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Quantum walks on graphs [?] are an extension of random walks to the quantum domain. Towards mixing, they can seemingly provide a quadratic speedup compared to reversible classical random walks. On the other hand, lifted random walks, as proposed by Diaconis et al. [?], are a non-quantum extension proven to deliver the same speedup [?]. However, the construction of lifted walks makes explicit use of multicommodity flows, rendering their construction NP-hard. The gain of using a quantum algorithm for mixing would thus strongly rely on the complexity of its construction; so far no constructive proof exists for quantum walks with mixing times quadratically improving the standard random walk, except for very specific graphs. The end goal of our research is hence to elucidate the key element(s) that provide acceleration in quantum walks with respect to lifted chain walks. As a first step in this contribution we highlight some similarities between the two procedures, at least on examples like the cycle and torus graphs, used previously to illustrate the acceleration of quantum walks. This shows that the two acceleration schemes on the standard random walk are closely related and can in fact learn from each other.

In one direction we show that a particular kind of quantum walk, called open quantum walks (as proposed by Attal et al. [?]), can further speed up mixing when applied to given lifted walks. We propose a straightforward yet unexplored quantization of lifted walks, constructible on all known examples. We show this does not imply a speedup using the general quantum walk mixing paradigm, but it does for the subclass of open quantum walks. Moreover, we show this subclass to be generally implementable in a non-quantum way, thus providing a non-quantum speedup for lifted walks. More specifically, by keeping some local degrees of freedom coherent, the quantization naturally introduces a feedback-like extension to the scheme.

In the other direction, we pose the question of whether quantum walks rely on similar principles or information as lifted walks do. Equivalently put, does the construction of a quantum walk generally require NP-hard information on the graph? We achieve a first confirming result in this direction, showing that this information can at least speed up a quantum walk. We base ourselves on a theorem at the heart of lifted walks, by Diaconis and Miclo [?]: *It is always possible to speed up mixing by introducing momentum*. Relating this observation to multicommodity flows on a graph allows for the speedup of lifted walks. We show a similar result applies to quantum walks. The introduction of a directional bias can further accelerate the mixing time. With this result we further underpin the observation made by Chandrashekar et al. [?] concerning quantum walk mixing time, and dare to pose it more boldly: *Unbalanced coins make quantum walks faster*.

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