Robust state estimation of P.E.M. fuel cell systems

Severus Constantin Olteanu, Abdel Aitouche, Lotfi Belkoura, Abdelaziz Nakrachi

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Olteanu S.C., Aitouche A., Belkoura L., Nakrachi A.
LAGIS UMR CNRS 8219
severus.olteanu@gmail.com

Keywords: PEMFC, State Estimation, Non-Linear Observer, Uncertainties.

ABSTRACT

In this paper a model based state estimation is developed using the Mean Value Theorem with Bounded Jacobian. The case of additive perturbations is also treated. This method is afterwards applied to a Fuel Cell Stack System(FCSS). The main type of fuel cell on which this work is concentrated is the Polymer Electrolyte Membrane(Proton Exchange Membrane) that uses a Nafilon 117 membrane [1]. The model presents some simplifications considering only pure Oxygen as input and also an ideal humidifying and cooling unit. The numerical problem of the disequilibrium between small mass values and big pressure values is resolved by constructing a symmetric system.

1. INTRODUCTION

In the last decade a growing interest can be seen in the Fuel Cell technology, presenting itself as one of the most efficient alternative power generation technologies. Fuel Cell Stack(FCS) development has generated much interest in both research and industrial areas such that, although still expensive for mass production, the expectations for the future show a change in this aspect. Fuel Cells (FCs), act as ideal power sources, being based on an inverse electrolysis, converting chemical energy into electrical power. They present: lower/zero emission, silent functioning, high efficiency, fast refill time in comparison to batteries. The type of FC analyzed here, Proton Exchange Membrane Fuel Cell(PEMFC) was invented at General Electric in 1960s is still the most popular membrane in use today [7]. A FC produces electricity, water, and heat, thereby eliminating pollution at the energy conversion. Although the research in the field of modeling and control of fuel cell systems has known a great boast, there is still a lack of FCS models appropriate for control and diagnosis research, especially in state space form, many diagnosis solutions make use of spectral analysis. Two types of models are known: steady state FCS models and dynamic FCS models. The former ones serve as a mean to design FC components and to choose the operating points. On another hand, even if they are not suitable for control and diagnosis studies, they are handy in establishing the effects of different parameters as pressure, temperature on the fuel cell voltage. Whereas, the latter are described by multiple variables and a strong coupling with profound dynamics. Some dedicated articles have been presented: -Turner et al [8] included the transient effect of fuel cell stack temperature in his dynamic model. -Pukrushpan et al [2] presented a nonlinear fuel cell system dynamic model suitable for control study. The flow characteristics and inertia dynamics of the compressor, the manifold dynamics, and consequently, the reactant partial pressures were included in the transient phenomena captured of the model he proposed.

In general, many real life system present some parameters that are hard to measure directly by means of sensors so these states are required to be estimated; For example in a Fuel Cell Stack there are many small cells grouped together in a stack and to measure the parameters of each incorporated cell is unsatisfactory. Also, observers can be built to estimate those variables that can generate residuals used to detect some eventual faults in the system. The observers need to be tolerant to perturbations. In literature, there are many methods for general state estimation for non-linear systems utilizing different methods some based on a Luenberger like observer and other; the effective theories for adaptation of a non-linear structure to a linear one show also a broad variety like Sliding mode observers(reference), observers based on Fuzzy Logic as Takagi-Sugeno approach where the system model is transformed into a sum of linear models, each of them scaled according to their weighting function(w_i) as in [6] and those that use the Mean value theorem approach. In many of the existing observers, the computations are manged using Linear Matrix Inequalities (L.M.I.s) to satisfy the stability of the estimation error's dynamics.

The paper is organized as follows: Section 2 will cover the PEMFCS modeling, after which Section 3 will treat the theory behind the observer, based on the Mean Value Theorem. After that in Section 4 the method will be applied to the PEMFCS. Finally conclusions and perspectives are drawn.

2. MODEL DEVELOPMENT

Modeling of a fuel cell system concerns multi-physical domains using electrochemical, thermo dynamical, electrical and fluid mechanical principles. In this paper, we focus on the study of the most generally used PEM fuel cell system which operates at low temperatures between 50 and 100 degrees Celsius. The fuel cell is a volumetric capacitor including two electrodes named as cathode (ca) and anode (an) which sandwich an electrolyte inside the proton exchange membrane with bipolar plates called Membrane Electrode Assemblies (MEA).The purpose of this work is to focus on the auxiliary elements, and less on the electrical part of the model(the dynamics being at a different scale): *Air compressor* (with an electrical DC motor); *Hydrogen tank (*with a control valve attached); *Supply manifold* (including the volume of pipes from the compressor plus the volume of the cooler and the volume of the humidifier); *Return Manifold*(the pipe volume at the Fuel Cell's exhaust); *Cooling Unit* (represents a circuit of de-ionized cooled water system). The application is concentrated on vehicle applicable Fuel cells (this would also take into consideration the adaptability towards embedded algorithms). Graphical description in figure below. The used model is based on [2]. More details can be found in [6]. The obtained system model can then be used in the end both for fault detection and control[4]. The State space representation is the following:

$$\begin{cases} \dot{x} = F_x(x)\cdot x + G_x(x)\cdot u + F_c; \\ y = C\cdot x; \end{cases} ;\ x \in R^n; y \in R^p; u \in R^m;$$

$$x = \begin{pmatrix} m_{O_2} \\ m_{H_2} \\ m_{sm} \\ p_{sm} \\ p_{rm} \end{pmatrix};\ u = \begin{pmatrix} I_{st} \\ A_{T,rm} \end{pmatrix};\ y = \begin{pmatrix} p_{sm} \\ p_{rm} \end{pmatrix};\ \textbf{y=direct measurable values;}\quad F_c = \begin{pmatrix} 0 \\ f_{cst4} \\ W_{cp} \\ f_{cst7}\cdot\left(1-\dfrac{1}{\eta_{cp}}\right) \\ 0 \end{pmatrix};\ C = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix};$$

$$F_x(x) = \begin{pmatrix} -f_{cst2} & 0 & 0 & k_{sm,out} & k_{ca,out} \\ 0 & -f_{cst5} & 0 & f_{cst3} & 0 \\ k_{sm,out}\cdot f_{cst1} & 0 & 0 & -k_{sm,out} & 0 \\ z_1\cdot f_{cst9} & 0 & 0 & \left(f_{cst11}\cdot z_2 - f_{cst8}\cdot z_1\right) & 0 \\ f_{cst6}\cdot f_{cst1} & 0 & 0 & 0 & -f_{cst6} \end{pmatrix};\quad G_x(x) = \begin{pmatrix} \left(-M_{O_2}\cdot\dfrac{n}{4\cdot F}\right) & 0 \\ \left(-M_{H_2}\cdot\dfrac{n}{2\cdot F}\right) & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & -f_{cst10}\cdot z_3 \end{pmatrix};$$

The model has 5 states, that includes the return manifold pressure, the supply manifold pressure and gas masses of Oxygen, Hydrogen and mass of Oxygen in the supply manifold; the pressures are considered measured. As inputs we consider the stack current and the return manifold evacuation nozzle's surface. With 'z', we specify the nonlinear terms (their values are presented in the Appendix). The other parameters are constant values also presented in the Appendix.

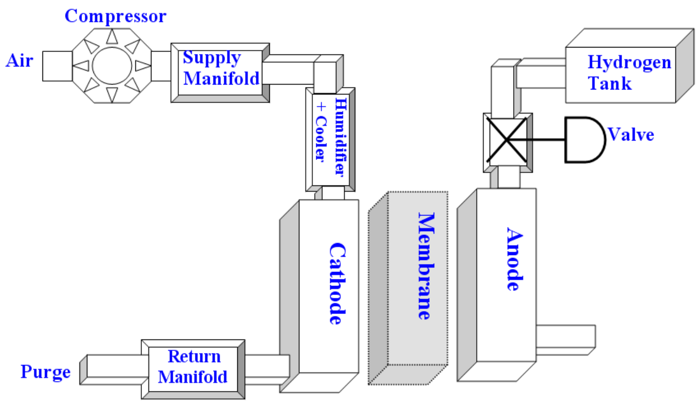


Fig.1 The PEMFC system with auxiliary elements

3. OBSERVER DEVELOPMENT

3.1 General Observer

A general Luenberger observer is adopted in this paper, observer for which it has been utilized the Mean Value Theorem / Bounded Jacobian approach to integrate the nonlinearities.
The system has to be of the special form, showed in parallel to the associated observer:

$$\begin{cases} \dot{x} = A \cdot x + \Phi(x) + g(y,u); \\ y = C \cdot x; \end{cases} \begin{cases} \dot{\hat{x}} = A \cdot \hat{x} + \Phi(\hat{x}) + g(y,u) + L \cdot (y - \hat{y}); \\ \hat{y} = C \cdot \hat{x}; \end{cases} \tag{1}$$

$\hat{x}$=estimated states; $L =$ Observer's Gain Matrix; y=Real Measured Output; $\hat{y}$=Estimated Output;
To calculate the gain matrix L we search for a P symmetric, K, so that the following LMISs present a solution for all i ,j <=number of system states:

$$\begin{cases} P(A + \bar{H}_{ij}^{\max}) + (A + \bar{H}_{ij}^{\max})^T P - C^T K^T - KC < 0; \\ P(A + \bar{H}_{ij}^{\min}) + (A + \bar{H}_{ij}^{\min})^T P - C^T K^T - KC < 0; \\ P > 0; \\ K > 0; \end{cases} \tag{2}$$

Where:

$$\begin{cases} h_{ij}^{\max} \geq \max(\frac{\partial \Phi_i}{\partial x_j}); h_{ij}^{\min} \leq \min(\frac{\partial \Phi_i}{\partial x_j}); \\ H_{ij}^{\max} = Z \cdot e_n(i) \cdot e_n^{\ T}(j) \cdot h_{ij}^{\max}; H_{ij}^{\min} = Z \cdot e_n(i) \cdot e_n^{\ T}(j) \cdot h_{ij}^{\min}; \\ e_n(i) = [0..1...0]^T; Z = n \cdot n; n = \text{number of states} \end{cases}$$

Proof: We start from the estimation error dynamics, that becomes:

$$\dot{\tilde{x}} = (A - L \cdot C) \cdot \tilde{x} + \tilde{\Phi}(\hat{x}); \text{ Where } \dot{\tilde{x}} = x - \hat{x}; \ \ \tilde{\Phi}(\hat{x}) = \Phi(x) - \Phi(\hat{x}); \tag{3}$$

The Lyapunov function candidate may be defined as
$V = \tilde{x}^T \cdot P \cdot \tilde{x}; P > 0; P \in \mathbb{R}^{n \times n}$ where „n" is the number of system states.

$$\dot{V} = \tilde{x}^T \cdot [(A - L \cdot C)^T \cdot P + P \cdot (A - L \cdot C)] \cdot \tilde{x} + \tilde{x}^T \cdot P \cdot \tilde{\Phi} + \tilde{\Phi}^T \cdot P \cdot \tilde{x}; \tag{4}$$

We want $\dot{V} < 0$; yet we know from the Mean value theorem [3] that :

$$\left[\Phi(x) - \Phi(\hat{x})\right] = \left[\left(\sum_{i,j}^{n,n} H_{ij}^{\max} \cdot \delta_{ij}^{\max}\right) + \left(\sum_{i,j}^{n,n} H_{ij}^{\min} \cdot \delta_{ij}^{\min}\right)\right](x - \hat{x}) \tag{5}$$

Where $\delta_{ij}^{\max} + \delta_{ij}^{\min} = 1$; each term being an unknown positive number. So we can write (4) as :

$$\begin{aligned} \dot{V} = \tilde{x}^T \cdot [&(A - L \cdot C)^T \cdot P + P \cdot (A - L \cdot C) + P\left(\sum_{i,j}^{n,n} H_{ij}^{\max} \cdot \delta_{ij}^{\max}\right) + \\ &+ P\left(\sum_{i,j}^{n,n} H_{ij}^{\min} \cdot \delta_{ij}^{\min}\right) + \left(\sum_{i,j}^{n,n} H_{ij}^{\max} \cdot \delta_{ij}^{\max}\right)^T P + \left(\sum_{i,j}^{n,n} H_{ij}^{\min} \cdot \delta_{ij}^{\min}\right)^T P] \cdot \tilde{x}; \end{aligned} \tag{6}$$

After some other computations, because $\delta_{ij}^{\max}+\delta_{ij}^{\min}=1$; we can write sums for all the terms in (6). Afterwards for eliminating the $\delta_{ij}^{\max},\delta_{ij}^{\min}$ terms that are unknown we make the restrictive hypothesis that if all the terms in a sum are less than 0 then their sum is les than 0. We consider $K=P\cdot L \Rightarrow L=P^{-1}\cdot K$; So we will reach the LMIs mentioned in (2).

3.2 Robustness

We can also take into consideration some additive perturbations both of the system dynamics and the output. This has been achieved in a similar manner with [9], by considering additive perturbations. This is resolved using H infinity performance. The gain matrix can be reached by finding a matrix P>0, K>0 and a lambda>0 scalar, so that the following LMIs have a solution:

$$\begin{cases}\begin{pmatrix}\left[H_{ij}^{\max}+A\right]^T\cdot P-C^T\cdot K^T+P\cdot\left[H_{ij}^{\max}+A\right]-K\cdot C+I & (P\cdot W_1-K\cdot W_2)\\ (P\cdot W_1-K\cdot W_2)^T & -\frac{1}{\lambda^2}\cdot I\end{pmatrix}<0\\ \begin{pmatrix}\left[H_{ij}^{\min}+A\right]^T\cdot P-C^T\cdot K^T+P\cdot\left[H_{ij}^{\min}+A\right]-K\cdot C+I & (P\cdot W_1-K\cdot W_2)\\ (P\cdot W_1-K\cdot W_2)^T & -\frac{1}{\lambda^2}\cdot I\end{pmatrix}<0\\ P>0\end{cases} \tag{7}$$

Proof: We start from the general system format presented below:

$$\begin{cases}\dot{x}=A\cdot x+\Phi(x)+g(y,u)+W_1\cdot w(t);\\ y=C\cdot x+W_2\cdot w(t);\end{cases} \tag{8}$$

$W_1=[E\quad 0];W_2=[0\quad D];w(t)=[w_1\quad w_2]^T$ to express it in the general state space format.

By using Lyapunov properties, the following conditions have to apply in order to reduce the effects of the perturbations upon the system: $\lim_{t\to\infty}\tilde{x}(t)=0$ for w(t)=0; and $\|\tilde{x}(t)\|_{L_2}\le\lambda^2\cdot\|w(t)\|_{L_2}$ for $w(t)\neq 0$ and $\tilde{x}(0)=0$;

So we have to find a positive scalar Lambda where

$$\dot{V}+\tilde{x}^T\cdot\tilde{x}-\lambda^2 w^T w<0 \tag{9}$$

The observer form remains (1), but the estimation error dynamics becomes:

$$\dot{\tilde{x}}=(A-LC)\cdot\tilde{x}+\tilde{\Phi}(\hat{x})+(W_1-L\cdot W_2)\cdot w;\ \dot{\tilde{x}}=\dot{x}-\dot{\hat{x}};\ \tilde{x}=x-\hat{x};\ \tilde{\Phi}(\hat{x})=\Phi(x)-\Phi(\hat{x}); \tag{10}$$

The Lyapunov function candidate may be defined as $V=\tilde{x}^T\cdot P\cdot\tilde{x};P>0;P\in\mathbb{R}^{n\times n}$; Considering also (11) we obtain:

$$\dot{V}=\tilde{x}^T\cdot[(A-L\cdot C)^T\cdot P+P\cdot(A-L\cdot C)]\cdot\tilde{x}+\tilde{x}^T\cdot P\cdot\tilde{\Phi}+\tilde{\Phi}^T\cdot P\cdot\tilde{x}+w^T\cdot(W_1-L\cdot W_2)^T\cdot P\cdot\tilde{x}+\tilde{x}^T\cdot P\cdot(W_1-L\cdot W_2)\cdot w; \tag{11}$$

From (11) and (9)=>

$$\begin{aligned}\dot{V}+\tilde{x}^T\cdot\tilde{x}-\lambda^2 w^T w&=\tilde{x}^T\cdot[(A-L\cdot C)^T\cdot P+P\cdot(A-L\cdot C)+I]\cdot\tilde{x}+\\&+\tilde{x}^T\cdot P\cdot\tilde{\Phi}+\tilde{\Phi}^T\cdot P\cdot\tilde{x}+w^T\cdot(W_1-L\cdot W_2)^T\cdot P\cdot\tilde{x}+\tilde{x}^T\cdot P\cdot(W_1-L\cdot W_2)\cdot w-\lambda^2\omega^T\omega;\end{aligned} \tag{12}$$

Writing it as a Matrix we will have:

$$\begin{pmatrix}\tilde{x}^T & w^T\end{pmatrix}\cdot\begin{pmatrix}[(A-L\cdot C)^T\cdot P+P\cdot(A-L\cdot C)+I] & [P\cdot(W_1-L\cdot W_2)]\\ \left[(W_1-L\cdot W_2)^T\cdot P\right] & -\lambda^2\cdot I\end{pmatrix}\cdot\begin{pmatrix}\tilde{x}\\ w\end{pmatrix}+\tilde{x}^T\cdot P\cdot\tilde{\Phi}+\tilde{\Phi}^T\cdot P\cdot\tilde{x}; \tag{13}$$

Yet from the mean value theorem[3], using (5) and knowing that $\delta_{ij}^{\max}+\delta_{ij}^{\min}=1$ we conclude :

$$\begin{pmatrix}[\sum_{i,j=1}^{n,n}\left\{\left[H_{ij}^{\max}+(A-L\cdot C)\right]^T\cdot P+P\cdot\left[H_{ij}^{\max}+(A-L\cdot C)\right]+I\right\}\cdot\delta_{ij}^{\max}+\sum_{i,j=1}^{n,n}\left\{\left[H_{ij}^{\min}+(A-L\cdot C)\right]^T\cdot P+P\cdot\left[H_{ij}^{\min}+(A-L\cdot C)\right]+I\right\}\cdot\delta_{ij}^{\min}] & [P\cdot(W_1-L\cdot W_2)]\\ \left[(W_1-L\cdot W_2)^T\cdot P\right] & -\lambda^2\cdot I\end{pmatrix}<0; \tag{14}$$

Using the Schur transformation of Matricial inequalities and rearranging the terms it resultes that

$$\begin{aligned}&\sum_{i,j=1}^{n,n}\left\{\left[H_{ij}^{\max}+(A-L\cdot C)\right]^T\cdot P+P\cdot\left[H_{ij}^{\max}+(A-L\cdot C)\right]+I+[P\cdot(W_1-L\cdot W_2)]\cdot\frac{1}{\lambda^2}\cdot I\cdot\left[(W_1-L\cdot W_2)^T\cdot P\right]\right\}\cdot\delta_{ij}^{\max}+\\&+\sum_{i,j=1}^{n,n}\left\{\left[H_{ij}^{\min}+(A-L\cdot C)\right]^T\cdot P+P\cdot\left[H_{ij}^{\min}+(A-L\cdot C)\right]+I+[P\cdot(W_1-L\cdot W_2)]\cdot\frac{1}{\lambda^2}\cdot I\cdot\left[(W_1-L\cdot W_2)^T\cdot P\right]\right\}\cdot\delta_{ij}^{\min}<0;\end{aligned} \tag{15}$$

Further, we can make an assumption, that is indeed restrictive concerning the solution, method already implemented in [3]: *Statement: If each term in the upper sums is negative then the whole negativity* *rests negative.Than we can transform the inequality in multiple inequalities(also taking into consideration* $\overline{\delta}_{ij}^{\max}, \overline{\delta}_{ij}^{\min} \geq 0$*).*For all i,j, using the inverse Schur format, and *considering* $K = P \cdot L \Rightarrow L = P^{-1} \cdot K$; we arrive at the wanted LMIs.

4. APPLICATION OF THE OBSERVER ON A PEMFC MODEL

4.1 General System Format

By applying the theory to our system we change the system format so that we have the general form required (1).

$$x = \begin{pmatrix} m_{O_2} \\ m_{H_2} \\ m_{sm} \\ p_{sm} \\ p_{rm} \end{pmatrix}; \; u = \begin{pmatrix} I_{st} \\ A_{T,rm} \end{pmatrix}; \; y = \begin{pmatrix} p_{sm} \\ p_{rm} \end{pmatrix};$$ **y=direct measurable values;**

$$A = \begin{pmatrix} -f_{cst2} & 0 & 0 & k_{sm,out} & k_{ca,out} \\ 0 & -f_{cst5} & 0 & f_{cst3} & 0 \\ k_{sm,out} \cdot f_{cst1} & 0 & 13.3 & -k_{sm,out} & 0 \\ 0 & 3.21 \cdot 10^8 & 5.4321 \cdot 10^7 & 69 & 0 \\ (f_{cst6} \cdot f_{cst1}) & 0 & 0 & 0 & -f_{cst6} \end{pmatrix}; \; g(y,u) = \begin{pmatrix} \left(-M_{O_2} \cdot \frac{n}{4 \cdot F}\right) \cdot I_{st} \\ \left(-M_{H_2} \cdot \frac{n}{2 \cdot F}\right) \cdot I_{st} \\ 0 \\ 0 \\ -f_{cst10} \cdot z_3 \cdot A_{Tm} \end{pmatrix}; \; Fc = \begin{pmatrix} 0 \\ f_{cst4} \\ W_{cp} \\ f_{cst7} \cdot \left(1 - \frac{1}{\eta_{cp}}\right) \\ 0 \end{pmatrix};$$

$$\Phi(x) = \begin{pmatrix} 0 \\ 0 \\ -13.3 \cdot m_{sm} \\ z_1 \cdot f_{cst9} \cdot m_{O_2} - 3.21 \cdot 10^8 \cdot m_{H_2} - 5.4321 \cdot 10^7 \cdot m_{sm} + (f_{cst11} \cdot z_2 - f_{cst8} \cdot z_1 - 69) \cdot p_{sm} \\ 0 \end{pmatrix}; \; C = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix};$$

The general observer schema will be in consequence:

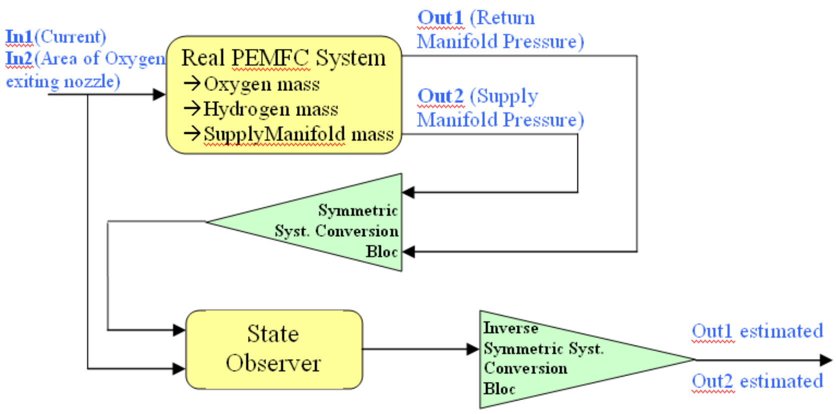


Fig.2 General Observer Schema adapted to our FC System

4.2 Observability

The condition of observability has to be satisfied by the virtual linear system determined by A and C and all $(A + H_{i,j}^{\max,\min})$. C = [0 0 0 1 0; 0 0 0 0 1]=> Observability_Matrix will not have a full rank, so we apply a procedure to force the observability. Also if we look at the eigenvalues of A(that will affect the consequent numerical computations), the results are scattered at different scales.

For this we add some terms to matrix A and then subtract the same terms from matrix $\Phi(x)$; A(3,3) and A(4,4) are chosen specifically to increase the A's Eigenvalues. While the others, to increase the rank of the Observability matrix of the pairs (A,C). Also the values chosen are carefully picked so that we can do a numerical modification to the LMIs so that the calculations will not have ill posed matrices. **A(4,2)=3.21*10^8; A(4,3)=5.4321*10^7; A(3,3)=13.3; A(4,3)=69;** And the eigenvalues of the new A are also normalized by the upper procedure.

4.3 Bounding the Jacobian Matrix

Now we calculate the jacobian, and so we reach the following matrix:

$$J\Phi(x)=\begin{pmatrix} 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\\ 0 & 0 & -13.3 & 0 & 0\\ z_1\cdot f_{cst9} & -3.21\cdot 10^8 & (-5.4321\cdot 10^7 - f_{cst9}\cdot z_{11}\cdot z_1 + f_{cst8}\cdot z_1^2) & \left(-69+f_{cst9}\cdot z_{11}+f_{cst11}\cdot(\frac{-1+\gamma}{\gamma})\cdot z_2 - 2\cdot f_{cst8}\cdot z_1\right) & 0\\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}; \quad (15)$$

We now find the minimum and maximum of each term. This has been done by means of simulating at different initial conditions. The following possibilities were tested:

$$\begin{aligned} &x1=(0.005 \vee 0.06) & & h_{4,1}\in(1.9422\mathbf{e}+\mathbf{007};2.7968e+\mathbf{009})\\ &x2=(10\wedge -4 \vee 2*10\wedge -3) & => \quad & h_{4,2}=-3.21\cdot 10^8\\ &x3=(0.01\vee 0.12) & & h_{4,3}\in(-1.3257\mathbf{e}+\mathbf{009};1.6837\mathbf{e}+\mathbf{0}10)\\ &x4=(5*10\wedge 4 \vee 6*10\wedge 5) & & h_{4,4}\in(-655.3592;159.9110)\\ &x5=(5*10\wedge 4\vee 5*10\wedge 5) & & h_{3,3}=\text{-}13.3 \end{aligned}$$

$$Z_H=5*5-nr\_of\_terms\_equl\_to\_0=25-20=5;$$

$$H^{\max}=5\cdot\begin{pmatrix} 0&0&0&0&0\\ 0&0&0&0&0\\ 0&0&-13.3&0&0\\ 2.7968\mathbf{e}+\mathbf{009} & -3.21\cdot 10^8 & 1.6837\mathbf{e}+010 & 159.9110 & 0\\ 0&0&0&0&0\end{pmatrix}; \quad H^{\min}=5\cdot\begin{pmatrix} 0&0&0&0&0\\ 0&0&0&0&0\\ 0&0&-13.3&0&0\\ 1.9422\mathbf{e}+\mathbf{007} & -3.21\cdot 10^8 & -1.3257\mathbf{e}+\mathbf{009} & -655.3592 & 0\\ 0&0&0&0&0\end{pmatrix}; \quad (16)$$

4.4 Building a Symmetric System

For the Luenberger Observer’s Gains, we need to perform a system of 8 inequalities. The problem is that the matrices are ill posed (amplitudes of different scales).To solve this we apply a mathematical numerical solution: We multiply the inequality left and right with a matrix on each side. The matrices will be diagonal, and the values of each of them has to be chosen, following all the possibilities of $A+H_i^{\min}$ respectively $A+H_i^{\max}$. In the end we choose(by means of experiment):

$$P^{-1}=\begin{pmatrix} 10^7&0&0&0&0\\ 0&10^7&0&0&0\\ 0&0&10^7&0&0\\ 0&0&0&10^{-1}&0\\ 0&0&0&0&1\end{pmatrix}; \quad P=\begin{pmatrix} 10^{-7}&0&0&0&0\\ 0&10^{-7}&0&0&0\\ 0&0&10^{-7}&0&0\\ 0&0&0&10&0\\ 0&0&0&0&1\end{pmatrix}; \quad (17)$$

The symmetry is to be taken into consideration $P=P^t$;

We apply the similarity transformation, which implies that if we consider a new state as being $\vec{x}=P^{-1}\cdot x$; then the system becomes :

$$\begin{cases} P^{-1}\cdot\dot{x}=P^{-1}\cdot A\cdot P\cdot P^{-1}\cdot x\cdot P+P^{-1}\cdot\Phi(x)+P^{-1}\cdot g(y,u)+P^{-1}\cdot Fc;\\ y=C\cdot P\cdot P^{-1}\cdot x;\end{cases} => \begin{cases}\dot{\vec{x}}=\vec{A}\cdot\vec{x}+P^{-1}\cdot\Phi(x)+\vec{g}(y,u)+\vec{F}_c;\\ \vec{y}=\vec{C}\cdot\vec{x};\end{cases} \quad (18)$$

If we make the notations: $\vec{x}=P^{-1}\cdot x;\ \vec{y}=y\ ;\vec{C}=C\cdot P;\ \vec{g}=P^{-1}\cdot g;\ \overset{\leftrightarrow}{(A)}=P^{-1}\cdot(A)\cdot P;\ \overset{\leftrightarrow}{(F_c)}=P^{-1}\cdot F_c;$

Important to be noted the behavior of $\Phi(x)$, dependent on x, and not the new state, because the transformation of x is not possible being situated in nonlinear equations. The now observer becomes:

$$\begin{cases}\dot{\hat{\vec{x}}}=\vec{A}\cdot\hat{\vec{x}}+P^{-1}\cdot\Phi(\hat{x})+\vec{g}(y,u)+L\cdot(\vec{y}-\hat{\vec{y}})+\vec{F}_c;\\ \hat{\vec{y}}=\vec{C}\cdot\hat{\vec{x}};\end{cases} \quad (19)$$

The estimation error dynamics is $\dot{\tilde{\vec{x}}} = (\vec{A} - L\cdot\vec{C})\cdot\tilde{\vec{x}} + \tilde{\Phi}(\hat{x});\ \tilde{\vec{x}} = \vec{x} - \hat{\vec{x}};\ \tilde{\Phi}(\hat{x}) = P^{-1}\cdot(\Phi(x) - \Phi(\hat{x}));$

Then the Lyapunov function candidate may be defined as $V = \tilde{\vec{x}}^T \cdot R \cdot \tilde{\vec{x}}; R > 0; R \in \mathbb{R}^{5\times 5} =>$

$$\dot{V} = \tilde{\vec{x}}^T \cdot [(\vec{A} - L\cdot\vec{C})^T \cdot R + R\cdot(\vec{A} - L\cdot\vec{C})]\cdot\tilde{\vec{x}} + \tilde{\vec{x}}^T \cdot R\cdot\tilde{\Phi} + \tilde{\Phi}^T \cdot R \cdot \tilde{\vec{x}} < 0; \tag{20}$$

Considering as previous the Mean Value Theorem(5) and also taking into account that Jacobian($P^{-1}\cdot\Phi(x)$) = $P^{-1}\cdot Jacobian(\Phi(x) - \Phi(\hat{x}))$; by oversimplifying the notations we will have $\tilde{\Phi}(\hat{x}) = P^{-1}\cdot(\Phi(x) - \Phi(\hat{x})) = P^{-1}\cdot\left[Sum(H\min,\max(x)\right]\cdot(x - \hat{x});$ But yet we need $\tilde{\vec{x}} = \vec{x} - \hat{\vec{x}}; =>$ $\tilde{\Phi}(\hat{x}) = P^{-1}\cdot\left[Sum(H\min,\max(x))\right]\cdot P\cdot P^{-1}(x - \hat{x}); => \tilde{\Phi}(\hat{x}) = \left\{P^{-1}\cdot\left[Sum(H\min,\max(x))\right]\cdot P\right\}\cdot\tilde{\vec{x}};$

This way we bring forth the notation: $(A\overset{\leftrightarrow}{+}H) = P^{-1}\cdot(A+H)\cdot P;$ In this manner from (34) we reach the inequalities:

$$\begin{cases} R(\vec{A} + \overleftrightarrow{H}_{ij}^{\max}) + (A + \overleftrightarrow{H}_{ij}^{\max})^T R - \vec{C}^T L^T R - RL\vec{C} < 0; \\ R(\vec{A} + \overleftrightarrow{H}_{ij}^{\min}) + (A + \overleftrightarrow{H}_{ij}^{\min})^T R - \vec{C}^T L^T R - RL\vec{C} < 0; \\ R > 0; \end{cases} \tag{21}$$

4.5 Applying robustness to the observer

Starting from the attached theory previously presented, we will have the system:

$$\begin{cases} \dot{x} = A\cdot x + \Phi(x) + g(y,u) + Fc + W_1\cdot \mathrm{w(t)}; \\ y = C\cdot x + W_1\cdot \mathrm{w(t)}; \end{cases} \tag{22}$$

where $W_2 = \begin{pmatrix} 0 & 0 & 0 & 0.05 & 0 \\ 0 & 0 & 0 & 0 & 0.05 \end{pmatrix}; W_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix};$

If we make the notations: $\vec{x} = P^{-1}\cdot x;\ \vec{y} = y\ ; \vec{C} = C\cdot P;\ \vec{g} = P^{-1}\cdot g;\ (\overleftrightarrow{A}) = P^{-1}\cdot(A)\cdot P;\ (\overleftrightarrow{F_c}) = P^{-1}\cdot F_c;\ (\overleftrightarrow{W_1}) = P^{-1}\cdot W_1;$ $(\overleftrightarrow{W_2}) = W_2;$ we will therefore obtain the LMIs (22):

$$\begin{cases} \begin{pmatrix} \left[\vec{H}_{ij}^{\max} + \vec{A}\right]^T \cdot R - \vec{C}^T\cdot K^T + R\cdot\left[\vec{H}_{ij}^{\max} + \vec{A}\right] - K\cdot\vec{C} + I & \left(R\cdot\vec{W}_1 - K\cdot\vec{W}_2\right) \\ \left(R\cdot\vec{W}_1 - K\cdot\vec{W}_2\right)^T & -\frac{1}{\lambda^2}\cdot I \end{pmatrix} < 0 \\ \begin{pmatrix} \left[\vec{H}_{ij}^{\min} + \vec{A}\right]^T \cdot R - \vec{C}^T\cdot K^T + R\cdot\left[\vec{H}_{ij}^{\min} + \vec{A}\right] - K\cdot\vec{C} + I & \left(R\cdot\vec{W}_1 - K\cdot\vec{W}_2\right) \\ \left(R\cdot\vec{W}_1 - K\cdot\vec{W}_2\right)^T & -\frac{1}{\lambda^2}\cdot I \end{pmatrix} < 0 \\ R > 0 \end{cases} \tag{23}$$

4.2 Results

The observer has been implemented in Matlab/Simulink environment. The inputs of the system are: Electrical current steps and step like variations of the Surface nozzle of the return manifold.

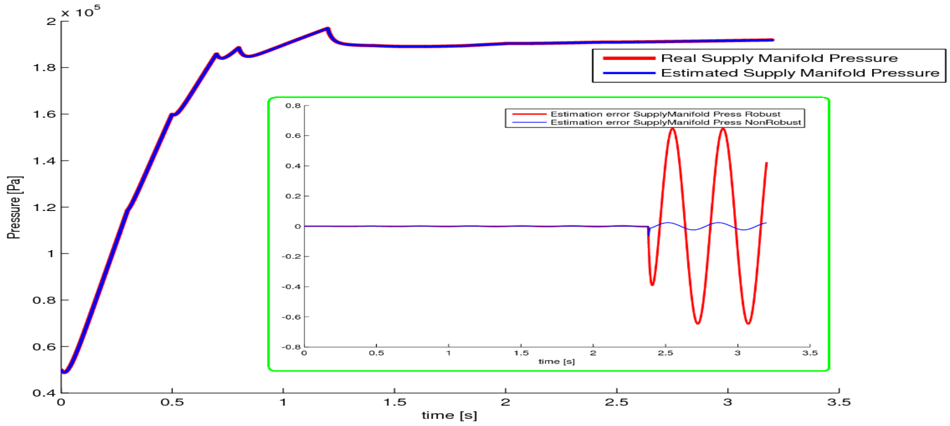
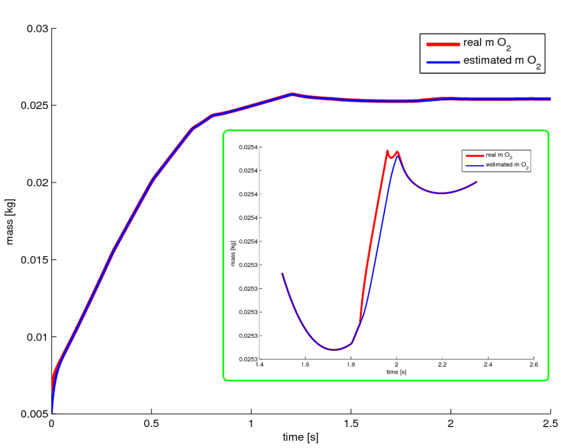


Fig.3 Evolution of the Pressure in the Supply Manifold

The evolution of the pressure in the supply manifold in Fig.3 shows the system's robustness(in the green rectangle it is shown the difference of the estimation error between the robust and non-robust case).



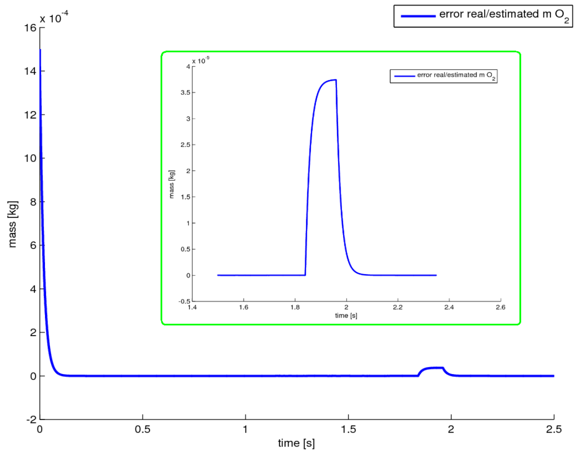
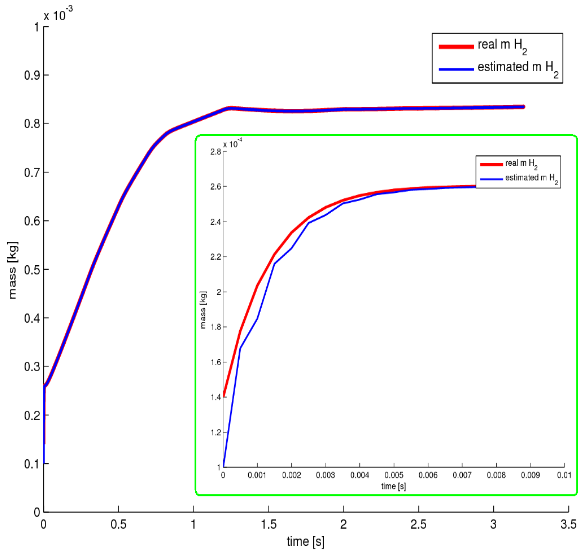


Fig.4a Mass of Oxygen, real and estimated evolution in the presence of initial state difference and a perturbation

Fig.4b Mass of Oxygen, estimation error evolution

For the first state(mass of Oxygen) with a modified initial state and perturbations we obtain the above results where we can see how the estimation follows the Real systems state value.

The other states' evolutions:



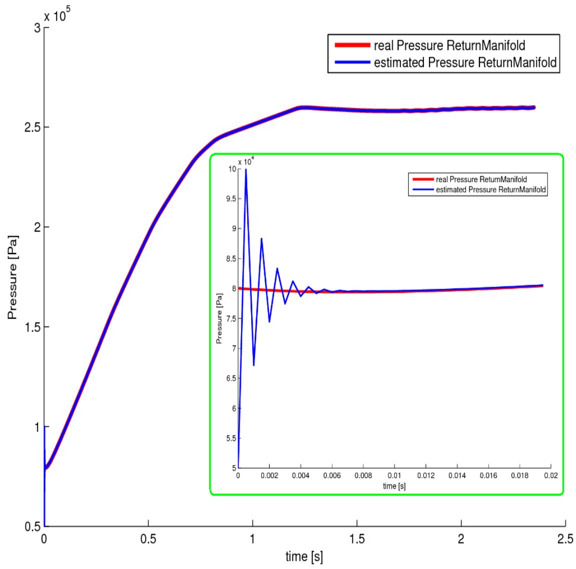


Fig.5 Hydrogen mass evolution (real/estimated)

Fig.6 Return Manifold Pressure evolution (real/estimated)

5. CONCLUSIONS AND PERSPECTIVES

Not many state space models exist in the scientific literature for the auxiliary elements. Many existing models are experimental or mathematical-physical but are not concentrated on the dynamics and more on the static equations implied by the Fuel Cell. Also this model possesses two interesting aspects: the high number of states and nonlinearities and also the high difference in magnitude of the parameters implied in the model (Pressures vary around 10^5 and Masses vary around 10^-3); these bring a generality to the estimation method so that it can be extended to a model of a higher degree of complexity. So this work brings an important contribution to F.C. observer development applicable even for a more complex and complete F.C. stack model.

As further development of this work, it is envisioned a validation of the system model and observer implementation using *AMESim* and after that, using a real F.C. Stack situated in an University in Lille. Other methods for estimation better suited for the Fuel Cell are to be analyzed.

APPENDIX

$$f_{cst1}=\frac{R_{O_2}\cdot T_{st}}{V_{ca}};\quad f_{cst2}=k_{sm,out}\cdot\frac{R_{O_2}\cdot T_{st}}{V_{ca}}+k_{ca,out}\cdot\frac{R_{O_2}\cdot T_{st}}{V_{ca}};\quad f_{cst3}=0.00092256030284\cdot 2.1\cdot 10^{-3};\quad f_{cst4}=2.84870103138421\cdot 2.1\cdot 10^{-3};$$

$$f_{cst5}=\frac{\mathrm{R_{H2}\cdot T_{st}}}{\mathrm{V_{an}}}\cdot 2.1\cdot 10^{-6};\ f_{cst6}=\frac{R_a\cdot T_{rm}\cdot k_{ca,out}}{V_{rm}};\ f_{cst7}=\frac{\gamma\cdot R_a}{V_{sm}}\cdot W_{cp}\cdot T_{atm};\ f_{cst8}=k_{sm,out}\cdot\frac{\gamma\cdot R_a}{R};\ f_{cst9}=\frac{k_{sm,out}\cdot R_{O_2}\cdot T_{st}\cdot\gamma\cdot R_a}{V_{ca}\cdot R};$$

$$f_{cst10}=\frac{R_aT_{rm}}{V_{rm}}\cdot\frac{C_{D,rm}}{\sqrt{\bar{R}\cdot T_{rm}}}\cdot\left(\frac{2\cdot\gamma}{\gamma-1}\right)^{\frac{1}{2}}\cdot p_{atm}^{\frac{1}{\gamma}};\quad f_{cst11}=\frac{f_{cst7}}{\eta_{cp}\cdot p_{atm}^{\frac{\gamma-1}{\gamma}}};$$

Initial states: ***m_O2 = 0.011; m_H2 = 0.001; m_sm = 0.011 + 0.0097; p_sm = 101325;p_rm = 101325;***

Nonlinear terms : $z_1=\frac{p_{sm}}{m_{sm}};\ z_2=p_{sm}^{\frac{-1}{\gamma}};\ z_{11}=\frac{m_{O2}}{m_{sm}};$

		Variable	Description
m_{O2}	Mass of Oxygen (kg)	R_{O2}	Mass of Oxygen (kg)
m_{H2}	Mass of Hydrogen (kg)	R_{H2}	Mass of Hydrogen (kg)
m_{sm}	Mass of gas accumulated in the manifold (kg)	Ra	Gas constant (J/mol.K)
p_{sm}	Pressure supply manifold (Pa)	R	Gas constant (J/kg.K)
p_{rm}	Pressure return manifold (Pa)	T_{atm}	Air temperature (K)
w_{cp}	Compressor speed (rad/s)	T_{rm}	Temperature return manifold(K)
I_{st}	Current in the stack (A)	$T_{cp,out}$	Ai outputted temperature (K)
T_{st}	Temperature in the stack (K)	P_{atm}	Air pressure (Pa)
γ	specific heat capacity of gas		

		Variable	Description
V_{ca}	Cathode volume (m^3)	C_p	Specific heat capacity of air (J/kg.K)
V_{an}	Anode volume (m^3)	F	Faraday number
V_{rm}	Return manifold volume (m^3)	M_{O2}	Molar masse of Oxygen (kg/mol)
V_{sm}	Supply manifold volume (m^3)	M_{H2}	Molar masse of Hydrogen(kg/mol)
W_{cp}	Compressor mass flow (kg/s)	$k_{sm,out}$	Supply manifold outlet flow constant (kg/s.Pa)
η_{cp}	Compressor efficiency	$k_{rm,out}$	Return manifold outlet flow constant (kg/s.Pa)
τ_{cm}, τ_{cp}	Torques of motor and compressor (N.m)	$A_{T,rm}$	Return manifold nozzle area (m^2)
η_{cp}	Compressor efficiency	$C_{D,rm}$	Discharge coefficient nozzle

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