



## Petri Net Synthesis

Eric Badouel, Luca Bernardinello, Philippe Darondeau

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# Petri Net Synthesis

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This book is dedicated to the memory of our friend Philippe Darondeau.



## Foreword

A Petri net synthesis problem can be seen as a model transformation from a behavioural to a structural description of a concurrent system. In its original version formulated by Andrzej Ehrenfeucht and myself, the behavioural description was given by transition systems and the structural description was given by elementary net systems. The solution we provided was based on the theory of regions of transition systems which also yielded an automatic synthesis of an elementary net system whose behaviour was captured by a given transition system.

Since then (1989) Petri net synthesis has become a very vivid and successful research area. The original approach was extended at both ends: the behavioural and the structural descriptions. The acceptable behavioural specifications were extended to, for example, formal languages and execution traces. The structural descriptions were extended first to Place/Transition nets and then to, for example, Petri Nets augmented with priorities or inhibitor arcs. Also, the types of operational semantics were extended to, for example, step sequence semantics (where several actions can be executed simultaneously) which can be constrained by various firing policies (such as maximal parallelism). The notion of a region (suitably adapted) proved to be robust in all these extensions—as a matter of fact it became a basic/central notion of the framework of Petri nets and beyond. All these research efforts were accompanied by the development of suitable algorithms for net synthesis.

Application areas of net synthesis currently cover, among others, the construction of distributed systems, supervisory control, process discovery, and the design of asynchronous circuits. One can already identify the influence of net synthesis research on the design of asynchronous circuits operating under relative timing, and process mining technologies aimed at understanding causal relationships in the behaviour of complex business systems.

Altogether Petri net synthesis has become a full-fledged, relevant, and exciting research area. This book provides an impressive in-depth account of a wide range of the existing results and techniques. It extensively covers the theory and the algorithms and it also provides a good insight into applications. The book is an impressive achievement that will be very valuable for both novices entering this research field and for researchers in this area as a rich and reliable reference as well as a source of relevant research problems.

The very high technical and presentation quality of the book is certainly due to the fact that all three authors made fundamental contributions to this research area. Unfortunately, Philippe Darondeau passed away much too early, so that he could not see the final product of this project. The book is a tribute to this outstanding and inspiring researcher.

Grzegorz Rozenberg  
Leiden, The Netherlands 2015





## Preface

The synthesis problem for Petri nets consists in building a Petri net satisfying a given behavioural specification. In its applications, quite often the initial specification is inherently sequential, and part of the problem consists in eliciting potential concurrence.

This book aims at giving a comprehensive and systematic survey of the synthesis problem and of region theory, which underlies its solution.

Region theory arose from the work by Ehrenfeucht and Rozenberg on 2-structures, starting from the late 1980s. A 2-structure is a special case of a labelled transition system, where labels denote actions producing state changes. A region in a 2-structure (or in a transition system) is a subset of states whose “frontier” is uniformly crossed, or not crossed, by all occurrences of the same label.

In the marking graph of an elementary Petri net, the extension of a place (or condition) is a region. Ehrenfeucht and Rozenberg showed that the set of all regions of a transition system serve as the places of a net whose marking graph is isomorphic to the given transition system, provided the latter satisfies a set of axioms, which can be checked by computing the set of regions.

Since then, the synthesis problem and the theory of regions have attracted several researchers. The original notions have been extended, generalized, and adapted to different classes of Petri nets, and to different initial specifications, other than transition systems.

The algebraic properties of several variants of the original notion of regions have been extensively studied, and some software tools have been developed which solve the synthesis problem.

Eric Badouel was first introduced to the theory of regions by a tutorial given by P.S. Thiagarajan at a Summer School in Gargnano, Italy, which was co-organized by Luca Bernardinello. In his talk P.S. Thiagarajan presented a joint work with Mogens Nielsen and Grzegorz Rozenberg on the use of regions to relate some models of concurrency. This was a source of inspiration for introducing a variant of regions for the characterization of trace automata defined by structural operational specifications. This work, done jointly with Philippe Darondeau, was the starting point for a long and fruitful collaboration.

Around the same period Jörg Desel and Wolfgang Reisig wrote a survey on the synthesis problem of Petri nets, Luca Bernardinello showed the central rôle played by minimal regions and Giorgio De Michelis applied net synthesis techniques in the context of workflow management systems.

In retrospect, the origins of this book can be traced back to a meeting held in 1992 in Leiden, The Netherlands, during a *REX Concurrency Day*, where the above-mentioned persons were invited by Grzegorz Rozenberg to exchange

ideas on region theory. This meeting marked the start of the collaboration on this subject among the three authors. Later, this collaboration was strengthened when Luca moved to Rennes to join Eric and Philippe for a year and a half.

A first comprehensive survey of the field was written by Eric Badouel and Philippe Darondeau in 1998, but the flow of publications on the subject remained steady, showing a combination of theoretical and application-oriented interest.

The vitality of the field, more than twenty years since its birth, is witnessed by the series of ART (Applications of Region Theory) workshops, held in 2010, 2011, and 2013. The topic has been nurtured by the contributions of many researchers including among others Eike Best, Benoît Caillaud, Josep Carmona, Jordi Cortadella, Raymond Devillers, Michael Kishinevsky, Jetty Kleijn, Alex Kondratyev, Maciej Koutny, Luciano Lavagno, Robert Lorenz, Marta Pietkiewicz-Koutny, and Alex Yakovlev.

Writing this book has followed a long process. It was started at the end of 2010 by Eric and Philippe. A first draft of the book was almost completed by the end of 2012, when Philippe had knowledge that he was suffering from a cancer that would leave him little time to live. At this point, Luca joined the team. Philippe passed away on Monday March 18th, 2013. He fought a courageous battle, and remained active until the very end. His pain was strong during his last days, but he wanted to complete his contribution to this book.

With the loss of a dear friend and colleague and the completion of this book we are aware that we are closing a period of our lives rich in experiences and emotions. We hope that this book will be an opportunity for others to open new pages.

As authors, we gratefully acknowledge the support by Springer, and particularly by Ronan Nugent, who has backed the book project throughout its drafting. A warm thank goes to the anonymous referees and to the copy editor, whose contribution has significantly improved the quality of the book.

Finally, a special thank goes to Grzegorz Rozenberg, who kindly agreed to write a foreword, and who has always been a source of inspiration and of encouragement.

Eric Badouel, Rennes, France  
Luca Bernardinello, Milano, Italy  
2015

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