



Comment on "Sequential Quasi-Monte Carlo Sampling"

Pierre l'Ecuyer

► **To cite this version:**

Pierre l'Ecuyer. Comment on "Sequential Quasi-Monte Carlo Sampling". Journal of the Royal Statistical Society: Series B, Royal Statistical Society, 2015. hal-01240158

HAL Id: hal-01240158

<https://hal.inria.fr/hal-01240158>

Submitted on 8 Dec 2015

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Comment on “Sequential Quasi-Monte Carlo Sampling”

Pierre L’Ecuyer

DIRO, Université de Montréal, Canada

Gerber and Chopin combine SMC with RQMC to accelerate convergence. They apply RQMC as in the array-RQMC method discussed below, for which convergence rate theory remains thin despite impressive empirical performance. Their proof of $o(N^{-1/2})$ convergence rate is a remarkable contribution.

Array-RQMC simulates an array of N dependent realizations of a Markov chain, using RQMC so the distribution of the N states has low discrepancy D_t with respect to its theoretical distribution, at any step t . If $\mathbf{x}_t = \Gamma_t(\mathbf{x}_{t-1}, \mathbf{u}_t)$, where $\mathbf{x}_{t-1} \sim \mathcal{U}([0, 1]^\ell)$ and $\mathbf{u}_t \sim \mathcal{U}([0, 1]^d)$, array-RQMC approximates the $(\ell + d)$ -dimensional integral $\mathbb{E}[D_t]$, using N RQMC points. It matches each state to a point whose first ℓ coordinates are near that point, and uses the next d coordinates for \mathbf{u}_t .

The *matching* step is crucial. If $\ell = 1$, just sort the states and points by their (increasing) first coordinate. If $\ell > 1$, one can map the states $\mathbf{x}_t \in [0, 1]^\ell \rightarrow [0, 1)$, then proceed as for $\ell = 1$. Gerber and Chopin do exactly this, using a Hilbert curve mapping. But the map $[0, 1]^\ell \rightarrow [0, 1)$ is not essential. E.g., if $\ell = 2$, one can sort the states and the points in $N^{-1/2}$ groups of size $N^{-1/2}$ by their first coordinate, then sort each group by the second coordinate. This extends to $\ell > 2$. With this multivariate sort, L’Ecuyer et al. (2009) observed an $\mathcal{O}(N^{-2})$ variance when pricing an Asian option with array-RQMC. I wonder what rate the Hilbert curve map can achieve for this example.

When $\mathcal{X} \subseteq \mathbb{R}^\ell$ instead of $[0, 1]^\ell$, Gerber and Chopin map $\mathbf{x}_t \rightarrow [0, 1]^\ell$ using a logistic transformation. The choice of transformation and its parameters may have a significant impact on the overall convergence. For the multivariate sort, no transformation is needed, it works directly in \mathbb{R}^ℓ . The Hilbert sort could also be adapted to work directly in \mathbb{R}^ℓ .

The SMC part uses importance sampling and particle resampling, a form of splitting (Kahn and Harris, 1949; L’Ecuyer et al., 2007) with fixed effort (N remains constant). Combinations of array-RQMC with several variants of splitting and with importance sampling were examined and tested by L’Ecuyer et al. (2007). Fixed-effort better fits array-RQMC than splitting variants that produce a random N at each step, and stratification improves the multinomial resampling process.

Array-RQMC was previously developed to estimate the expectation of a function of the state, not in the context of filtering for parameter estimation and for estimating posterior densities. But the same ideas apply. L’Ecuyer et al. (2008) proved an $\mathcal{O}(n^{-3/2})$ convergence rate for the variance for $\ell = 1$ and special types of RQMC points. Faster rates have been observed empirically in examples. Proving those rates in a general setting (under appropriate conditions) is challenging and deserves further research effort.

References

- Kahn, H. and Harris, T. E. (1949) Estimation of particle transmission by random sampling. In *Monte Carlo Method*, vol. 12 of *Applied Mathematics Series*, 27–30. National Bureau of Standards.
- L'Ecuyer, P., Demers, V. and Tuffin, B. (2007) Rare-events, splitting, and quasi-Monte Carlo. *ACM Transactions on Modeling and Computer Simulation*, **17**, Article 9.