes described by discrete time stochastic state-space models subject to parameter changes, and that at every time instant one of the working modes is active. If the active mode is known all the time, then the hybrid system can be treated as a (discontinuous) time varying system, and in this case some fault diagnosis methods designed for time varying systems can be applied (Chung and Speyer (1998); Chen and Speyer (2000); Chen et al. (2003); Zhong et al. (2010); Zhang and Basseville (2014)). It is more challenging to study hybrid systems whose active mode is unknown. For hybrid systems with a deterministic mode switching mechanism, fault diagnosis has been studied in (Bemporad et al. (1999)) through the mixed logic dynamic formalism (see also Ferrari-Trecate et al. (2002)), in (Belkhiat et al. (2011)) with state observers, and in (Wang et al. (2013)) with a bond graph-based approach. When the deterministic mode switching mechanism can be determined from observations within a finite time, a hybrid observer-based method has been proposed in (Wang et al. (2007)). Stochastically switching hybrid systems are considered in (Cinquemani et al. (2004)), with each monitored fault modeled as one of the modes of the hybrid system. More generally, particle filters can be applied to stochastic hybrid systems for fault diagnosis (Guo et al. (2013)), but such solutions are numerically expensive.

For the hybrid systems considered in this paper, each mode is described by a state-space model subject to Gaussian noises, and the mode switching mechanism is characterized by a Markov model. Such systems exhibit stochastic behaviors both in each mode and during mode transitions. In this framework, actuator faults are modeled as parameter changes, typically the gain losses of actuators. The magnitude of each parameter change, when it occurs, is an *unknown* real value, possibly belonging to some bounded interval. It is thus impossible to model such faults as modes of a hybrid system, as each fault would correspond to a particular value of the parameter change magnitude and infinitely many modes would have to be considered.

In the *fault-free case*, the hybrid systems considered in this paper have been well studied for the problem of *state estimation* (Blom and Bar-Shalom (1988); Bar-Shalom et al. (2001); Hwang et al. (2006)), notably with the well-known Interacting Multiple Model (IMM) estimator (Bar-Shalom et al. (2001)). Such results cannot be directly applied to the systems considered in this paper for state estimation, because of unknown parameter changes. On the other

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hand, for time varying state-space systems, efficient joint state-parameter estimation methods exist, for instance the *adaptive Kalman filter*, also known as adaptive observer for continuous time systems (Zhang (2002); Li et al. (2011)). The *main idea* of the present paper is to combine the IMM estimator and the adaptive Kalman filter, in order to design an *adaptive IMM estimator* for joint state-parameter estimation of hybrid systems. Then the resulting algorithm can be directly applied to the actuator fault diagnosis problem considered in this paper.

2. PROBLEM STATEMENT

The stochastic hybrid system considered in this paper is modeled at two levels. At the top level, the system has a finite number of working modes. At each time instant, one of the modes is active, and random transitions between different modes are characterized by a Markov model. At the bottom level, each mode of the system is described by a stochastic linear state-space model subject to actuator faults formulated as parameter changes.

2.1 Markov transition model

Assume that a hybrid system has r working *modes*, labeled by $M_1, M_2, \ldots, M_r$. At the initial time instant $k = 0$, the *prior probability* that (the mode labeled by [1]) M_j is active is

$$P\{M_j\} = \mu_j(0) \tag{1}$$

with known probabilities $\mu_1(0), \mu_2(0), \ldots, \mu_r(0)$ satisfying

$$\sum_{j=1}^{r} \mu_j(0) = 1.$$

The mode switching mechanism is characterized by a Markov process with the transition probabilities

$$P\{M_j(k)|M_i(k-1)\} = p_{i,j}. \tag{2}$$

where k is the *discrete time instant*, and $p_{i,j}$ are known transition probabilities independent of k and satisfying

$$\sum_{j=1}^{r} p_{i,j} = 1 \quad \text{for} \quad i = 1, 2, \ldots, r.$$

2.2 Stochastic state-space mode model

In each of the possible r modes, say M_j, the considered hybrid system is described by the linear state-space model

$$\begin{aligned} x(k) &= A(M_j)x(k-1) + B(M_j)u(k-1) + w(k) \\ &\quad + \Phi(k-1; M_j)\theta \qquad (3a) \\ y(k) &= C(M_j)x(k) + v(k) \qquad (3b) \end{aligned}$$

where $x(k) \in \mathbb{R}^n$ is the state, $u(k) \in \mathbb{R}^l$ the input, $y(k) \in \mathbb{R}^m$ the output, $A(M_j), B(M_j), C(M_j)$ are mode-dependent matrices of appropriate sizes, $w(k) \in \mathbb{R}^n, v(k) \in \mathbb{R}^m$ are mutually independent white Gaussian noises of covariance matrices $Q(M_j) \in \mathbb{R}^{n\times n}$ and $R(M_j) \in \mathbb{R}^{m\times m}$ respectively, and the term $\Phi(k; M_j)\theta$ represents actuator faults with a known matrix sequence $\Phi(k; M_j) \in \mathbb{R}^{n\times p}$ and a constant (or piecewise constant) vector $\theta \in \mathbb{R}^p$. At the initial time instant $k = 0$, the initial state $x(0)$, under the assumption of each possible mode M_j, is assumed to be a Gaussian random vector

$$x(0) \sim \mathcal{N}(\hat{x}^j(0|0), P^j(0|0)). \tag{4}$$

A typical example of actuator faults represented by $\Phi(k; M_j)\theta$ is in the case where θ corresponds to actuator gain loss coefficients. Assume that each of the l actuators, say the one corresponding to the q-th component of the vector $u(k)$, is affected by a gain loss represented by a coefficient $(1 - \theta_q)$, where θ_q is the q-th component of θ. Then the input term changes from its nominal form $B(M_j)u(k)$ to the faulty form

$$B(M_j)(I_l - \text{diag}(\theta))u(k) = B(M_j)u(k) - B(M_j)\text{diag}(u(k))\theta$$

where I_l is the $l \times l$ identity matrix. In this case, $p = l$ and

$$\Phi(k; M_j) = -B(M_j)\text{diag}(u(k)). \tag{5}$$

2.3 Actuator fault diagnosis

The problem of actuator fault diagnosis considered in this paper is to characterize actuator parameter changes (typically gain loss coefficients associated with $\Phi(k; M_j)$ as expressed in (5)) by estimating them from the mode prior probabilities $\mu_j(0)$, the mode transition probabilities $p_{i,j}$, the mode-dependent matrices $A(M_j), B(M_j), C(M_j), Q(M_j), R(M_j), \Phi_k(M_j)$, and the input-output data sequences $u(k), y(k)$.

In this considered framework, the actual active mode M_j at each time instant k is *unknown*. This is the main cause of difficulty for stochastic hybrid system fault diagnosis compared to the case of single mode (non hybrid) systems. As a matter of fact, if the active mode was known at each time instant k, then the considered hybrid system would be equivalent to a *time varying* state-space system, for which the actuator fault diagnosis problem, similar to the one formulated in this paper, has already been studied. See, for instance, (Zhang and Basseville (2014)).

3. INTERACTING MULTIPLE MODEL ESTIMATOR FOR STATE ESTIMATION

In this section, one of the two basic elements for developing the proposed fault diagnosis method, the well-known *Interacting Multiple Model* (IMM) estimator for hybrid system state estimation, is shortly recalled.

The problem of state estimation, for the considered *fault-free* ($\theta = 0$ in (3)) stochastic hybrid system, is to characterize the probability distribution of the state vector $x(k)$, from the mode prior probabilities $\mu_j(0)$, the mode transition probabilities $p_{i,j}$, the mode-dependent matrices $A(M_j), B(M_j), C(M_j), Q(M_j), R(M_j)$, and the input-output data sequences $u(k), y(k)$.

3.1 Optimal multiple model estimator and simplifications

In general, mode transitions can happen at every time instant k. To ease the introduction of multiple model estimators, let us first consider a much simpler case: the considered system always remains in one of the r possible modes, but the actual mode is unknown. This is not really a hybrid system and will be referred to as a *static unknown*

[1] For shorter statements, "the mode labeled by M_j" is often simply written as "M_j" in this paper.

mode system. In this case, as the actual mode is unknown, we have to try each of them. Under the assumption of each possible mode, say M_j, the system is characterized by the faut-free ($\theta = 0$) stochastic state-space model (3), to which the Kalman filter can be applied for stat estimation. Therefore, at every time instant, r Kalman filters are run in parallel, each assuming a particular working mode. Each of these Kalman filters provides a state estimation. The overall state estimation can be made from a weighted average of the r state estimates with weighting coefficients equal to the posterior probabilities of the r possible modes given input-output observations. This algorithm is known as the *static multiple model estimator* (Bar-Shalom et al. (2001)).

Now let us consider true hybrid systems with mode transitions that may happen at each time instant k. In this case, in principle it is no longer sufficient to run r Kalman filters. As mode transitions can happen at every time instant, *all the possible mode sequences* up to the current instant k should be considered. Within the instants from 1 to k, there are r^k different possible mode sequences. In principle, for the *optimal dynamic multiple model estimator*, r^k Kalman filters should be run in parallel, each corresponding to one of the possible mode sequences. Following this approach, *the number of Kalman filters increases exponentially* with k. In practice it is not reasonable to implement such solutions, it is then necessary to make simplifications, leading to heuristic solutions, notably the IMM estimator.

3.2 The IMM estimator

At each time instant $k = 1, 2, 3, \ldots$, the IMM estimator performs one iteration composed of the same computation steps based on the input-output data, on the mode-dependent system matrices, and for $k = 1$ on the initialization data *or* for $k > 1$ on the results of the last iteration.

The computation steps at iteration k are as follows.

- Compute the mixing probabilities for mixing the state estimates and covariances of the last iteration.
- Compute the mixed state estimates and covariance matrices from the last iteration of the r parallel Kalman filters weighted by the mixing probabilities.
- For each of the r assumed active modes at instant k, perform a Kalman filter iteration delivering a state estimate and a covariance matrix.
- Update mode probabilities from the likelihoods of the r Kalman filters.
- Deliver the algorithm output of the current iteration, by averaging the estimates of the r Kalman filters weighted with the updated mode probabilities.

See (Bar-Shalom et al. (2001)) for more details about the IMM estimator.

4. TIME VARYING SYSTEM ADAPTIVE KALMAN FILTER

In this section, the second basic element for developing the proposed fault diagnosis method, the *adaptive Kalman filter*, is shortly introduced.

Consider time varying state-space systems in the form of

$$\begin{aligned} x(k) &= A(k)x(k-1) + B(k)u(k-1) + w(k) \\ &\quad + \Phi(k-1)\theta && \text{(6a)} \\ y(k) &= C(k)x(k) + v(k). && \text{(6b)} \end{aligned}$$

Here it is assumed that the *time varying* system matrices $A(k), B(k), C(k), \Phi(k-1)$ and noise covariance matrices $Q(k), R(k)$ are known at every time instant k. The stochastic hybrid systems formulated in Section 2 are also state-space systems with system and covariance matrices evolving in time, but the evolution characterized by a Markov model is *not* known exactly at each time instant.

It is also assumed for system (6) that the initial state $x(0)$ is a random vector following the Gaussian distribution $\mathcal{N}(\hat{x}(0|0), P(0|0))$, and an initial guess of the unknown parameter vector θ, namely $\hat{\theta}(0)$, is given.

The adaptive Kalman filter is designed for joint estimation of the state vector $x(k)$ and the parameter vector θ of system (6). If the parameter vector $\theta \in \mathbb{R}^p$ was known in (6), then the basic Kalman filter would be applicable to its state estimation. In order to perform joint state-parameter estimation, the adaptive Kalman filter presented below incorporates a parameter estimation mechanism, complimenting the part originating from the basic Kalman filter.In addition to the variables originating from the basic Kalman filter, the new algorithm involves new matrices $\Upsilon(k) \in \mathbb{R}^{n\times p}, \Omega(k) \in \mathbb{R}^{m\times p}, S(k) \in \mathbb{R}^{p\times p}, \Gamma(k) \in \mathbb{R}^{p\times m}$ and a forgetting factor $\lambda \in (0, 1)$. In particular, the recursively computed $\Upsilon(k)$ and $S(k)$ need initializations $\Upsilon(0) = 0$ (the $n \times p$ zero matrix) and $S(0) = \alpha I_p$ with some $\alpha > 0$.

At each time instant k, the adaptive Kalman filter performs the following computations.

$$\begin{aligned}
P(k|k-1) &= A(k)P(k-1|k-1)A^T(k) + Q(k) && \text{(7a)} \\
\Sigma(k) &= C(k)P(k|k-1)C^T(k) + R(k) && \text{(7b)} \\
K(k) &= P(k|k-1)C^T(k)\left[\Sigma(k)\right]^{-1} && \text{(7c)} \\
P(k|k) &= (I_n - K(k)C(k))P(k|k-1) && \text{(7d)} \\
\Upsilon(k) &= (I_n - K(k)C(k))A(k)\Upsilon(k-1) \\
&\quad + (I_n - K(k)C(k))\Phi(k-1) && \text{(7e)} \\
\Omega(k) &= C(k)A(k)\Upsilon(k-1) + C(k)\Phi(k-1) && \text{(7f)} \\
S(k) &= \frac{1}{\lambda}S(k-1) - \frac{1}{\lambda}S(k-1)\Omega^T(k)\big(\lambda\Sigma(k) \\
&\quad + \Omega(k)S(k-1)\Omega^T(k)\big)^{-1}\Omega(k)S(k-1) && \text{(7g)} \\
\Gamma(k) &= S(k)\Omega^T(k)\left(\lambda\Sigma(k) + \Omega(k)S(k)\Omega^T(k)\right)^{-1} && \text{(7h)} \\
\tilde{y}(k) &= y(k) - C(k)\big(A(k)\hat{x}(k-1|k-1) \\
&\quad - B(k)u(k-1) - \Phi(k-1)\hat{\theta}(k-1)\big) && \text{(7i)} \\
\hat{\theta}(k) &= \hat{\theta}(k-1) + \Gamma(k)\tilde{y}(k) && \text{(7j)} \\
\hat{x}(k|k) &= A(k)\hat{x}(k-1|k-1) + B(k)u(k-1) \\
&\quad + \Phi(k-1)\hat{\theta}(k-1) + K(k)\tilde{y}(k) \\
&\quad + \Upsilon(k)(\hat{\theta}(k) - \hat{\theta}(k-1)). && \text{(7k)}
\end{aligned}$$

Similar algorithms corresponding to the continuous-time counterpart of this adaptive Kalman filter are more frequently studied (Zhang (2002); Li et al. (2011)). Because of the noises in the state and output equations, the state and parameter estimation errors do not converge to zero when k tends to infinity. If the noises are ignored, the estimation errors converge exponentially to zero (Guyader and Zhang (2003)). In the presence of the noises, by taking mathematical expectations of all the terms in the error equations, the mathematical expectations of the estimation errors converge exponentially to zero.

5. ADAPTIVE IMM ESTIMATOR FOR HYBRID SYSTEM ACTUATOR FAULT DIAGNOSIS

Now let us go back to the problem of hybrid system actuator fault diagnosis formulated in Section 2 through equations (1)-(4).

If in (3) the mode-dependent matrices $A(M_j), B(M_j), C(M_j), Q(M_j), R(M_j), \Phi(k; M_j)$ were known at each time instant (in other words, *if the active mode was known* at every time instant), then the system would fit into the framework of time varying systems formulated in (6), and it would be possible to directly apply the adaptive Kalman filter. Of course, in the presently considered case of stochastic hybrid systems, the active mode is unknown. Like in the case of optimal state estimator discussed in Section 3.1 for fault-free hybrid systems, in principle it is possible to try *all the possible mode sequences* up to the current instant k and to combine somehow all the resulting state and parameter estimates. Again the number of possible mode sequences (r^k) increases exponentially with time.

Let us follow the same ideas as in the basic IMM estimator to avoid the exponentially increasing complexity of the optimal estimator. At each time instant, instead of considering all the possible past mode sequences, summarize the past estimates with weighted averages by using *mixing probabilities* $\mu_{i|j}(k-1|k-1)$, and run r parallel adaptive Kalman filters, each assuming a different currently active mode M_j. The state and parameter estimates of the hybrid system are then obtained by taking the weighted average of the r estimates delivered by the r adaptive Kalman filters, with weights equal to the posterior probabilities of the corresponding modes.

At each instant k, the *adaptive IMM estimator* for joint state-parameter estimation consists of the following steps.

Calculation of mixing probabilities

The mixing probabilities for mixing results of the last iteration (of instant $k-1$) are computed as

$$\mu_{i|j}(k-1|k-1) = \frac{1}{\bar{c}_j} p_{i,j} \mu_j(k-1) \qquad (8)$$

where $\mu_j(0)$ (for instant $k=1$) are mode prior probabilities, $\mu_j(k-1)$ (for $k>1$) are mode probabilities updated at instant $k-1$ (see the *Mode probability update* step below), and

$$\bar{c}_j = \sum_{i=1}^{r} p_{i,j} \mu_i(k-1).$$

Intermediate results mixing

During the last iteration (at instant $k-1$), r adaptive Kalman filters were run in parallel, each assuming a different active mode at instant $k-1$, yielding state estimates $\hat{x}^i(k-1|k-1)$, parameter estimates $\hat{\theta}^i(k-1)$, the matrices $P^i(k-1|k-1)$, $S^i(k-1)$, $\Upsilon^i(k-1)$, all indexed by the corresponding assumed mode M_i.

The mixed quantities are then computed as

$$\begin{aligned}
\hat{\hat{x}}^j(k-1|k-1) &= \sum_{i=1}^{r} \hat{x}^i(k-1|k-1)\mu_{i|j}(k-1|k-1) \\
\hat{\hat{\theta}}^j(k-1) &= \sum_{i=1}^{r} \hat{\theta}^i(k-1)\mu_{i|j}(k-1|k-1) \\
\hat{\hat{S}}^j(k-1) &= \sum_{i=1}^{r} S^i(k-1)\mu_{i|j}(k-1|k-1) \\
\hat{\hat{\Upsilon}}^j(k-1) &= \sum_{i=1}^{r} \Upsilon^i(k-1)\mu_{i|j}(k-1|k-1) \\
\hat{\hat{P}}^j(k-1|k-1) &= \sum_{i=1}^{r} \mu_{i|j}(k-1|k-1)\{P^i(k-1|k-1) \\
&\quad + [\hat{x}^i(k-1|k-1) - \hat{\hat{x}}^j(k-1|k-1)] \\
&\quad \cdot [\hat{x}^i(k-1|k-1) - \hat{\hat{x}}^j(k-1|k-1)]^T\}.
\end{aligned}$$

Mode-matched filtering

For each of the possible modes M_j at instant k, an adaptive Kalman filter is implemented as follows.

$$\begin{aligned}
P^j(k|k-1) &= A(M_j)\hat{\hat{P}}^j(k-1|k-1)A^T(M_j) + Q(M_j) \\
\Sigma^j(k) &= C(M_j)P^j(k|k-1)C^T(M_j) + R(M_j) \\
K^j(k) &= P^j(k|k-1)C^T(M_j)\left[\Sigma^j(k)\right]^{-1} \\
P^j(k|k) &= (I_n - K^j(k)C(M_j))P^j(k|k-1) \\
\Upsilon^j(k) &= (I_n - K^j(k)C(M_j))A(M_j)\hat{\hat{\Upsilon}}^j(k-1) \\
&\quad + (I_n - K^j(k)C(M_j))\Phi(k-1; M_j) \\
\Omega^j(k) &= C(M_j)A(M_j)\hat{\hat{\Upsilon}}^j(k-1) \\
&\quad + C(M_j)\Phi(k-1; M_j) \\
S^j(k) &= \frac{1}{\lambda}\hat{\hat{S}}^j(k-1) - \frac{1}{\lambda}\hat{\hat{S}}^j(k-1)[\Omega^j(k)]^T \\
&\quad \cdot \left(\lambda\Sigma^j(k) + \Omega^j(k)\hat{\hat{S}}^j(k-1)[\Omega^j(k)]^T\right)^{-1} \\
&\quad \cdot \Omega^j(k)\hat{\hat{S}}^j(k-1) \\
\Gamma^j(k) &= S^j(k)[\Omega^j(k)]^T\left(\lambda\Sigma^j(k)\right. \\
&\quad \left. + \Omega^j(k)S^j(k)[\Omega^j(k)]^T\right)^{-1} \\
\tilde{y}^j(k) &= y(k) - C(M_j)\big(A(M_j)\hat{\hat{x}}^j(k-1|k-1) \\
&\quad - B(M_j)u(k-1) - \Phi(k-1; M_j)\hat{\hat{\theta}}^j(k-1)\big) \\
\hat{\theta}^j(k) &= \hat{\hat{\theta}}^j(k-1) + \Gamma^j(k)\tilde{y}^j(k) \\
\hat{x}^j(k|k) &= A(M_j)\hat{\hat{x}}^j(k-1|k-1) + B(M_j)u(k-1) \\
&\quad + \Phi(k-1; M_j)\hat{\hat{\theta}}^j(k-1) + K^j(k)\tilde{y}^j(k) \\
&\quad + \Upsilon^j(k)(\hat{\theta}^j(k) - \hat{\hat{\theta}}^j(k-1)).
\end{aligned}$$

The likelihood of the mode M_j, given the input-output data up to instant k, is evaluated through the innovation $\tilde{y}^j(k)$

$$\Lambda_j(k) = \frac{1}{\sqrt{(2\pi)^m \det(\Sigma^j(k))}} \cdot \exp\left(-\frac{1}{2}(\tilde{y}^j(k))^T(\Sigma^j(k))^{-1}\tilde{y}^j(k)\right).$$

Mode probability update

The mode probabilities are updated as

$$\mu_j(k) = \frac{1}{c}\Lambda_j(k)\bar{c}_j$$

with

$$\bar{c}_j = \sum_{i=1}^{r} p_{i,j}\mu_i(k-1)$$
$$c = \sum_{j=1}^{r} \Lambda_j(k)\bar{c}_j.$$

Algorithm outputs

The outputs of the adaptive IMM estimator at instant k are the state estimate

$$\hat{x}(k|k) = \sum_{j=1}^{r} \hat{x}^j(k|k)\mu_j(k)$$

and the parameter estimate

$$\hat{\theta}(k|k) = \sum_{j=1}^{r} \hat{\theta}^j(k|k)\mu_j(k).$$

Like the basic IMM estimator, this adaptive algorithm is also based on heuristic simplifications of the optimal estimator to avoid the exponentially increasing complexity.

6. NUMERICAL EXAMPLE

In order to illustrate the proposed solution for actuator fault diagnosis in hybrid systems, let us consider a system with 4 modes ($r = 4$). Each mode is described by a third order state-space model ($n = 3$) subject to white Gaussian noises, with one input ($l = 1$) and 2 outputs ($m = 2$). The mode-dependent system matrices $A(M_j), B(M_j), C(M_j)$ are randomly generated such that each mode is stable (the eigenvalues of $A(M_j)$ are inside the unit cycle), observable and controllable. The noise covariances matrices are chosen as $Q(M_j) = 0.1I_3$ and $R(M_j) = 0.05I_2$ for all the 4 modes. The mode transition probabilities $p_{i,j}$ are randomly generated. During each simulation trial, a gain loss of 50%, corresponding to a jump of θ (a scalar parameter, i.e., $p = 1$) from 0 to 0.5 at the time instant $k = 500$ is simulated, and the simulation runs till $k = 1000$. The adaptive IMM estimator proposed in this paper is then applied to the simulated system for joint state-parameter estimation. The result of parameter estimation for one of the trials is presented in Fig. 1, and that of state estimation in Fig. 2.

In these figures, the results obtained with the adaptive Kalman filter (see Section 4) are also presented as a reference for the purpose of comparison. *In practice the adaptive Kalman filter is not applicable*, as its computations require the true mode transition sequence, which

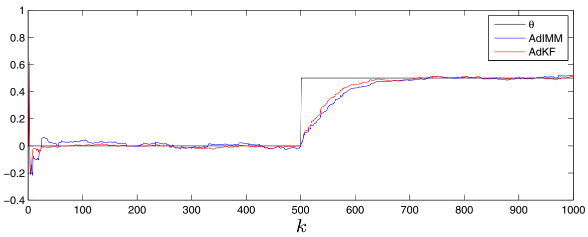

Fig. 1. The simulated "true" parameter (θ, black), parameter estimates by the adaptive IMM estimator (adIMM, blue) and by the adaptive Kalman filter (AdKF, red).

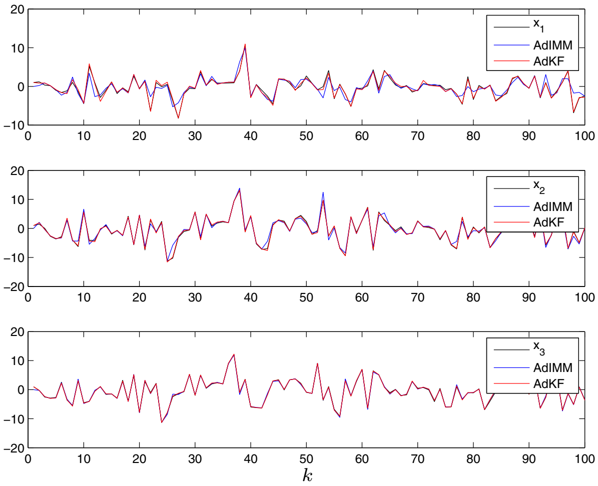

Fig. 2. State estimates by the adaptive IMM estimator (adIMM) and by the adaptive Kalman filter (AdKF), zoomed for $0 \leq k \leq 100$.

is unknown in practice. Because the adaptive IMM estimator uses less information, it cannot perform as well as the adaptive Kalman filter, but yet the results are quite similar.

In order to statistically evaluate the performance of the proposed method, 1000 simulated trials are performed, each corresponding to a different random realization of mode transition probabilities, mode-dependent system matrices, state and output noises. At each time instant k, the histogram of the parameter estimation error based on the 1000 simulated trials is generated, and all the histograms are depicted as a 3D illustration in Fig. 3 for the adaptive IMM estimator, and in Fig, 4 for the adaptive Kalman filter, again for the purpose of comparison. The histograms are normalized so that they are similar to probability density functions.

7. CONCLUSION

Based on the existing basic IMM estimator for hybrid system state estimation and on the adaptive Kalman filter for time varying system joint state-parameter estimation, a new algorithm, the *adaptive IMM estimator* has been proposed in this paper for actuator fault diagnosis in stochastic hybrid systems. In the presented numerical illustrations, the results of the adaptive IMM estimator are quite close to those of the adaptive Kalman filter, which represents a performance upper bound that cannot

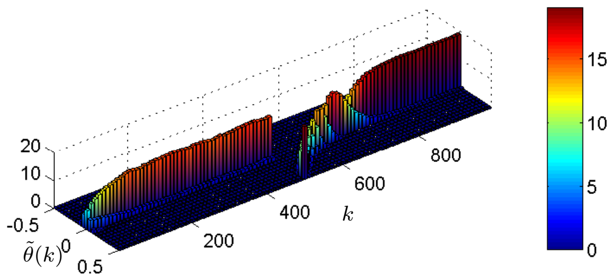

Fig. 3. Histogram per instant k of the parameter estimation error of the adaptive IMM estimator.

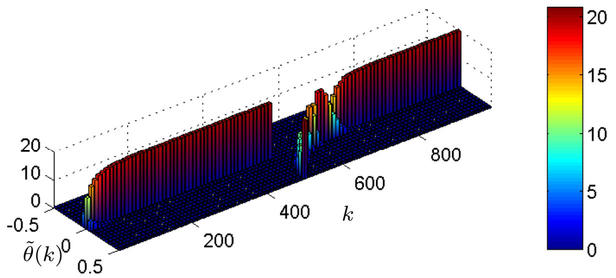

Fig. 4. Histogram per instant k of the parameter estimation error of the adaptive Kalman filter.

be attained in practice. The computational burden of the adaptive IMM estimator is essentially equal to that of the adaptive Kalman filter multiplied by the number of modes.

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