

## Reduced order model from a new stochastic decomposition of fluid flow

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## UNCECOMP 2015

# Reduced order model from a new stochastic decomposition of fluid flow

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25 mai 2015

# PLAN

## STOCHASTIC FLUID DYNAMIC MODEL

Classical Large Eddy approach

Stochastic Navier-Stokes model

## REDUCED MODEL

## RESULTS

## CONCLUSION

# CLASSICAL LARGE EDDY APPROACH

Numericians approach :

$$\frac{\partial \bar{T}}{\partial t} + \nabla \cdot (\bar{T} \bar{u}) = \nabla \cdot (-\overline{u' T'})$$

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# CLASSICAL LARGE EDDY APPROACH

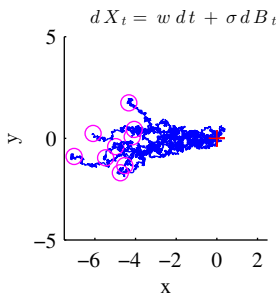
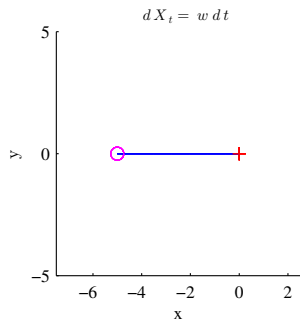
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$\frac{dB_t}{dt}$  = white noise, and  $B_t$  = Brownian motion

# STOCHASTIC NAVIER-STOKES MODEL

Notations :

- ▶  $u = w + \frac{\sigma dB}{dt}$
- ▶  $w$  continuous and differentiable w.r.t. time : large scales
- ▶  $\frac{\sigma dB}{dt}$  Gaussian and discontinuous w.r.t. time : small scales

$$\sigma(x, t) \frac{dB_t}{dt} \triangleq \int_{\Omega} \underbrace{\check{\sigma}(x, z, t)}_{\text{kernel}} \underbrace{\frac{dB_t}{dt}(z)}_{\text{white noise}} dz$$

- ▶ Local covariance matrix

$$a(x, t) \triangleq \sigma(x, t) \sigma^t(x, t) = \int_{\Omega} \check{\sigma}(x, z, t) \check{\sigma}^t(x, z, t) dz$$



# STOCHASTIC NAVIER-STOKES MODEL

## Theorem

$$\frac{\partial w_i}{\partial t} + (w^* \cdot \nabla) w_i = \nabla \cdot \left( \frac{a}{2} \nabla w_i \right) - \frac{1}{\rho} \frac{\partial p}{\partial x_i} + \nu \Delta w,$$

$$\text{with } w^* = w - \frac{(\nabla \cdot a)^T}{2}$$

Applications :

- ▶ Large eddies simulation
- ▶ Uncertainty quantification
- ▶ Filtering
- ▶ Downscaling
- ▶ Mixing diagnostics

...

# PLAN

STOCHASTIC FLUID DYNAMIC MODEL

REDUCED MODEL

Principle / Notations

Additional reduction : time sub-sampling

Our approach

Estimation of  $a$

RESULTS

CONCLUSION

## PRINCIPLE / NOTATIONS

- ▶ Galerkin projection from POD modes :

$$u(x, t) \approx \sum_{i=0}^n b_i(t) \phi_i(x)$$

- ▶ Chronos :  $b_i$

- ▶  $\frac{\partial u}{\partial t} = I + L(u) + C(u, u)$  (a PDE) becomes :

$$\forall i, \frac{db_i}{dt} = \left( \int_{\Omega} \phi_i \cdot I \right) + \sum_{p=0}^n \left( \int_{\Omega} \phi_i \cdot L(\phi_p) \right) b_p + \sum_{p,q=0}^n \left( \int_{\Omega} \phi_i \cdot C(\phi_p, \phi_q) \right) b_p b_q \quad (\text{ODEs})$$

# ADDITIONAL REDUCTION : TIME SUB-SAMPLING

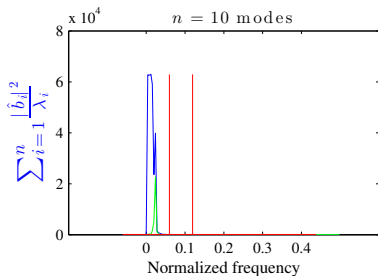
## Large time scale approach

we keep only necessary information for the resolved modes :

$$\frac{db_i}{dt} = i_i + l_{\bullet i} b + b^T c_{\bullet \bullet i} b$$

Nyquist-Shanon criterion for sub-sampling :

$$\frac{f_s}{2} = 2 \max_i f_{max}(b_i) \text{ meets Shanon.}$$



# OUR APPROACH

Our approach :

▶  $u = w + \frac{\sigma dB}{dt}$

▶  $w = \sum_{i=0}^n b_i \phi_i$   
(resolved modes)

▶  $v_{res} = u - w = \sum_{i=n+1}^N b_i \phi_i$  a realization of  $\frac{\sigma dB}{dt}$   
(aliased  $\Rightarrow$  partially decorrelated in time)

# ESTIMATION OF $a$ CONSTANT IN TIME

Local covariance matrix estimated by :

$$a(x) \approx \Delta t \frac{1}{N} \sum_{k=1}^N v_{res}(x, t_k) v_{res}^T(x, t_k)$$

## ESTIMATION OF $a$ DEPENDING OF TIME

We look for a decomposition of  $a = \sum_{i=0}^n b_i(t)z_i(x)$ .

Estimation based on Stochastic Calculus (Genon-Catalot 1992) :

$$z_i(x) = \int_0^T \frac{b_i}{T\lambda_i} a dt \approx \Delta t \frac{1}{N} \sum_{k=1}^N \frac{b_i(t_k)}{\lambda_i} v_{res}(x, t_k) v_{res}^T(x, t_k)$$

# PLAN

STOCHASTIC FLUID DYNAMIC MODEL

REDUCED MODEL

**RESULTS**

Data

Reconstruction of temporal modes

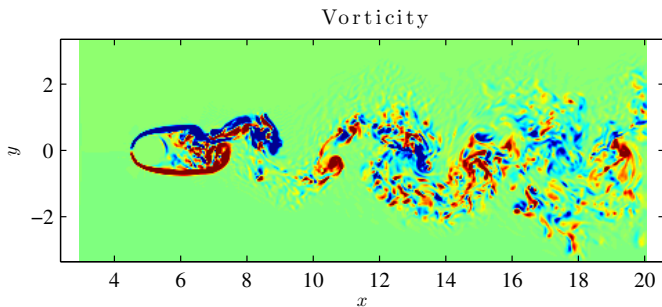
CONCLUSION



# DATA

## Wake behind a cylinder

- ▶ at Reynolds 3900 : 3D flow



## RECONSTRUCTIONS OF TEMPORAL MODES

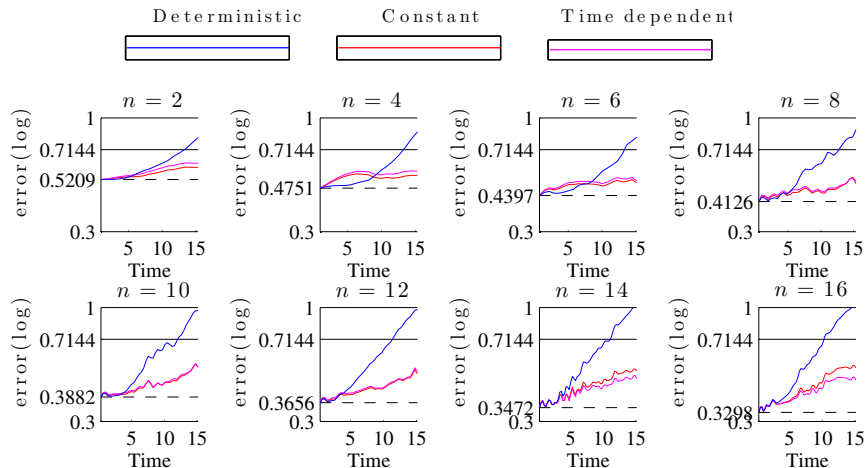


FIGURE :  $Re=3900$  (3D flow),  $a$  constant and in the span of *Chronos*  $b_i$ .

# PLAN

STOCHASTIC FLUID DYNAMIC MODEL

REDUCED MODEL

RESULTS

CONCLUSION

# SUMMARY

- ▶ Complete fluid dynamics model  
based on Physics and Stochastic Calculus :  
Generalization of empirical approaches
- ▶ Local covariance matrix of the small-scales  $a(x, t)$   
constant or projected on *Chronos*
- ▶ Good results without tuning :  
Stabilization and correction of the frequency drift

## FURTHER WORK

- ▶ Test on DNS data with more available snapshots
- ▶  $a = \sum_{i \leq j} b_i b_j z_{ij}(x) \rightarrow$  Inverse problem estimation
- ▶ Generalization :  
 $w$  continuous but not differentiable w.r.t. time  
 Energy conserving model for UQ

$$\begin{aligned}
 \frac{\partial w_i}{\partial t} + (w^* \cdot \nabla) w_i &= \nabla \cdot \left( \frac{a}{2} \nabla w_i \right) - \frac{1}{\rho} \frac{\partial p}{\partial x_i} + \nu \Delta w \\
 &\quad - \underbrace{\left( \frac{\sigma dB_t}{dt} \cdot \nabla \right) w_i}_{\text{multiplicative noise}} + \underbrace{\nu \Delta \frac{\sigma_i dB_t}{dt}}_{\text{additive noise}}
 \end{aligned}$$

# QUESTIONS

Thank you for your attention

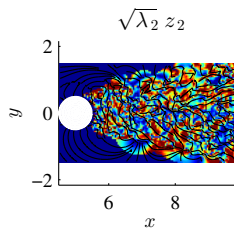
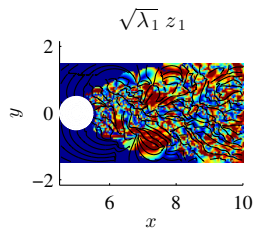
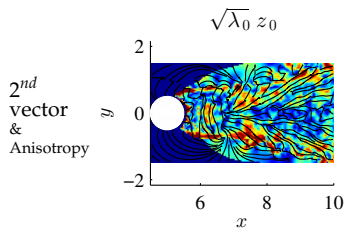
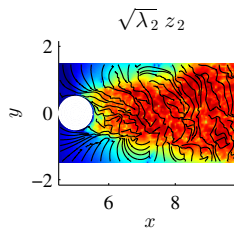
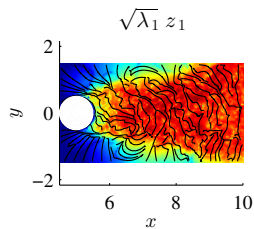
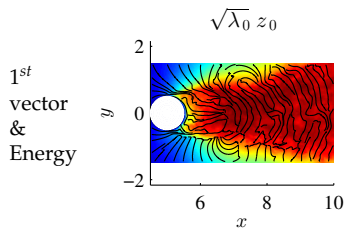
ESTIMATION OF  $a$  DEPENDING OF TIME

$$\forall x \in \mathbb{R}^d, d\tilde{X}_t^x \stackrel{\Delta}{=} \sigma(x, t)dB_t$$

$$\begin{aligned} z_i(x) &= \int_0^T \frac{b_i(t)}{\lambda_i T} a(x, t) dt \\ &= \int_0^T \frac{b_i(t)}{\lambda_i T} d \langle \tilde{X}^x, (\tilde{X}^x)^t \rangle_t \quad (\text{since } a \stackrel{\Delta}{=} \sigma \sigma^t) \\ &= \mathbb{P} - \lim_{\Delta t \rightarrow 0} \sum_{t_k=0}^T \frac{b_i(t_k)}{\lambda_i T} \left( \tilde{X}_{t_{k+1}}^x - \tilde{X}_{t_k}^x \right) \left( \tilde{X}_{t_{k+1}}^x - \tilde{X}_{t_k}^x \right)^t \end{aligned}$$

$$v_{res}(x, t) \Delta t \quad \text{a realization of} \quad d\tilde{X}_t^x \stackrel{\Delta}{=} \sigma(x, t)dB_t$$

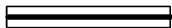
- ▶ Estimator with good statistical properties (Genon-Catalot 1992) (when the projection subset is well chosen)

TURBULENCE MODES  $z_i$  : EIGENVALUES/EIGENVECTORS : RE=3900

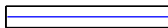


# RECONSTRUCTIONS OF TEMPORAL MODES

Real values



Deterministic



Stochastic

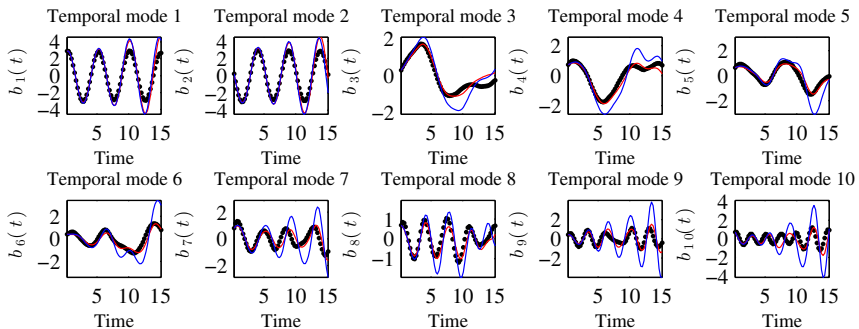
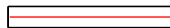


FIGURE :  $n=10$ ,  $Re=3900$  (3D flow),  $a$  constant in time.