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Michèle Basseville * **Qinghua Zhang** **

* *IRISA/CNRS, Campus de Beaulieu, 35042 Rennes Cedex, France (e-mail: michele.basseville@irisa.fr)*
** *INRIA Rennes Bretagne Atlantique, Campus de Beaulieu, 35042 Rennes Cedex, France (email: qinghua.zhang@inria.fr)*

Abstract: We recently proposed and investigated a statistical approach to fault detection and isolation (FDI) for linear time-varying (LTV) systems subject to parametric additive faults with time-varying profiles, combining a generalized likelihood ratio (GLR) test and minmax tests with a new recursive filter that cancels out the dynamics of the monitored fault effects. In this paper we extend that approach to the case of LTV systems subject to both parametric and non-parametric additive faults. Two solutions for handling such cases are proposed, assuming either constant or slowly varying parametric faults.

Keywords: Linear time-varying systems; additive faults with time-varying profiles; recursive filters; GLR test.

1. INTRODUCTION

Fault detection and isolation (FDI) for industrial systems has been requiring research efforts for technical, economic, and environmental reasons, in particular using model-based methods (Hwang et al. (2010)). Model-based approaches to FDI problems have been mostly studied for linear time invariant (LTI) systems; see e.g. Blanke et al. (2006), Chen and Patton (1999), Ding (2008), Frank (1990), Gertler (1998), Isermann (1997), Isermann (2005), Patton et al. (2000), and references therein. In many applications, however, the time-varying and/or nonlinear properties of the monitored system cannot be neglected. Some studies have been focused on nonlinear system FDI, see e.g. Berdjag et al. (2006), Bokor and Szabó (2009), De Persis and Isidori (2001), Fliess et al. (2004), but these results are often developed under restrictive assumptions.

Another approach to dealing with nonlinear systems uses linearization along the actual or nominal trajectory of the monitored system. In general the working point of a state-space system depends on the state vector and the input. In practice the true state trajectory is often unknown, thus the nonlinear system model is linearized around the nominal state trajectory, except the case of linear parameter varying (LPV) systems mentioned below. This linearization generally results in linear time-varying (LTV) systems, leading to FDI methods for LTV systems, which are more powerful than those for their LTI counterparts usually related to the linearization around a single working point. Finally, nonlinear control systems have been widely studied with the LPV approach; see Bokor and Balas (2004), Lopes dos Santos et al. (2011), Tóth et al. (2011), and the special issues Lovera et al. (2011) and Edwards et al. (2014). In this case, the working point is fully defined by a known scheduling variable, there is no need to linearize around the nominal state trajectory. FDI problems for LTV systems have been addressed using three main approaches known as fault detection filters, observers, and parity relations (Zhang and Basseville (2014), Graton et al. (2014a), Graton et al. (2014b)).

In most FDI methods for LTV systems, additive *non-parametric* faults are considered (Chen and Patton (1996), Chen et al. (2003), Li and Zhou (2009), Varga (2012)). Here the term “non-parametric” means that each fault is assumed to be an *arbitrary unknown* function of time, unlike *parametric faults* characterized by (rare) changes in a parameter vector, like those considered in Zhang and Basseville (2014). It seems that existing FDI methods for LTV systems consider *either* parametric faults *or* non-parametric faults. In this paper we extend the results of Zhang and Basseville (2014) to the case of LTV systems subject to *both* parametric *and* non-parametric additive faults. Two solutions for handling such a case are proposed.

The first solution assumes that the parametric fault vector is (possibly piecewise) constant, and involves an *unknown input* Kalman filter that rejects the non-parametric one. By analyzing the innovation sequence of this *unknown input* Kalman filter, parametric faults are detected and isolated through statistical tests, no matter if the non-parametric faults are present or not. The innovation analysis and the application of statistical tests are similar to those in Zhang and Basseville (2014). To monitor the non-parametric faults, a *standard* Kalman filter ignoring both types of faults is also run in parallel to the *unknown input* Kalman filter. As long as the statistical test based on the unknown input Kalman filter does not detect anything, the innovation of the standard Kalman filter can be used for detecting the onset of the non-parametric fault.

The second solution assumes that the parametric fault vector is slowly time-varying. Based on the analysis of the innovation of the standard Kalman filter designed for the fault-free system and applied to the possibly faulty system, a tracking algorithm is used to follow the slowly varying parametric fault by assuming the absence of the non-parametric fault. If the non-parametric fault occurs, the dysfunction of the tracking algorithm allows its detection.

The paper is organized as follows. The considered FDI problem is stated in Section 2 and the faults signatures are investigated. The first solution is described in Section 3 and the second one in Section 4. Some conclusions and directions for future work are drawn in Section 5.

2. FDI PROBLEM STATEMENT AND FAULT SIGNATURES

In this section we define the considered system type and fault types introducing both parametric and non-parametric faults. Then we address the issue of the computability of the signature of both types of faults on the innovation of a linear filter. Such a fault signature is used throughout the paper for different filter instances.

2.1 LTV system with both non-parametric and parametric faults

The considered fault-free stochastic multiple-input multiple-output (MIMO) LTV systems are of the form

$$\begin{cases} X_{k+1} = F_k\, X_k + G_k\, U_k + W_k \\ Y_k = H_k\, X_k + J_k\, U_k + V_k \end{cases} \tag{1}$$

where X_k is the n-dimensional state vector, U_k is the l-dimensional input, Y_k is the p-dimensional output, F_k, G_k, H_k, J_k are bounded time-varying matrices of appropriate sizes, and W_k and V_k are two independent white Gaussian noise sequences with time-varying covariance matrices Q_k and R_k, respectively. The initial state condition X_0 is assumed to be a Gaussian random vector independent of W_k and V_k, with mean $\widehat{X}_0$ and covariance P_0. The matrix pair (F_k, H_k) is assumed uniformly observable, and the matrix pair $(F_k, Q_k^{1/2})$ is assumed uniformly controllable for ensuring filter stability (Jazwinski (1970)).

The LTV system is supposed to be subject to two types of additive faults, non-parametric faults modeled by $E_k f_k$ and parametric faults represented by $\Psi_k \theta_k$:

$$\begin{cases} X_{k+1} = F_k X_k + G_k U_k + W_k + E_k f_k + \Psi_k \theta_k \\ Y_k = H_k X_k + J_k U_k + V_k \end{cases} \tag{2}$$

where the term $E_k f_k$ represents the non-parametric fault, with the q-dimensional fault profile vector f_k, the $n \times q$ incidence matrix E_k; and the term $\Psi_k \theta_k$ represents the parametric fault, with the m-dimensional (constant or slowly varying) fault vector θ_k, and the $n \times m$ bounded profile matrix Ψ_k. The two time-varying matrices E_k and Ψ_k are assumed known, but f_k and θ_k are unknown.

In (2), $E_k f_k$ and $\Psi_k \theta_k$ typically represent actuator faults. The difference between the two terms lies in the assumptions on f_k and θ_k based on the amount of available *a priori* information. While f_k is assumed to be any *arbitrary* unknown sequence (no *a priori* information at all), θ_k is either assumed constant (then written simply as θ) or slowly varying (in a sense to be defined below). See (Zhang and Basseville (2014)[Sec.2]) for a discussion of particular and practical cases represented by the fault model (2), involving possibly constant or piece-wise constant fault profiles or fault vectors, and (Zhang and Basseville (2014)[Sec.5]) for the case of parametric faults in both state and output equations. As explained in (Basseville (1998)), the case of non additive faults, not considered here, is fully different in nature and more difficult. Finally, note that the modeling framework (2) encompasses multiple faults.

In the two following subsections, we investigate the effect of each of these faults on the innovation of a linear filter. This effect is of course linear, but a number of additional properties of the fault signatures can be outlined.

2.2 Faults signature on the innovation of a linear filter

We now investigate the effects of both the parametric fault $\Psi_k \theta_k$ and the non-parametric fault $E_k f_k$ on the innovation sequence of a linear filter. These computations generalize those of Zhang and Basseville (2014) made for the particular case of a constant fault θ and in the absence of the non-parametric fault $E_k f_k$.

Let us consider a linear filter (state estimator) designed for the fault-free system (1) in the form of

$$\widehat{X}_{k+1} = F_k \widehat{X}_k + G_k U_k + F_k \mathcal{K}_k (Y_k - J_k U_k - H_k \widehat{X}_k) \tag{3}$$

In the special case of the Kalman filter, the notations $\widehat{X}_k$ and P_k defined here correspond to the one-step ahead state prediction and its covariance, or more clearly

$$\widehat{X}_k \triangleq \widehat{X}_{k|k-1}, \quad P_k \triangleq P_{k|k-1}, \tag{4}$$

where the notation P_k (related to the usual notation $P_{k|k-1}$ in the Kalman filter) is also used in this paper. Also in the case of the Kalman filter, where with filter gain $\mathcal{K}_k = K_k$ is known as the Kalman gain the time-varying matrix $F_k(\mathbf{I} - \mathcal{K}_k H_k)$ defines an exponentially stable LTV system, under the assumptions of uniform observability and controllability (Jazwinski (1970)).

Despite the fact that the filter (3) is designed for the fault-free system (1), it is applied to the possibly faulty system (2).

The state prediction error and the output error (the latter is known as the *innovation* in the Kalman filter literature) are defined as:

$$\widetilde{X}_k \triangleq X_k - \widehat{X}_k \tag{5}$$

$$\varepsilon_k \triangleq Y_k - J_k\, U_k - H_k\, \widehat{X}_k. \tag{6}$$

The behavior of these error sequences is analyzed next. Following (2), (3), (5) and (6), it is straightforward to check that $\widetilde{X}_k$ and ε_k satisfy the following recursions:

$$\widetilde{X}_{k+1} = F_k\, (\mathbf{I} - \mathcal{K}_k\, H_k)\, \widetilde{X}_k - F_k\, \mathcal{K}_k\, V_k + W_k + E_k\, f_k + \Psi_k\, \theta_k \tag{7}$$

$$\varepsilon_k = H_k\, \widetilde{X}_k + V_k \tag{8}$$

Let $\Gamma_k \in \mathbb{R}^{n \times m}$ be recursively defined by

$$\Gamma_{k+1} = F_k\, (\mathbf{I} - \mathcal{K}_k\, H_k)\, \Gamma_k + \Psi_k\ , \quad \Gamma_0 = 0. \tag{9}$$

Define the following linear combination of $\widetilde{X}_k$ and θ_k:

$$\eta_k \triangleq \widetilde{X}_k - \Gamma_k\, \theta_k \tag{10}$$

with Γ_k defined in (9). It follows from (7) and (10) that:

$$
\begin{aligned}
\eta_{k+1} \triangleq & \widetilde{X}_{k+1} - \Gamma_{k+1}\ \theta_{k+1} \\
& = F_k\ (\mathbf{I} - \mathcal{K}_k\ H_k)\ (\eta_k + \Gamma_k\ \theta_k) - F_k\ \mathcal{K}_k\ V_k + W_k \\
& \quad +\ \Psi_k\ \theta_k - \Gamma_{k+1}\ \theta_{k+1} \\
& = F_k\ (\mathbf{I} - \mathcal{K}_k\ H_k)\ \eta_k - F_k\ \mathcal{K}_k\ V_k + W_k \\
& \quad +\ E_k\ f_k - \Gamma_{k+1}\ (\theta_{k+1} - \theta_k) \\
& \quad +\ [F_k(\mathbf{I} - \mathcal{K}_k\ H_k)\ \Gamma_k + \Psi_k - \Gamma_{k+1}]\ \theta_k \qquad (11)
\end{aligned}
$$

The bracketed term in (11) vanishes because of the first part of (9). Thus the recursion for η_k becomes:

$$
\begin{aligned}
\eta_{k+1} = & F_k\ (\mathbf{I} - \mathcal{K}_k\ H_k)\ \eta_k - F_k\ \mathcal{K}_k\ V_k + W_k \\
& +\ E_k\ f_k - \Gamma_{k+1}\ (\theta_{k+1} - \theta_k) \qquad (12)
\end{aligned}
$$

On the other hand, $\widetilde{X}_k^0$ and ε_k^0, the state and output prediction errors in the fault-free case, are also governed by equations similar to (7) and (8) with $\theta_k \equiv 0$. Accordingly,

$$
\begin{aligned}
\widetilde{X}_{k+1}^0 &= F_k\ \ (\mathbf{I} - \mathcal{K}_k\ H_k)\ \ \widetilde{X}_k^0 - F_k\ \mathcal{K}_k\ V_k + W_k \qquad (13) \\
\varepsilon_k^0 &= H_k\ \widetilde{X}_k^0 + V_k \qquad (14)
\end{aligned}
$$

To investigate the parametric and non-parametric faults effects on the innovation sequence ε_k (6) through (12), we now distinguish two cases for the parametric fault θ_k.

2.2.1 Constant parametric fault. Let ζ_k be defined by

$$
\zeta_{k+1} = F_k\ (\mathbf{I} - \mathcal{K}_k\ H_k)\ \zeta_k + E_k\ f_k, \quad \zeta_0 = 0. \qquad (15)
$$

In the case of a constant parametric fault vector, namely when $\theta_k = \theta$ for all $k \geq 0$, equation (12) becomes:

$$
\eta_{k+1} = F_k\ (\mathbf{I} - \mathcal{K}_k\ H_k)\ \eta_k - F_k\ \mathcal{K}_k\ V_k + W_k + E_k f_k \quad (16)
$$

By appropriately choosing the initial value $\widehat{X}_0$ so that $\eta_0 = \widetilde{X}_0^0$, (13), (15) and (16) lead to

$$
\eta_k = \widetilde{X}_k^0 + \zeta_k \qquad (17)
$$

for all $k \geq 0$. It then follows from (8), (10) and (17) that:

$$
\begin{aligned}
\varepsilon_k &= H_k\ (\eta_k + \Gamma_k\ \theta) + V_k \\
&= H_k\ \eta_k + V_k + H_k\ \Gamma_k\ \theta \\
&= H_k\ \widetilde{X}_k^0 + V_k + H_k\ \zeta_k + H_k\ \Gamma_k\ \theta \qquad (18)
\end{aligned}
$$

Hence, thanks to (14), we get

$$
\varepsilon_k = \varepsilon_k^0 + H_k\ \Gamma_k\ \theta + H_k\ \zeta_k. \qquad (19)
$$

It is not surprising that the effects on the innovation sequence of the parametric and non-parametric additive faults, namely $\Psi_k\theta$ and $E_k f_k$, through $H_k\Gamma_k\theta$ and $H_k\zeta_k$ respectively, are both additive. Nevertheless, it is worth noticing the difference between the effects of the two types of faults: Γ_k is computed recursively through (9) which is independent of the unknown fault parameter vector θ, but the recursive definition of ζ_k based on (15) does depend on the unknown fault profile f_k. This means that Γ_k can be effectively computed as part of a FDI algorithm (through the stable linear filter (9)), but ζ_k is completely unknown.

Note that, under the assumed uniform observability and controllability conditions, the time-varying matrix $F_k(\mathbf{I} - \mathcal{K}_k H_k)$ defines an exponentially stable LTV system, therefore the recursive definitions of Γ_k and ζ_k are both based on stable linear filters. This stability property implies that, if Ψ_k and f_k are bounded, so are Γ_k and ζ_k.

This is exploited and investigated further in Section 3.

2.2.2 Time-varying parametric fault. We now assume that θ_k is slowly time-varying, namely that:

$$
\theta_{k+1} = \theta_k + e_k \qquad (20)
$$

where e_k, the increment of θ_k, is small in the sense that:

$$
\|e_k\| \leq \delta \qquad (21)
$$

for some small value $\delta > 0$ and for all $k = 1, 2, 3, \ldots$. If a mathematical model of the evolution of e_k was assumed, it would be possible to design an algorithm for tracking θ_k with some convergence property. In practice it is often difficult to build accurate models for parameter evolutions. Here it is simply assumed that $\|e_k\| \leq \delta$ for some small $\delta > 0$. Notice that a bounded e_k does not imply a bounded θ_k, but only limits the evolution speed of θ_k.

Because $e_k \neq 0$ in this case, but $\|e_k\| \leq \delta$ instead, it results from (12), (15), (20) and (13) that:

$$
\eta_k = \widetilde{X}_k^0 + \zeta_k + \delta_k \qquad (22)
$$

where δ_k is recursively defined by

$$
\delta_{k+1} = F_k\ (\mathbf{I} - \mathcal{K}_k\ H_k)\ \delta_k - \Gamma_{k+1}\ e_k, \quad \delta_0 = 0. \qquad (23)
$$

As in the previous case, the matrix gain Γ_{k+1} is bounded, and the time-varying matrix $F_k\ (\mathbf{I} - \mathcal{K}_k\ H_k)$ defines an exponentially stable LTV system. Moreover, it results from (21) and (23) that δ_k is bounded by a bound proportional to δ. It then follows from (8), (10) and (22) that:

$$
\begin{aligned}
\varepsilon_k &= H_k\ (\eta_k + \Gamma_k\ \theta_k) + V_k \\
&= H_k\ \eta_k + V_k + H_k\ \Gamma_k\ \theta_k \\
&= H_k\ \widetilde{X}_k^0 + V_k + H_k\ \Gamma_k\ \theta_k + H_k\ \zeta_k + H_k\ \delta_k \qquad (24)
\end{aligned}
$$

Hence

$$
\varepsilon_k = \varepsilon_k^0 + H_k\ \Gamma_k\ \theta_k + H_k\ \zeta_k + H_k\ \delta_k. \qquad (25)
$$

These results are summarized as follows.

Proposition 1. The effects of the parametric fault $\Psi_k\theta_k$ and the non-parametric fault $E_k f_k$ on the innovation ε_k of a linear filter as formulated in (3) are reflected by the additive terms $H_k\Gamma_k\theta_k + H_k\delta_k$ for the former and $H_k\zeta_k$ for the latter, in addition to the innovation ε_k^0 of the same filter applied to the fault-free system, as expressed in (25). In particular, for a constant parametric fault vector θ_k, the expression in (25) is simplified to that of (19).

This is exploited and investigated further in Sections 3-4.

3. FIRST SOLUTION: REJECTING THE NON-PARAMETRIC FAULT

We now assume that the (parametric) fault vector is (possibly piecewise) constant, and we propose to use an unknown input Kalman filter that rejects the non-parametric fault $E_k f_k$. Based on the results in subsection 2.2, the idea is then to detect the parametric fault with the same GLR test as in Zhang and Basseville (2014) and, as long as that test does not detect anything, to use the innovation of the standard Kalman filter, designed for the fault-free system and run in parallel with the unknown input Kalman filter, for detecting the onset of the non-parametric fault.

3.1 Kitanidis filter for rejecting the non-parametric fault

A filter that ignores the parametric fault $\Psi_k\theta$ (assumes $\theta = 0$) and rejects the non-parametric fault $E_k f_k$ is an unknown input Kalman filter (UI-KF), for instance the Kitanidis unbiased minimum-variance filter (Kitanidis (1987)). The innovation of this UI-KF is affected by the parametric fault $\Psi_k\theta$ as analyzed above, *but not affected by the non-parametric fault $E_k f_k$* rejected by the UI-KF.

Using (4), for the parametric fault-free model:

$$\begin{cases} X_{k+1} = F_k\ X_k\ + G_k\ U_k\ + W_k\ + E_k\ f_k \\ \quad Y_k = H_k\ X_k\ + J_k\ U_k\ + V_k \end{cases}, \tag{26}$$

the Kitanidis filter that rejects $E_k f_k$ writes:

$$\widehat{X}_{k+1} = F_k\widehat{X}_k + G_kU_k + F_kL_k(Y_k - J_kU_k - H_k\widehat{X}_k) \tag{27}$$

$$\begin{aligned} L_k &= K_k + (I - K_k\ H_k)\ E_{k-1}\dots \\ &\quad \dots(E_{k-1}^T H_k^T \Sigma_k^{-1} H_k E_{k-1})^{-1} E_{k-1}^T H_k^T \Sigma_k^{-1} \end{aligned} \tag{28}$$

$$\begin{aligned} K_k &= P_k\ H_k^T\ \Sigma_k^{-1} \\ P_{k+1} &= F_k\ (I - L_k\ H_k)\ P_k\ (I - L_k\ H_k)^T F_k^T \\ &\quad +\ F_k\ L_k\ R_k\ L_k^T\ F_k^T + Q_k \\ \Sigma_k &= H_k\ P_k\ H_k^T + R_k \end{aligned}$$

The state prediction in (27) has the form (3) considered in subsection 2.2, with a gain $\mathcal{K}_k = L_k$ defined in (28). Thus the results in subsection 2.2 concerning the additive effect of the parametric fault on the filter innovation apply.

3.2 Monitoring a constant parametric fault

In case of constant θ, the effect of the parametric fault $\Psi_k\theta$ on the innovation sequence is as in (19), but since the Kitanidis filter rejects the non-parametric fault $E_k f_k$, the last term vanishes ($H_k\zeta_k = 0 \forall k \geq 0$), thus (19) writes:

$$\varepsilon_k = \varepsilon_k^0 + H_k\ \Delta_k\ \theta, \tag{29}$$

where:

$$\Delta_{k+1} = F_k\left(\mathbf{I} - L_k\ H_k\right)\Delta_k + \Psi_k\ ,\quad \Delta_0 = 0 \tag{30}$$

which is a particular case of (9) with $\mathcal{K}_k = L_k$, L_k in (28). The signature of the fault $\Psi_k\theta$ on ε_k is thus the same as in Zhang and Basseville (2014)[Sec.3], up to the replacement of the Kalman gain K_k with the Kitanidis gain L_k in (28).

Now, for being allowed to detect the parametric fault $\Psi_k\theta$ with the GLR test as in Zhang and Basseville (2014)[Sec.4], we first have to show that the innovation sequence $(\varepsilon_k^0)_k$ of the Kitanidis filter is a Gaussian white noise.

Since the noises V_k and W_k in (1) are white, Gaussian and independent, and the considered filter is linear, it results from (13)-(14) that each innovation ε_k^0 is a zero-mean Gaussian distributed vector, provided that $\widetilde{X}_0^0$ is Gaussian and independent of V_k. In the case of the classical Kalman filter applied to the fault-free system (1) (no fault rejection considered), it is also known that the innovation sequence ε_k^0 is a white noise. The white noise property seems unknown in the literature for the Kitanidis filter or other similar filters rejecting faults of the form $E_k f_k$. Because such a result is important for statistical hypothesis testing, let us establish it through the following.

Proposition 2. Assuming the state and output noises W_k and V_k to be mutually independent white Gaussian noises and also independent of the initial state estimate $\widehat{X}_0$, the innovation sequence ε_k^0 of the Kitanidis filter is a zero-mean Gaussian white noise, with covariance $\mathrm{E}[\varepsilon_k^0(\varepsilon_k^0)^T] = H_kP_kH_k^T + R_k$, where P_k is the state estimation error covariance and R_k the output noise covariance at time k.

Proof. The proof of the zero-mean Gaussian distribution of ε_k^0 is trivial, as noted above before Proposition 2. It thus remains to prove the whiteness of ε_k^0. The Kitanidis filter (27) is an unbiased minimum-variance state estimator for system (26) (Kitanidis (1987)). Under the considered linear Gaussian assumptions, the unbiased minimum-variance state estimate $\widehat{X}_k$ is unique and equal to

$$\widehat{X}_k = \mathrm{E}[X_k | Z_0^{k-1}], \tag{31}$$

where $Z_0^k \triangleq \{\widehat{X}_0\} \cup \{(U_j, Y_j)\ :\ j = 0, 1, 2, \dots, k\}$.

Note that, in this proof and for the sake of simplicity, we omit the superscript $\cdot^0$. Remind that

$$\widetilde{X}_k = X_k - \widehat{X}_k = X_k - \mathrm{E}[X_k | Z_0^{k-1}], \tag{32}$$

then, according to Lemma 1 in Appendix A, for all $j = 0, 1, 2, \dots, k-1$,

$$\mathrm{E}[\widehat{X}_0\widetilde{X}_k^T] = 0,\quad \mathrm{E}[Y_j\widetilde{X}_k^T] = 0,\quad \mathrm{E}[U_j\widetilde{X}_k^T] = 0. \tag{33}$$

Some simple computations lead to

$$\mathrm{E}[\varepsilon_j^0(\varepsilon_k^0)^T] = \mathrm{E}\left\{[H_j\widetilde{X}_j + V_j][H_k\widetilde{X}_k + V_k]^T\right\} \tag{34}$$

$$\begin{aligned} &= H_j\mathrm{E}[\widetilde{X}_j\widetilde{X}_k^T]H_k^T + \mathrm{E}[V_jV_k^T] \\ &\quad + \mathrm{E}[V_j\widetilde{X}_k^T]H_k^T + H_j\mathrm{E}[\widetilde{X}_jV_k]. \end{aligned} \tag{35}$$

Let us first consider the case $j < k$.

Because V_k is a white noise, the second term on the right-hand side (RHS) of (35) writes

$$\mathrm{E}[V_jV_k^T] = 0. \tag{36}$$

Yet for $j < k$, it follows from (26) and (27), respectively, that X_j and $\widehat{X}_j$, and thus $\widetilde{X}_j = X_j - \widehat{X}_j$, are independent of V_k. Therefore, the last term on the RHS of (35) writes

$$H_j\mathrm{E}[\widetilde{X}_jV_k] = 0. \tag{37}$$

For $j < k$, $\widehat{X}_j$ computed via (27) is a linear combination of $\widehat{X}_0, U_0, U_1, \dots, U_{j-1}, Y_0, Y_1, \dots, Y_{j-1}$. Thus from (33):

$$\mathrm{E}[\widehat{X}_j\widetilde{X}_k^T] = 0, \tag{38}$$

and consequently

$$\mathrm{E}[\widetilde{X}_j\widetilde{X}_k^T] = \mathrm{E}[(X_j - \widehat{X}_j)\widetilde{X}_k^T] = \mathrm{E}[X_j\widetilde{X}_k^T]. \tag{39}$$

Adding the first and third terms on the RHS of (35) and using (39) leads to

$$\begin{aligned} &H_j\mathrm{E}[\widetilde{X}_j\widetilde{X}_k^T]H_k^T + \mathrm{E}[V_j\widetilde{X}_k^T]H_k^T \\ &= \mathrm{E}[(Y_j - J_jU_j)\widetilde{X}_k^T]H_k^T && (40) \\ &= 0 && (41) \end{aligned}$$

where the last equality is again based on (33).

Then it is concluded that, for $j < k$, the innovations are decorrelated, namely $\varepsilon_j^0(\varepsilon_k^0)^T = 0$. By symmetry the same result is also proved for $j > k$. Therefore, the sequence $(\varepsilon_k^0)_k$ is indeed a white noise. The whiteness property of ε_k^0 justifies its name "innovation", as it carries the new information provided by the current observation Y_k with respect to past observations.

Now let us compute the covariance matrix of ε_k^0:

$$\Sigma_k = \mathrm{E}[\varepsilon_k^0(\varepsilon_k^0)^T] \tag{42}$$

$$= \mathrm{E}\left\{[H_k\widetilde{X}_k + V_k][H_k\widetilde{X}_k + V_k]^T\right\} \tag{43}$$

$$= H_k\mathrm{E}[\widetilde{X}_k\widetilde{X}_k^T]H_k^T + \mathrm{E}[V_kV_k^T] + \mathrm{E}[V_k\widetilde{X}_k^T]H_k^T + H_k\mathrm{E}[\widetilde{X}_kV_k]. \tag{44}$$

Since X_k, $\widehat{X}_k$, and thus $\widetilde{X}_k$, are independent of V_k, the last two terms on the RHS of (44) write $\mathrm{E}[V_k\widetilde{X}_k^T]H_k^T = 0$, $H_k\ \mathrm{E}[\widetilde{X}_kV_k] = 0$. By definition $\mathrm{E}[\widetilde{X}_k\widetilde{X}_k^T] = P_k$ and $\mathrm{E}[V_kV_k^T] = R_k$, therefore

$$\Sigma_k = H_k\ P_k\ H_k^T + R_k. \tag{45}$$

□

Thus the detection and the isolation of the parametric faults $\Psi_k\theta$ can be achieved by applying the GLR and minmax tests to the innovation sequence ε_k of the Kitanidis filter as in (Zhang and Basseville (2014)[Sec.4]), should the non-parametric fault E_kf_k affect the system or not.

3.3 Monitoring the non-parametric fault

As long as the GLR test that monitors the parametric fault $\Psi_k\theta$ does not detect anything, it is possible to monitor the onset of the non-parametric fault E_kf_k based on the innovation of the standard Kalman filter designed for the fault-free model (1) and run in parallel with the Kitanidis filter. This filter writes:

$$\begin{aligned}
\widehat{X}_{k+1} &= F_k\widehat{X}_k + G_kU_k + F_kK_k(Y_k - J_kU_k - H_k\widehat{X}_k)\\
K_k &= P_k\ H_k^T\ \Sigma_k^{-1}\\
P_{k+1} &= F_k(I - K_k\ H_k)P_k\ F_k^T + Q_k\\
\Sigma_k &= H_k\ P_k\ H_k^T + R_k
\end{aligned} \tag{46}$$

A simple test on the energy of the innovation of this Kalman filter allows to deal with the case where $\dim(f_k) \geq \dim(Y_k)$, namely testing a Gaussian white noise against an arbitrary signal. More sophisticated tests might be considered in the case where $\dim(f_k) < \dim(Y_k)$.

4. SECOND SOLUTION: ADAPTING TO THE PARAMETRIC FAULT

We now describe the second proposed solution to the FDI problem stated in Section 2. We assume that the parametric fault vector θ_k is slowly time-varying in a manner described by (20)-(21). Based on the analysis of the innovation of the standard Kalman filter designed for the fault-free system and applied to the possibly faulty system, a tracking algorithm is used to follow the slowly varying parametric fault vector by assuming the absence of the non-parametric fault. If the non-parametric fault does occur, the dysfunction of the tracking algorithm allows its detection.

In 2.2.2 it is shown that the effect of such a time-varying parametric fault on the innovation of a linear filter with gain $\mathcal{K}_k$ is additive and involves three terms (see (25)):

$$\varepsilon_k = \varepsilon_k^0 + H_k\ \Gamma_k\ \theta_k + H_k\ \zeta_k + H_k\ \delta_k \tag{47}$$

with $\Gamma_k, \zeta_k, \delta_k$ defined in (9), (15), (23), respectively.

By setting the filter gain $\mathcal{K}_k$ to the standard Kalman gain K_k, this result shows that ε_k, $H_k\Gamma_k$ and θ_k are related by a linear algebraic equation, up to a white noise term ε_k^0 and a small unknown disturbance $H_k\delta_k$ due to the unknown slowly time-varying behavior of θ_k, and possibly an extra term $H_k\zeta_k$ if the non-parametric fault E_kf_k is affecting the system. Based on this result and the assumption that θ_k is *slowly* varying, classical tracking algorithms can be applied to follow the evolution of θ_k. When the non-parametric fault E_kf_k is affecting the system and implies $H_k\zeta_k \neq 0$ (possibly with large values, unlike $H_k\delta_k$ which is always small), the tracking algorithm is in trouble, thus the dysfunction of the tracking algorithm allows the detection of the non-parametric fault E_kf_k.

Here we propose to use the recursive least squares (RLS) algorithm with a forgetting factor for tracking θ_k (this is one of the possibilities) and to use the resulting error for detecting the onset of the non-parametric fault. More precisely, let $0 < \lambda < 1$ be a forgetting factor. The RLS algorithm for tracking θ_k in (47) writes (Ljung (1999)):

$$\begin{aligned}
\widehat{\theta}_k &= \widehat{\theta}_{k-1} + \mathcal{L}_k\left(\varepsilon_k - H_k\ \Gamma_k\ \widehat{\theta}_{k-1}\right)\\
S_k &= \left(\lambda\ \Sigma_k + H_k\ \Gamma_k\ \mathcal{P}_{k-1}\ \Gamma_k^T\ H_k^T\right)^{-1}\\
\mathcal{L}_k &= \mathcal{P}_{k-1}\ \Gamma_k^T\ H_k^T\ S_k\\
\mathcal{P}_k &= \lambda^{-1}\left[\mathcal{P}_{k-1} - \mathcal{P}_{k-1}\ \Gamma_k^T\ H_k^T\ S_k\ H_k\ \Gamma_k\ \mathcal{P}_{k-1}\right]
\end{aligned}$$

where Σ_k is the innovation covariance in (46), and with the initial conditions $\widehat{\theta}_0 \triangleq 0, \mathcal{P}_0 \triangleq \mathbf{I}$. Consider the error:

$$\mathcal{E}_k \triangleq \varepsilon_k - H_k\ \Gamma_k\ \widehat{\theta}_k$$

When the non-parametric fault E_kf_k is absent, in (47) we have $H_k\zeta_k = 0$ for all $k \geq 0$, and the term $H_k\delta_k$ is considered to be a small disturbance. In this case, the RLS algorithm with an appropriately chosen forgetting factor λ is able to track the slowly varying θ_k. Upon the occurrence of the non-parametric fault, however, we have $H_k\zeta_k \neq 0$, and the RLS algorithm can non longer correctly track θ_k, leading to large error values $\mathcal{E}_k$. Monitoring the energy of $\mathcal{E}_k$ thus allows to detect the onset of the non-parametric fault.

5. CONCLUSION

In this paper, we have addressed the FDI problem for LTV systems subject to both parametric and non-parametric additive faults. We have extended a recent statistical approach that combines a GLR test and minmax tests with a recursive filter that cancels out the dynamics of parametric additive fault effects. Two solutions for handling both parametric non-parametric additive faults have been proposed.

The first solution assumes that the parametric fault vector is (possibly piecewise) constant, and involves a Kitanidis filter that rejects the non-parametric fault. No matter if the non-parametric fault is present or not, the innovation sequence of that filter has been shown to be a white noise and to reflect the parametric faults through changes in its mean vector, allowing for the use of GLR and minmax FDI tests as in Zhang and Basseville (2014), with weaker assumptions than usual on the stability of the monitored system and the number of required sensors. The onset of the non-parametric faults is detected based on the energy of a standard Kalman filter ignoring both types of faults and run in parallel to the Kitanidis filter.

The second solution assumes that the parametric fault vector is slowly time-varying, and involves a RLS algorithm tracking those variations. The onset of the non-parametric faults is detected based on the energy of the tracking error of the RLS filter.

Future investigations include experiments on simulated and real cases to confirm the relevance and assess the performances of the two proposed solutions. The considered FDI problem is limited to additive faults, based on linearization of nonlinear systems.

Appendix A. ON CONDITIONAL EXPECTATIONS

In this appendix, we recall one useful property of conditional expectations. The reader is referred to (Feller (1966)) for the proof.

Lemma 1. Let X and Y be two vectors of real-valued random variables, then for any real-valued function $g(Y)$ such that $\mathrm{E}[g(Y)]$ is finite,

$$\mathrm{E}[(X - \mathrm{E}[X|Y])g(Y)] = 0. \tag{A.1}$$

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