Spatio-Temporal Linear Response of Spiking Neuronal Network Models

UNIVERSITÉ DE GENÈVE
FACULTÉ DES SCIENCES

informatics mathematics
Inria

Rodrigo Cofre*, Bruno Cessac

*rodrigo.cofre@unige.ch

1 Introduction

We study the impact of a weak time-dependent external stimulus on the collective statistics of spiking responses in neuronal networks. We extend the current knowledge, assessing the impact over firing rates and cross correlations, to any higher order spatio-temporal correlation [1]. Our approach is based on Gibbs distributions (in a general setting considering non stationary dynamics and infinite memory) [2] and linear response theory. The linear response is written in terms of a correlation matrix, computed with respect to the spiking dynamics without stimulus. We give an example of application in a conductance based integrate-and fire model.

2 Spiking Neuronal Network Models

We consider models of networks of noisy integrate-and fire neurons:

$$C_k \frac{dV_k}{dt} + g_k(t,\omega)V_k = i_k(t,\omega) \longrightarrow P[\omega(n)|\omega_{\infty}^{n-1}] \longrightarrow \mu$$

$$\log(P[\omega(n)|\omega_{\infty}^{n-1}]) = \phi(\omega_{\infty}^{n})$$

Dependence in the previous spikes

Spike Pattern

Network Spike History

Gibbs measure

Gibbs potential

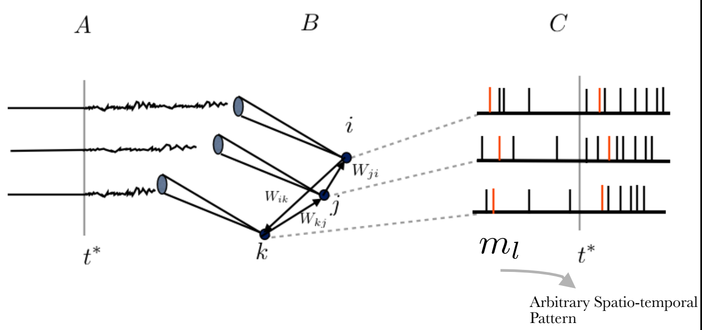
These models produce random spikes whose statistics can be computed.

The Question

Is it possible to quantify the impact of a weak time-dependent external stimulus on the collective statistics of spiking responses of a neuronal network model as a function of its parameters?

3 The Idea

We first consider a neuronal network model without stimulus (spontaneous regime). At time t* we stimulate the network and evaluate the statistical response.



A: Time dependent stimuli after t*.
B: Network of Integrate and Fire Neurons.
C: Spiking responses due to spontaneous neuronal dynamics and time dependent stimulus. In red an arbitrary spatio-temporal pattern m_l that is compared statistically before and after the stimuli presentation.

4 Our Results

For a large class of models it is possible to decompose the corresponding Gibbs potential as a sum under two regimes: spontaneous (no stimulus) and evoked (weak time dependent stimulus)

$$\phi(t,\omega) = \phi^{(sp)}(\omega) + \delta\phi(t,\omega) \quad (1)$$

From this decomposition we obtain:

$$\mu_t[m_l] = \mu_0^{(sp)}[m_l] + \sum_{s=t^*}^{t} C^{(sp)}(m_l, \delta\phi(s,\omega))$$

where m_l is an observable (i.e. firing rate, pairwise correlation or any other higher order spatio-temporal correlation) and $C^{(sp)}$ is the correlation under the spontaneous measure $\mu_0^{(sp)}$.

Example: 1-time step pairwise correlation

Consider m_l as the spikes fired by a pair of neurons (k,j) in a neuronal network model where j fires one time step after k.

$$\mu_t[\omega_k(0)\omega_j(1)] = \mu_0^{(sp)}[\omega_k(0)\omega_j(1)] + \sum_{s=t^*}^{t} C^{(sp)}(\omega_k(0)\omega_j(1), \delta\phi(s,\omega))$$

5 Example

Neuronal Network Model: We consider as an example of application of our results the conductance based integrate-and fire model [3].

For this example we have explicitly computed and decomposed the Gibbs potential as a function of the parameters (1):

$$\phi^{(sp)}(\omega) \rightarrow$$

Is a function of the parameters of this model, independent of the external stimulus.

number of neurons in the network

$$\delta\phi(t,\omega) = \sum_{k=1}^{N} \delta\phi_k(t,\omega)$$

function independent of the external stimulus

time dependent external stimulus

flow of the neuronal network differential equation

$$\delta\phi_k(t,\omega) = \frac{H_k(t,\omega)}{C_k\sigma_k(t-1,\omega)} \int_{\max\{t^*,\tau_k(t-1,\omega)\}}^{t} i_k(t_1)\Gamma_k(t_1,t-1,\omega)dt_1$$

integrated noise

last firing time of neuron k up to time t-1

6 Conclusions

Our result is a version of the so-called fluctuation-dissipation theorem in statistical physics. It holds for spatio-temporal Gibbs distributions with infinite range potential. The main advantage of our approach is that we can evaluate beyond firing rates and cross correlations[4], any higher order spatio-temporal correlation.

References

[1] R. Cofre and B. Cessac, PRE, 89, 2014.
[2] R. Cofre and B. Cessac, Chaos, Solitons and Fractals, 50, 2013.
[3] M. Rudolph and A. Destexhe, Neural Computation, 2006.
[4] J. Trousdale, Y. Hu, E. Shea-Brown, and K. Josic, PLoS Comput Biol,8(3), 2012.