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1 Introduction

E-commerce has become nowadays a common practice and worldwide sales are forecast to hit \$ 1.5 trillion by the end of 2014 [3]. Companies operating in this sector need to propose efficient services for attractive prices [1]. This can be obtained by an efficient organization and management of the customer demand, from the moment it is ordered to when it is delivered at the customer's place.

In this paper we consider a packaging and shipping problem that occur in a warehouse. The objective is to optimize the process to *prepare* the customer demands. The preparation of a demand consists in different sequential phases: first, the objects that form the order need to be picked up in the storehouse, then they need to be inserted in standard packages and finally charged in appropriate vehicles.

For simplicity, we associate each demand with a volume. The volume represents the number of packages needed to contain all the objects ordered by the customer.

In addition, the customer can choose among different delivery options, called *channels*, (for example normal or express delivery). Each option is associated with different trucks that guarantee different delivery delays (for example the normal channel deliver within 3-5 business days or the quick channel within 24 hours). Each service corresponds to a certain price payed by the customer.

Our problem consists in determine the daily operations of the warehouse. Each day is structured as follows: first, we suppose that it is composed by S shifts. and a shift by $\bar{T}$ periods. During each shift, a certain number of workers is in charge of the demand preparation. The workers can be *fixed* or *temporary*. The second can be hired for only a shift, when the workload is estimated to be high. On the other side, they cost more than a fixed worker. Workers are associated with a *productivity*, i.e., the number of packages they can prepare during a time period. It is supposed that the productivity of a fixed worker is higher than the productivity of a temporary worker.

The number of workers of each shift is a decision variable of the problem and then need to be set. The preparation plan of each period needs to be determined, namely, for each demand, the exact number of packages that need to be prepared at each period needs to be set. Note that a demand with a volume greater than one can be prepared during different and not necessarily consequent periods.

Once the demand has been processed, i.e., all its packages have been treated, they need to be loaded in the vehicles. Vehicles need to be present at the warehouse docks to be charged. The warehouse is equipped with a limited number of docks that bounds the number of vehicles that can be simultaneously charged. Note, that each vehicle should contain only packages of demands associated with a specific delivery channel. Then, if the plan for a certain period envisages the preparation of demands associated with different channels, at least one truck for each needs to be present at the docks.

Note that since demands can be prepared during different periods, their packages can be charged in different vehicles. On the other side, packages of the same demand must be delivered by the same channel. Moreover, in each period, the number of packages that can be prepared cannot exceed the number of workers times their productivity or the number of vehicles at the docks times their capacity.

We consider the possibility of demand *re-affectation*: it is possible to deliver a demand with another channel than the one chosen by the customer. In that case a penalty needs to be paid. This penalty represents the extra cost needed to deliver the demand (for example with a quicker channel), or quantifies the dissatisfaction of the customer for a possible late delivery. Re-affectation takes place when it allows to hire less employees or to use less vehicles.

In addition, the problem we consider is a multi-day problem. In particular, we need to plan the operations for a certain current day j, but we suppose that a forecast on the whole demand of the following days is available. We then introduce the possibility to postpone in the following days the preparation of certain demands if this is beneficial. As a counterpart, a penalty is payed for this delay in the preparation. As an example, given the available information, some demands associated with the normal delivery channel can be delivered with the quick channel of the day after, if this allows to hire less workers or to use less vehicle.

The problem looks for an operational plan that minimizes the cost of hiring the workers (fixed and temporary) and the cost of re-affecting or postponing deliveries and the number of used vehicles. In addition, considering that the number of docks is usually a critical resource, we look for an plan that minimizes its usage (called trucks management policy).

2 A model for the packaging and shipping problem

In this section we formally introduce the problem and we propose a model for it. In particular, we consider a planning horizon made of H days, indexed $\mathcal{H} = \{1, \dots, H\}$. For each day $h \in \mathcal{H}$ we need to prepare a number of demands D_h (indexed $\mathcal{D}_h = \{1, \dots, D_h\}$). Demands that are revealed on day h need to be prepared in one of the following $\bar{h} = 0, \dots, \bar{H}$ days, indexed in $\bar{\mathcal{H}}$ and delivered by one of the available channels $v = 1, \dots, V$, indexed in $\mathcal{V}$. Days are divided in T periods, indexed in $\mathcal{T}$. Each day is moreover divided in S shifts, indexed in $\mathcal{S} = \{1, \dots, S\}$. Each shift is divided in $\bar{T}$ periods. We have $T = S\bar{T}$.

Each demand $d \in \mathcal{D}_h$ of day h is characterized by: the type of channel v_{hd}; the volume n_{hd}, i.e. the number of packages it is composed; the period t_{hd} at which the demand becomes known; the preparation time coefficient g_{hd} (based on the time needed to prepare one unit of volume); the penalty $p_{hd}^{\bar{h}v}$ for re-affectation of the demand to channel v and/or postponement to day $\bar{h}$. Each of the V available channels is characterized by: the number r_v of trucks available; the departure period h_v (no more vehicles associated to channel v will be available after h_v). All the vehicles are supposed to have the same capacity Q. Each shift s of day h is characterized by: the starting period t^{st}_{hs}; the ending period t^e_{hs}; the cost c_{hs} of a fix worker; the cost $\bar{c}_{hs}$ of a temporary worker; the productivity b_{hs} of a fix employees; the productivity $\bar{b}_{hs}$ of a temporary employees; the maximum number $e^{\max}_{hs}$ of fix employees allowed to work during shift s. Finally $N_{\max}$ is the number of trucks that can be simultaneously charged, i.e., the number of available docks at the depot. Several decisions need to be taken. For each day $h \in \mathcal{H}$, for each $\bar{h} \in \bar{\mathcal{H}}$, for each $d \in \mathcal{D}_h$, for each channel $v \in \mathcal{V}$ and for each shift $s \in \mathcal{S}$ we have the following decision variables:

$x^{\bar{h}v}_{hd}$	equals 1 if the demand d of day h is prepared in day $h+\bar{h}$ and affected to channel v
$f^{\bar{h}vt}_{hd}$	number of packages of demand d prepared in period t of day $h+\bar{h}$ and affected to channel v
y^{vt}_h	equals 1 if the number of available trucks for channel v during day h at period t is not zero
w^{vt}_h	number of utilized trucks at day h in period t for channel v
u^{vt}_h	number of new trucks at day h in period t for channel v
k^{vt}_h	residual capacity of trucks at day h in period t for channel v
z_{hs}	number of fix workers working on shift s of day h
$\bar{z}_{ts}$	number of temporary workers working on shift s of day h

$$(PSP) \quad \min \sum_{h\in\mathcal{H}} \sum_{d\in\mathcal{D}_h} \sum_{\bar{h}\in\bar{\mathcal{H}}} \sum_{v\in\mathcal{V}} p^{\bar{h}v}_{hd} n_{hd} x^{\bar{h}v}_{hd} + \sum_{h\in\mathcal{H}} \sum_{s\in\mathcal{S}} (c_{hs} z_{hs} + \bar{c}_{hs}\bar{z}_{hs}) + \sum_{h\in\mathcal{H}} \sum_{v\in\mathcal{V}} \sum_{t\in\mathcal{T}} (\gamma_1 u^{vt}_h + \gamma_2 w^{vt}_h) \tag{1}$$

$$\text{s.t.} \sum_{\bar{h}\in\bar{\mathcal{H}}} \sum_{v\in\mathcal{V}} \sum_{t\in\mathcal{T}} f^{\bar{h}vt}_{hd} = n_{hd} \quad \forall h \in \mathcal{H},\ \forall d \in \mathcal{D}_h \tag{2}$$

$$\sum_{\bar{h}\in\bar{\mathcal{H}}} \sum_{v\in\mathcal{V}} x^{\bar{h}v}_{hd} = 1 \quad \forall h \in \mathcal{H},\ \forall d \in D_h \tag{3}$$

$$\sum_{t\in\mathcal{T}} f^{\bar{h}vt}_{hd} \le x^{\bar{h}v}_{hd} n_{hd} \quad \forall h \in \mathcal{H},\ \forall d \in D_h,\ \forall \bar{h} \in \bar{\mathcal{H}},\ \forall v \in \mathcal{V} \tag{4}$$

$$\sum_{d\in\mathcal{D}_{h-\bar{h}}} \sum_{\bar{h}\in\bar{\mathcal{H}}} f^{\bar{h}vt}_{(h-\bar{h})d} + k^{vt}_h = k^{v(t-1)}_h + Q_v u^{vt}_h \quad \forall h \in \mathcal{H},\ \forall v \in \mathcal{V},\ \forall t \in \mathcal{T}, t \le h_v \tag{5}$$

$$\sum_{t=t}^{t_v} u^{vt}_h \le \alpha y^{vt}_h \quad \forall h \in \mathcal{H},\ \forall v \in \mathcal{V},\ \forall t \le h_v \tag{6}$$

$$y^{vt}_h \le \sum_{t=t}^{t_v} u^{vt}_h \quad \forall h \in \mathcal{H},\ \forall v \in \mathcal{V},\ \forall t \le t_v \tag{7}$$

$$Q_v u^{vt}_h + k^{v(t-1)}_h \le Q_v w^{vt}_h + Q_v(1 - y^{vt}_h) \quad \forall h \in \mathcal{H},\ \forall v \in \mathcal{V},\ \forall t \le t_v \tag{8}$$

$$Q_v u^{vt}_h + k^{v(t-1)}_h - k^{v(t_v-1)}_h \le Q_v w^{vt}_h \quad \forall h \in \mathcal{H},\ \forall v \in \mathcal{V},\ \forall t \le t_v \tag{9}$$

$$\sum_{v\in\mathcal{V}} w^{vt}_h \le N_{\max} \quad \forall h \in \mathcal{H},\ \forall t \in \mathcal{T} \tag{10}$$

$$\sum_{\bar{h}\in\bar{\mathcal{H}}} \sum_{d\in\mathcal{D}_{h-\bar{h}}} \sum_{v\in\mathcal{V}} g_{h-\bar{h}} f^{\bar{h}vt}_{(h-\bar{h})d} \le b_{hs} z_{hs} + \bar{b}_{hs}\bar{z}_{hs} \quad \forall h \in \mathcal{H},\ \forall s \in \mathcal{S},\ \forall t = t^{st}_s, \ldots, t^e_s \tag{11}$$

$$z_{hs} \le e^{\max}_{hs} \quad \forall h \in \mathcal{H},\ \forall s \in \mathcal{S} \tag{12}$$

$$\bar{z}_{hs} \le z_{hs} \quad \forall h \in \mathcal{H},\ \forall s \in \mathcal{S} \tag{13}$$

$$x^{\bar{h}v}_{hd} \in \{0,1\}\ \forall h \in \mathcal{H},\ \forall d \in \mathcal{D}_h,\ \forall \bar{h} \in \bar{\mathcal{H}},\ \forall v \in \mathcal{V} \tag{14}$$

$$y^{vt}_h \in \{0,1\}\ \forall h \in \mathcal{H},\ \forall v \in \mathcal{V},\ \forall t \in \mathcal{T} \tag{15}$$

$$z_{hs}, \bar{z}_{hs} \in \{0,1\}\ \forall h \in \mathcal{H},\ \forall s \in \mathcal{S} \tag{16}$$

$$f^{\bar{h}vt}_{hd} \in \mathbb{Z}^+\ \forall h \in \mathcal{H},\ \forall d \in \mathcal{D}_h,\ \forall \bar{h} \in \bar{\mathcal{H}},\ \forall v \in \mathcal{V},\ \forall t \in \mathcal{T} \tag{17}$$

$$w_h^{vt}, k_h^{vt}, u_h^{vt} \in \mathbb{Z}^+ \;\forall h \in \mathcal{H},\; \forall v \in \mathcal{V},\; \forall t \in \mathcal{T} \tag{18}$$

$$\tag{19}$$

The objective function (1) is to minimize the cost to prepare all the demands. This cost is given by the sum of the penalization costs due to re-affectation and/or postponement of demands and the cost of the employees. The third term counts the used vehicles and the docks occupation. Coefficients γ_1 and γ_2 are user defined values. Constraints (2) impose that all the packages that form a demand are affected to the same channel of the same day. Constraints (3) impose that each demand is affected to only one channel and served during only one day. Constraints (4) impose that no packages of a demand are treated if the demand is not managed that day. On the other side, they force not to manage more than the packages that constitute the demand the day it is managed. Constraints (5) impose capacity of trucks to be respected on each period of each day and for each different channel. Constraints (6) and (7) manage the arrival of new trucks based on the number of packages that still need to be prepared. Constraints (8) and (9) manage the arrival and departure of trucks at docks. Constraints (10) impose a limit on the number of trucks available for each channel. Constraints (11) impose the number of packages to be prepared in each period should not exceed the capacity of the employees. Constraints (12) impose a limit on the number of fix employees. Constraints (13) impose the number of temporary employees to not be bigger than the fix employees. Constraints (14)–(18) define the variables.

3 Solution method

We propose a matheuristic, i.e., a heuristic algorithm containing steps where a MILP is solved ([2]), for the complex problem described in Section 1. In particular we solve the problem with a two-phase algorithm. In the first phase we solve a relaxation of the model (PSP) where we do not consider the trucks management policy, i.e., we do not look for a solution that minimizes vehicles usage and docks occupation. In particular Constraints (5)–(10)) are replaced by

$$\sum_{v \in \mathcal{V}} \sum_{\bar{h} \in \bar{\mathcal{H}}} \sum_{d \in \mathcal{D}_{h-\bar{h}}} f_{(h-\bar{h})d}^{\bar{h}vt} \le N_{\max} Q + VQ \quad \forall h \in \mathcal{H} \;\forall t \le t_v \tag{20}$$

$$\sum_{t \in \mathcal{T}} \sum_{\bar{h} \in \bar{\mathcal{H}}} \sum_{d \in \mathcal{D}_{h-\bar{h}}} \sum_{v \in \mathcal{V}} f_{(h-\bar{h})d}^{\bar{h}vt} \le |\mathcal{T}| Q N_{\max} \quad \forall h \in \mathcal{H} \tag{21}$$

In this phase we determine the number of workers that are needed in each period. In the second phase we consider the whole problem where the number of workers hired at each period are not anymore decision variables: they are fixed according to the values found in the first phase of the algorithm. The algorithm has been tested on real-size instances. Data are provided by a major logistic company. Encouraging results are obtained in a reasonable amount of time.

References

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