L2-Orthogonal ST-Code Design for Multi-h CPM with fast Decoding

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Abstract

⊕ CPM: favorable trade-off between power and bandwidth efficiency.

⊕ Multi-h CPM: generalization to further decrease the need for bandwidth.

⊖ Difficult decoding in multi-path environments with no diversity.

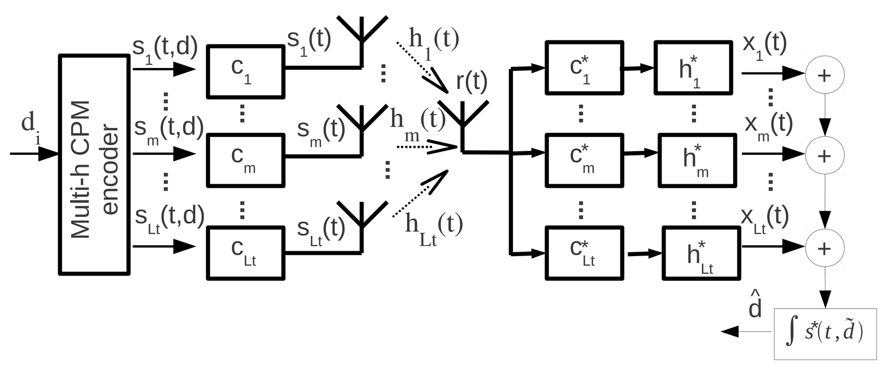
How to overcome these limitations:

⊕ **IDEA:** To combine CPM with Space-Time Block Coding (STBC).

→ Non trivial extension to L^2-orthogonal Space-Time codes provides full diversity and better spectral compactness.[1][2]

→ Decoding complexity greatly decreased.[1][2]

The System Model



M-ary multi-*h* MIMO system of $L_t \times L_r$ antennas.

$$\mathbf{r}(t,\mathbf{d}) = \mathbf{H}\mathbf{s}(t,\mathbf{d}) + \mathbf{n}(t)$$

$\mathbf{r}(t,\mathbf{d})_{(L_r \times 1)}$, $\mathbf{s}(t,\mathbf{d})_{(L_t \times 1)}$

$\mathbf{H}_{(L_r \times L_t)}$ with elements $h_{n,m}$

$\mathbf{n}(t)$ AWGN.

The vector of transmitted signals:

$$\mathbf{s}(t,\mathbf{d}) = \left[\mathbf{s}_l(t,\mathbf{d}) \dots \mathbf{s}_{L_t}(t,\mathbf{d})\right]^T$$

The baseband general form [3]

$$s(t,\mathbf{d}) = \sqrt{\frac{E_s}{T}} \exp(j\phi(t,\mathbf{d}))$$

The information-carrying phase function

$$\phi(t,\mathbf{d}) = 2\pi \sum_{k=0}^{K-1} h_{[k]} d_k q(t-(k-1)T)$$

modulation indices $h_1, \dots, h_H$, cycle in time with period *H* as:

$$[k] = mod(k, H) + 1$$

$h_{[k]}$ quotient between two relative prime integers.

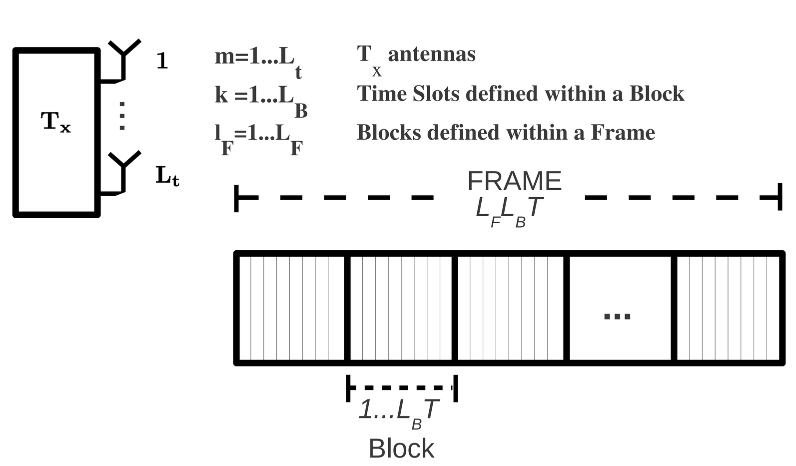
$$H_k = (h_1, h_2, \dots, h_k) = \left(\frac{p_1}{q}, \frac{p_2}{q}, \dots, \frac{p_k}{q}\right)$$

The phase continuity is ensured by the phase pulse $q(t)$,

$$q(t) = \begin{cases} 0 & t < 0 \\ \bar{q}(t) & elsewhere \\ 1/2 & t \geq \gamma T \end{cases}$$

$\bar{q}(0) = 0$, $\bar{q}(\gamma T) = 1/2$ and $\lim_{t \to \tau} \bar{q}(t) = \bar{q}(\tau)$ for $0 \leq \tau < \gamma T$. γ is the overlapping factor and the symbol index is given by $k = \lceil t/T \rceil$

L^2 Orthogonality



Based on $\| \mathbf{s} \|_{L^2}^2 = \int \mathbf{s}(t)\mathbf{s}^H(t)dt$, each block fulfills the condition

$$\int_{(l_f-1)L_BT}^{l_FL_BT} \mathbf{s}(t,\mathbf{d})\mathbf{s}^H(t,\mathbf{d})dt = \mathbf{I}$$

The phase continuity is ensured by an antenna dependent phase memory:

$$\theta_{m,k} = \theta_{m,k-1} + \frac{h_{[k-\gamma]}}{2} d_{m,k-\gamma} + c_{m,k}(T) - c_{m,k}(0).$$

Auto-correlation coefficients cancel and cross-correlation coefficients between antennas put to 0

$$\sum_{k=1}^{L_B} \int_{(k-1)T}^{kT} \exp(j2\pi[\theta_{m,k} + \sum_{i=k-\gamma+1}^{k} h_{[i]} d_{m,i} q(t-(i-1)T) + c_{m,k}(t) - \theta_{m',k} - \sum_{i=k-\gamma+1}^{k} h_{[i]} d_{m',i} q(t-(i-1)T) - c_{m',k}(t)])dt = 0$$

To ease the design, two assumptions are introduced,

1. $c_{m,k}(t) = c_{m,k'}(t)$
2. $d_{m,k} = d_{m',k}$

For any arbitrary number of transmit antennas, we introduce correction functions $c_{m,k}$ as:

$$c_m^{lin}(t) = \frac{m-1}{L_BT} t \quad \text{for} \quad (k-1)T < t < kT$$

For $m = 1, \dots, L_t$ and the transmitted signal takes the form of

$$s_m(t,\mathbf{d}) = s(t,\mathbf{d}) \exp(j2c_m^{lin}(t))$$

L^2 Decoding

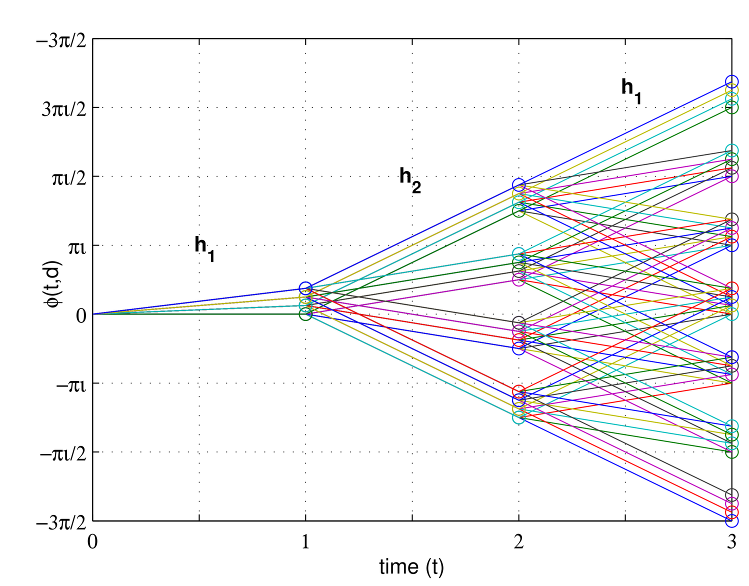


FIG. 1: Inner-block phase transition for $h_1 = \frac{1}{4}$ and $h_2 = \frac{3}{4}$.

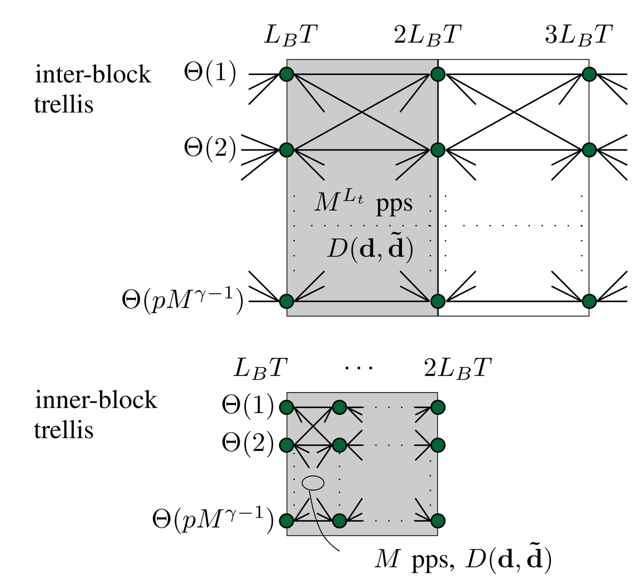


FIG. 2: Simplified detection with $k = 1$ (pps-paths per state).

Classical correlation based multi-*h* detector expressed blockwise:

$$D(\mathbf{d},\tilde{\mathbf{d}}|\Theta(k)) = \sum_{k=1}^{L_F} \sum_{n=1}^{N_k} \int_{kTL_B}^{(k+1)TL_B} Re\left\{ r(t, \mathbf{d}_{1_{[n]}}, \dots, \mathbf{d}_{L_B[n]}) \cdot \left(\sum_{m=1}^{L_t} h_m^* s_m^*(t, \theta, \tilde{\mathbf{d}}_{1_{[n]}}, \dots, \tilde{\mathbf{d}}_{L_B[n]}) \right) \right\} dt$$

⊖ **Impractical:**

$pM^{(L_tL_BN_s+\gamma-1)}$ matched filters of length N_sL_BT employed $pM^{\gamma-1}$ times.

pM^{L_B-1} states and $M^{L_BL_T}$ symbols to decode with N_s samples each.

⊕ **Orthogonality:** the cross correlation terms are canceled out with blockwise decoding.

$$D_B(\mathbf{d},\tilde{\mathbf{d}}|\Theta(k)) = \sum_{m=1}^{L_T} \int_{(l-1)T}^{lT} Re\{r(t,\mathbf{d})h_m^* c_m^*(t) s^*(t,\tilde{\mathbf{d}})\}dt.$$

To get decoding with complexity growing linearly w.r.t number of transmit antennas, we introduce

$$x(t,\mathbf{d}) = r(t,\mathbf{d}) \sum_{m=1}^{L_T} h_m^* c_m^*(t).$$

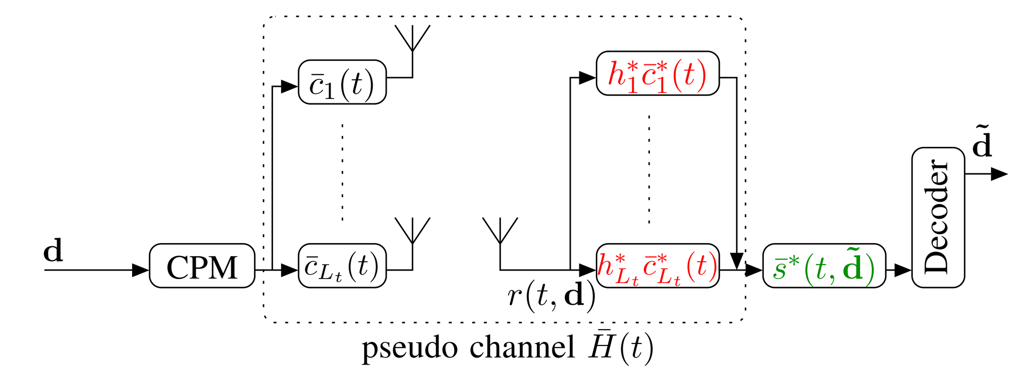
A simplified expression for a classical multi-*h* decoder [4] then is given by

$$D(\mathbf{d},\tilde{\mathbf{d}}|\Theta(k)) = \int_{(l-1)T}^{lT} Re\{x(t,\mathbf{d}) \cdot s(t,\tilde{\mathbf{d}})_{|\phi_l}\}dt$$

Hence our detector based on the correlations gives us an expression which maximizes the minimum distance as

$$D_r(\mathbf{d},\tilde{\mathbf{d}}|\Theta(k)) = \underset{\Theta(1)\to\Theta(pM^{\gamma-1})}{\arg\max} \left\{ \int_{(l-1)T}^{lT} Re\{x(t,\mathbf{d})s(t,\tilde{\mathbf{d}})\}dt \right\}.$$

The simplified detector can now be represented in a similar form as in [5],



Simulation Results

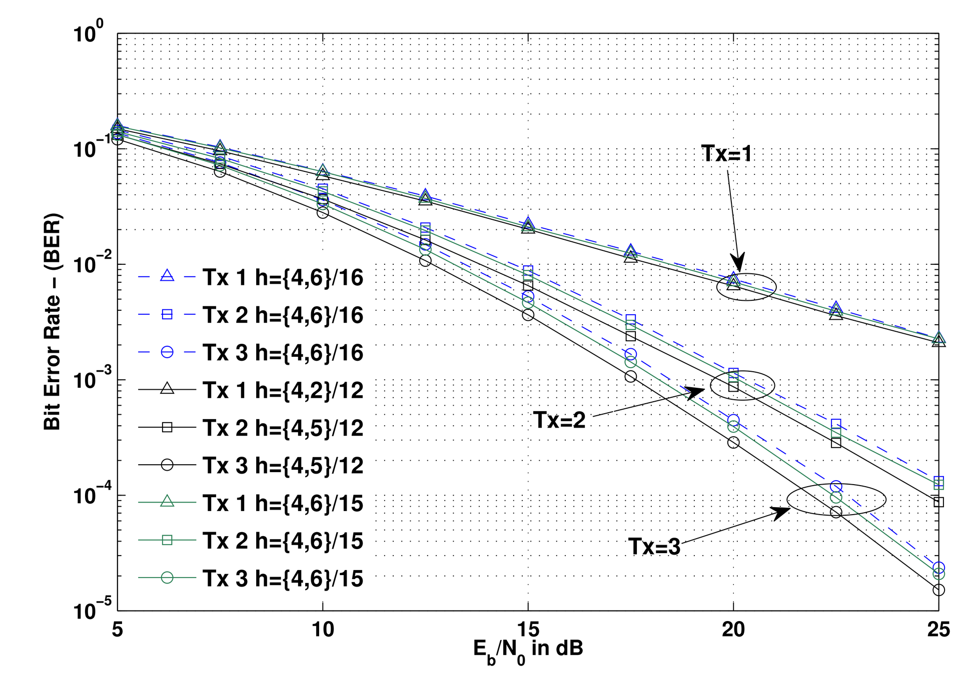


FIG. 3: BER for $T_x = 1, 2, 3$, $L_B = 2$ and $h_i = \{4, 5\}/12$, $h_i = \{4, 6\}/16$, $h_i = \{4, 6\}/15$.

Conclusions

√ L^2-orthogonal STBC provide full diversity by the construction of orthogonal waveforms.

√ We have shown that the inner trellis is equivalent to the inter-trellis for multi-*h* CPM.

√ By choosing $H \leq L_B$ we get the largest minimum distance in a block.

√ L^2-orthogonal STBC satisfies the needs for energy and spectral efficiency.

√ Decoding complexity increases as PM instead of PMM^{L_T} .

References

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[2] M. Hisojo, J. Lebrun, and L. Deneire. L2-Orthogonal ST-Code design for multi-h CPM with fast decoding. *ICC proceedings*, 2013.

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[4] J. B. Anderson and D. P. Taylor. A bandwidth-efficient class of signal-space codes. *IEEE. Trans. Inf. Theory*, 1978.

[5] M. Hesse, J. Lebrun, and L. Deneire. L2-Orthogonal ST-Code design for CPM. *IEEE Trans. Commun.*, 2011.