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Upper bound on the performance of an order statistics-based decoder.*

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Abstract. In modern telecommunication systems severe interference caused by various factors can affect the performance of the system drastically. In this case robust reception techniques are to be applied in order to mitigate the performance degradation. In this paper a single user order statistics-based receiver is considered. For the system employing the receiver in question a vector channel model and a decoder are introduced. For the proposed decoder an upper bound on the probability of incorrect decision (i.e. the probability of the fact that decoding will result in erroneous or denial decision) is given.

Keywords: performance, upper bound, order statistics, interference mitigation

1 Introduction.

Interference mitigation is one of the key issues in modern telecommunication systems design. This is mainly due to the fact that interference can be caused by different factors: authorized users' activity in a multiple access system (multi-user interference, MUI), signals transmitted by the users of other telecommunication systems operating within the same frequency bands or intentional jamming. If the interference is severe traditional reception techniques turn out to be ineffective due to low reliability of the computed decision statistics. Recently several robust order-statistics based reception techniques were proposed to solve the problem [1–3]. Unfortunately up to the present moment no analytical description has been proposed for these reception techniques. On the other hand a number of channel models were proposed to address the problem of multiuser system description or jamming environment description [4, 5, 7]. Due to relative simplicity of the models in question estimates on the performance of the communication systems utilizing corresponding channels were obtained (e.g. for the A channel model presented in [4] asymptotic Shannon channel capacity has been

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derived in [8, 9] for various transmission strategies and explicit code constructions approaching the capacity of this channel were presented for both jamming scenario [5] and multiple access systems [6].) Unfortunately these channel models are not very realistic and thus the applicability of the results obtained for the respective channel models to real-life communication systems description is limited. Further a reception technique proposed in [3] is considered. For the aforesaid reception technique a channel model is proposed providing a more realistic description of the real-life communication scenarios than the one given by the channel models considered in [4–9]. For the coded transmission via the channel that can be described by the proposed model a decoder will be introduced. For the proposed decoder an upper bound on the probability of incorrect decision (i.e. the probability of the fact that decoding will result in an erroneous or denial decision) will be given.

2 Channel model and real life communication systems

Let us consider the following channel model: the input of the channel is a length q binary vector of Hamming weight 1 and the corresponding output is a length q binary vector of Hamming weight α . Let us designate the input vector corresponding to a certain time instant as \bar{X}_s and the corresponding output vector as \bar{Y}_s . It will be assumed that with probability p the vector \bar{Y}_s covers \bar{X}_s i.e.

$$\bar{X}_s \wedge \bar{Y}_s = \bar{X}_s \quad (1)$$

where \wedge designates an element-wise conjunction. Correspondingly with probability $\tilde{p} = 1 - p$

$$\bar{X}_s \wedge \bar{Y}_s \neq \bar{X}_s. \quad (2)$$

We shall assume that for any \bar{X}_s all the output vectors meeting the condition (1) are equiprobable and the same holds for the output vectors meeting the condition (2) (it should be noted however that the method that will be used here to obtain an upper bound can be employed even if this assumption does not hold). For the sake of brevity this channel model will be referred to as (α, p) channel. Below real-life communication systems that can be modeled by the (α, p) channel model will be considered.

Let us consider a user transmitting information via a channel split into Q identical nonoverlapping subchannels. These subchannels can be allocated either in the time domain (in this case each subchannel can be e.g. a time slot with a certain number in a TH-IR-UWB system employing PPM [10, 11]) or in the frequency domain (in this case each subchannel can be e.g. a subcarrier in a system employing OFDM [12]).

Further it will be assumed that the user under consideration transmits q -ary symbols. Whenever a user is to transmit a q -ary symbol it places 1 in the position of the vector \bar{x}_g corresponding to the symbol in question within the scope of the mapping in use (in what follows it will be assumed that all the positions

of the vector are enumerated from 1 to Q and all the elements of the finite field in use are enumerated from 1 to q . Moreover for the sake of simplicity (but without loss of generality) we shall consider the simplest form of such mapping: within the scope of this mapping the k th subchannel corresponds to the symbol number k . Thus it will be assumed that each vector \bar{x}_g can be represented as $\bar{x}_g = \begin{bmatrix} \bar{X}_g \\ \bar{Z} \end{bmatrix}$ where \bar{X}_g is the length q vector with one non-zero entry corresponding to the symbol under consideration, and \bar{Z} is an all-zero vector of length $Q - q$. Then a random permutation of the aforesaid vector is performed and the resulting vector $\bar{\chi}_g = \pi_g(\bar{x}_g)$ is sent via the channel in use (permutations are selected equiprobably from the set of all possible permutations and the choice is performed whenever a symbol is to be transmitted); i.e. a signal is sent via the subchannel corresponding to the only non-zero entry of the vector $\bar{\chi}_g$. Hereinafter it will be assumed that K interfering signals are transmitted via the channel in use throughout the period of time within which the user under consideration transmits. This model can be used to describe a multiple access system where all the users transmit using the same method that has been described above (uncoordinated transmission in a multiple access system) or a communication system that is jammed by another communication system operating in the same channel (if the subchannels via which the interfering signals are transmitted are chosen without replacement since different interfering users can choose the same subchannels) or a communication system under intentional jamming (if the subchannels via which the interfering signals are transmitted are chosen with replacement since the jammer can transmit jamming signals via different subchannels.)

Note that the receiver is assumed to be synchronized with the transmitter of the user. Therefore all the permutations done within the scope of transmission of the codeword in question are known to the user. The receiver measures energies at the outputs of all subchannels (let us designate the vector of the measurements as $\bar{\omega}_g$ where g is the number of the transmitted vector) and applies inverse permutation to each vector $\bar{\omega}_g$ corresponding to the respective vector thus reconstructing the initial order of elements and obtaining vector $\bar{w}_g = \pi_g^{-1}(\bar{\omega}_g)$ where each vector \bar{w}_g can be represented in the following form $\bar{w}_g = \begin{bmatrix} \bar{\omega}_g \\ \bar{\theta}_g \end{bmatrix}$, where $\bar{\omega}_g$ is a length q column vector and $\bar{\theta}_g$ is a length $Q - q$ column vector. Let us consider a matrix $W = [\bar{w}_1, \bar{w}_2, \dots, \bar{w}_n]$ and the submatrix $\Omega = [\bar{\omega}_1, \bar{\omega}_2, \dots, \bar{\omega}_n]$ (that is the submatrix corresponding to the q first rows of the matrix W). Please note that the matrix Ω contains all the information about the codeword sent by the user under consideration. Let us assume that for any vector $\bar{\omega}_i$ there is a corresponding vector $\tilde{\omega}_i$ obtained by sorting the elements of the vector $\bar{\omega}_i$ in the descending order. For any given value of the parameter α ($0 < \alpha < q$) the elements of the decision vector \bar{Y}_i corresponding to the received vector $\bar{\omega}_i$ are then given by

$$Y_i(j) = \begin{cases} 1 & \omega_i(j) \geq \tilde{\omega}_i(\alpha) \\ 0 & \omega_i(j) < \tilde{\omega}_i(\alpha) \end{cases} \quad (3)$$

Thus the receiver simply assigns 1's to the entries corresponding to the elements of the vector $\bar{\omega}_i$ that are greater than the α th element of the ordered series $\tilde{\omega}_i$ obtained by sorting the vector $\bar{\omega}_i$ in the descending order and zeros to the remaining elements of \bar{Y}_i . Therefore each vector \bar{Y}_i has Hamming weight α . Since the positions of the nonzero entries in the vector \bar{Y}_s (i.e. the numbers of subchannels chosen by the receiver) depend only on the distribution of the received vector elements and the user under consideration employs random permutations equiprobably chosen from the set of all possible permutations for any \bar{X}_s all the output vectors meeting the condition (1) are equiprobable and the same holds for the output vectors meeting the condition (2). Therefore the single user reception in the scenario under consideration can be described by (α, p) channel. The channel under consideration chooses α "best" (in a certain sense) subchannels (within the scope of the example under consideration the receiver chooses α subchannels with the greatest energy values.) Thus the output of the channel under consideration at each time instant is the list of the numbers of subchannels that could have been used for signal transmission by the user under consideration. Please note that the subchannels via which the interfering signals are transmitted need not be chosen randomly since the receiver employs inverse permutations. The only important condition is that the permutations that are employed by the user should not be known to the interfering users.

Let us now consider major dissimilarities between the proposed model and the channel models considered in [4, 5, 7]. In [4] two channel models: the A-channel model (also referred to as "a channel without intensity information") and the B-channel model (also referred to as "a channel with intensity information") were introduced. Both channel models can be interpreted in terms of a multiuser binary vector channel (even though the authors of ([4]) used another interpretation). The output of the A-channel is an element-wise disjunction of the input vectors, i.e. it is assumed that if a signal has been transmitted via a certain subchannel it will be detected at the receiver side. Thus the A-channel model implies that the probability of a miss (i.e. the probability of the fact that the signal sent via a certain subchannel will not be detected by the receiver) is equal to zero. This assumption is unrealistic since in real-life systems the output of each subchannel is influenced by the additive background noise. The output of the B-channel is the arithmetic sum of the input vectors which is also unrealistic since in real-life systems the resulting signal in each subchannel is a vector sum of the signals transmitted by the users and the output of each subchannel is influenced by the additive background noise. In [5] a J-channel model similar to the A-channel considered in [4] has been introduced. Similarly to the A-channel the J-channel is a binary vector channel and the output of this channel is an element-wise disjunction of the input vector and the vector corresponding to a jamming signal. Therefore similarly to the A-channel model the J-channel model is unrealistic. Thus, the A-channel, B-channel and J-channel can be considered rather crude models of the real-world channels. A more realistic model has been introduced in [7]. The model in question boils down to a serial concatenation of a disjunctive vector channel and a vector BSC (i.e. each output of the disjunc-

tive vector channel is inverted with probability ε). In contrast to the A-channel model in this model the probability of a miss is nonzero. However in this case the probability of a miss does not depend on the number of signals transmitted via the subchannel and is equal to the probability of a false detection which is not the case in real-life systems. The proposed channel model on the other hand can be employed to describe various transmission scenarios (including uncoordinated multiple access and jamming) in various real-life channels (including fading channels) since for each transmission scenario and physical channel model the value of the probability p for the equivalent (α, p) channel model can be obtained.

Since the information provided by the channel at each time instant is not sufficient to restore the transmitted symbol with high reliability an error correcting code is to be used. Thus in what follows it will be assumed that the information that is to be transmitted by the user under consideration is encoded into a codeword of a $C_q(n, k, d)$ code. Thus the model under consideration corresponds either to coded FH OFDMA (if the subchannels are allocated within the frequency domain) or coded PPM-TH-IR UWB system (if the subchannels are allocated within the time domain) with noncoherent detection. Within the scope of a certain codeword reception the receiver is to receive n vectors corresponding to the codeword in question.

In the next section a decoder that uses matrix $Y = [\bar{Y}_1, \bar{Y}_2, \dots, \bar{Y}_n]$ to restore the transmitted codeword will be introduced.

3 The MaxSum decoder.

Let us assume that the user under consideration transmits a codeword v_m and within the scope of the mapping in use this vector is mapped into a matrix $X^m = [\bar{X}_1, \bar{X}_2, \dots, \bar{X}_n]$. For the sake of convenience let us define a row vector V_m of the indices corresponding to the symbols of the codeword v_m :

$$\forall s = 1 : n \quad X_s^m(l) = \begin{cases} 1 & l = V_m(s) \\ 0 & l \neq V_m(s) \end{cases} \quad (4)$$

where $X_s^m(l)$ is the l th element of the s th column of the matrix X^m and $V_m(s)$ is the s th element of the vector V_m .

In what follows we shall assume that the decision statistic for the m th codeword is given by:

$$S_m = \sum_{s=1}^n Y_s(V_m(s)) \quad (5)$$

where $Y_s(V_m(s))$ is the $V_m(s)$ th element of the vector \bar{Y}_s .

The decoding rule boils down to choosing codeword number $m^* = \arg \max_m (S_m)$ if m^* is unique (i. e. $\forall m = 1 : M, m \neq m^* \quad m^* > m$, where $M = |C|$) otherwise a denial decision is taken. In what follows an upper bound for the probability of the fact that the decoding will not result in a correct decision (i.e. the probability that decoding will result in erroneous or denial decision) will be obtained. For

the sake of brevity we shall further on refer to this probability as the probability of incorrect decision and the decoder in question will be referred to as the MaxSum decoder.

4 Upper bounds on the incorrect decision probability.

Let S_i and S_j be the decision statistics corresponding to the codewords v_i and v_j respectively ($\forall i \neq j, d_{ij} = d_H(v_i, v_j)$).

Let us designate the set of positions in which the codewords v_i and v_j differ with $D_{\{i,j\}} (D_{\{i,j\}} : s \in D_{\{i,j\}} \leftrightarrow v_i(s) \neq v_j(s), 1 \leq s \leq n)$ Let us consider the value:

$$\Delta_{ij} = S_i - S_j = \sum_{s \in D_{\{i,j\}}} \xi_{ij}(s) \quad (6)$$

where $\xi_{ij}(s) = Y_s(V^i(s)) - Y_s(V^j(s))$ is the partial difference corresponding to the s -th column of the matrix Y (i.e. the vector \bar{Y}_s), $Y_s(z)$ is the value of the z -th position in the vector \bar{Y}_s . Please note, that the partial differences $\xi_{ij}(s)$ and $\xi_{ij}(s')$ are mutually independent (for any $s \neq s'$) since the value of $\xi_{ij}(s)$ depends on the position of the nonzero values in s -th column of the matrix Y and does not depend on the values of the elements of other columns. Moreover

$$\xi_{ij}(s) \in \{-1, 0, 1\} \quad \forall s \in D_{\{i,j\}} \text{ and } |\Delta_{ij}| \leq |D_{\{i,j\}}| = d_{ij} \quad (7)$$

Let us assume that i -th codeword has been transmitted. According to the decoding rule described above correct decoding will occur if

$$\Delta_{ij} > 0 \quad \forall i \neq j, j \in [1, M] \quad (8)$$

holds.

In order to find the upper bound on the incorrect decision probability we are aiming at finding the distribution of the values $\xi_{ij}(s)$. Let us consider a certain input vector \bar{X}_s corresponding to the s th symbol of the codeword \bar{v}_j transmitted by the user under consideration. Let us designate the position of the nonzero entry in the vector under consideration as $k = V_m(s)$. Please note that the probability of the fact that the output vector \bar{Y}_s covers the corresponding input vector \bar{X}_s is the probability of the fact that the k element of the vector \bar{Y}_s is equal to 1:

$$p = P(\bar{X}_s \wedge \bar{Y}_s = \bar{X}_s) = P(Y_s(k) = 1) \quad (9)$$

Let us consider the g -th position of the vector \bar{Y}_s corresponding to the s -th symbol of the codeword \bar{v}_j . Furthermore let us designate the set of binary vectors \bar{f} of length q and Hamming weight α such that $f(k) = a$ and $f(g) = b$ as $S(\bar{f}, q, \alpha | f(k) = a, f(g) = b)$ and the set of binary vectors f of length q and Hamming weight α such that $f(k) = a$ as $S(\bar{f}, q, \alpha | f(k) = a)$ and the cardinalities of the respective sets as

$$N(\bar{f}, q, \alpha | f(k) = a, f(g) = b) = |S(\bar{f}, q, \alpha | f(k) = a, f(g) = b)| \text{ and } \\ N(\bar{f}, q, \alpha | f(k) = a) = |S(\bar{f}, q, \alpha | f(k) = a)| \text{ respectively. Since the positions}$$

of the nonzero entries in the vector \bar{Y}_s (i.e. the numbers of subchannels chosen by the receiver) depend on the distribution of the received vector elements and the user under consideration employs random permutations equiprobably chosen from the set of all possible permutations, all the $N(\bar{f}, q, \alpha | f(k) = 1) = C_{q-1}^{\alpha-1}$ possible output vectors are equiprobable and therefore:

$$P(\bar{Y}_s(k) \in S(\bar{f}, q, \alpha | f(k) = 1) | \bar{X}_s(k) = 1) = \frac{p}{C_{q-1}^{\alpha-1}} \quad (10)$$

Thus the conditional probabilities for this case are given by:

$$\begin{aligned} P(\xi_{ij}(s) = 1 | \bar{X}_s^m \wedge \bar{Y}_s = \bar{X}_s^m) &= P(Y_s(k) = 1 | X_s(k) = 1, Y_s(g) = 0) = \\ &= \sum_{\bar{Y}_s: \bar{Y}_s(g)=0} P(\bar{Y}_s(k) \in S(\bar{f}, q, \alpha | f(k) = 1) | \bar{X}_s(k) = 1) = \\ &= p \frac{N(\bar{Y}_s, q, \alpha | Y_s(k)=1, Y_s(g)=0)}{N(\bar{Y}_s, q, \alpha | Y_s(k)=1)} = p \frac{C_{q-2}^{\alpha-1}}{C_{q-1}^{\alpha-1}} = p \frac{q-\alpha}{q-1} \end{aligned} \quad (11)$$

and

$$\begin{aligned} P(\xi_{ij}(s) = 0 | \bar{X}_s^m \wedge \bar{Y}_s = \bar{X}_s^m) &= P(Y_s(k) = 1 | X_s(k) = 1, Y_s(g) = 1) = \\ &= \sum_{\bar{Y}_s: \bar{Y}_s(g)=1} P(\bar{Y}_s(k) \in S(\bar{f}, q, \alpha | f(k) = 1) | \bar{X}_s(k) = 1) = \\ &= p \frac{N(\bar{Y}_s, q, \alpha | Y_s(k)=1, Y_s(g)=1)}{N(\bar{Y}_s, q, \alpha | Y_s(k)=1)} = p \frac{C_{q-2}^{\alpha-2}}{C_{q-1}^{\alpha-1}} = p \frac{\alpha-1}{q-1} \end{aligned} \quad (12)$$

Similarly one can claim

$$P(\bar{Y}_s(k) \in S(\bar{f}, q, \alpha | f(k) = 1) | \bar{X}_s(k) = 0) = \frac{1-p}{C_{q-1}^{\alpha}} \quad (13)$$

Therefore

$$\begin{aligned} P(\xi_{ij}(s) = 0 | \bar{X}_s^m \wedge \bar{Y}_s \neq \bar{X}_s^m) &= P(Y_s(k) = 0 | X_s(k) = 1, Y_s(g) = 0) = \\ &= \sum_{\bar{Y}_s: \bar{Y}_s(g)=0} P(\bar{Y}_s(k) \in S(\bar{f}, q, \alpha | f(k) = 0) | \bar{X}_s(k) = 1) = \\ &= p \frac{N(\bar{Y}_s, q, \alpha | Y_s(k)=0, Y_s(g)=0)}{N(\bar{Y}_s, q, \alpha | Y_s(k)=0)} = p \frac{C_{q-2}^{\alpha}}{C_{q-1}^{\alpha}} = (1-p) \frac{q-\alpha-1}{q-1} \end{aligned} \quad (14)$$

and

$$\begin{aligned} P(\xi_{ij}(s) = -1 | \bar{X}_s^m \wedge \bar{Y}_s \neq \bar{X}_s^m) &= P(Y_s(k) = 0 | X_s(k) = 1, Y_s(g) = 1) = \\ &= \sum_{\bar{Y}_s: \bar{Y}_s(g)=1} P(\bar{Y}_s(k) \in S(\bar{f}, q, \alpha | f(k) = 0) | \bar{X}_s(k) = 1) = \\ &= p \frac{N(\bar{Y}_s, q, \alpha | Y_s(k)=0, Y_s(g)=1)}{N(\bar{Y}_s, q, \alpha | Y_s(k)=0)} = p \frac{C_{q-2}^{\alpha-1}}{C_{q-1}^{\alpha}} = (1-p) \frac{\alpha}{q-1} \end{aligned} \quad (15)$$

Therefore the distribution of $\xi_{ij}(s)$ for any s is given by:

$$\begin{aligned} p(\xi_{ij}(s) = 1) &= p \frac{q-\alpha}{q-1} \\ p(\xi_{ij}(s) = 0) &= \frac{q-\alpha-1}{q-1} + p \frac{2\alpha-q}{q-1} \\ p(\xi_{ij}(s) = -1) &= (1-p) \frac{\alpha}{q-1} \end{aligned} \quad (16)$$

and the expectation of this value is given by

$$E(\xi_{ij}(s)) = p \frac{q-\alpha}{q-1} - \left((1-p) \frac{\alpha}{q-1} \right) = p \frac{q}{q-1} - \frac{\alpha}{q-1} = \mu. \quad (17)$$

Thus each decision statistic

$$\Delta_{ij} = S_i - S_j = \sum_{s \in D_{\{i,j\}}} \xi_{ij}(s)$$

is the sum of independent bounded values and its expectation is given by

$$E(\Delta_{ij}) = \mu d_{ij}.$$

Thus applying Hoeffding inequality [13] we obtain:

$$p(\Delta_{ij} \leq E(\Delta_{ij}) - t) \leq e^{-\frac{t^2}{2d_{ij}}} \quad (18)$$

$$p(\Delta_{ij} \leq 0) \leq e^{-\frac{\mu^2 d_{ij}}{2}}. \quad (19)$$

Since $d_{ij} \geq d$ applying union bound we can claim that the incorrect decision probability can be upper bounded by:

$$p_I = \sum_{i=1}^{M-1} p(\Delta_{ij} \leq 0) \leq (M-1) e^{-\frac{\mu^2 d}{2}}. \quad (20)$$

A more tight bound can be obtained if the $C_q(n, k, d)$ code in use is a linear code with a known weight spectrum. Then the upper bound is given by:

$$p_I = \sum_{i=1}^{M-1} p(\Delta_{ij} \leq 0) \leq \sum_{w=d}^n A(w) e^{-\frac{\mu^2 w}{2}}. \quad (21)$$

5 Discussion and future work.

Let us now consider the obtained bound in its simplest form (20). In fact we shall use a crude approximation of (20):

$$p_I < M e^{-\frac{\mu^2 d}{2}} = e^{\ln(M) - \frac{\mu^2 d}{2}} = e^{k \ln q - \frac{\mu^2 d}{2}}. \quad (22)$$

Thus p_I decreases with d as long as

$$k \ln q - \frac{d\mu^2}{2} < 0 \quad (23)$$

holds, which yields

$$\mu > \sqrt{\frac{2k \ln q}{d}}. \quad (24)$$

where μ is given by (17). Therefore inequality (24) guarantees that for any fixed α the probability of incorrect decision decreases with d as long as $p(\alpha)$ satisfies

$$p(\alpha) > \left(\sqrt{\frac{2k \ln q}{d}} + \frac{\alpha}{q-1} \right) \frac{q-1}{q}. \quad (25)$$

Thus very low rate codes (e.g. repetition codes) are to be used in the system employing the proposed decoder. Since both the right and the left part of inequality (24) depend on α it is important to find explicit estimates on $p(\alpha)$. This problem is out of the scope of this paper and is a subject for future work.

6 Conclusion.

Hereinabove the problem of finding upper bounds on the performance characteristics of an order-statistics based single user receiver operating under severe interference has been considered. To solve this problem an (α, p) channel model has been introduced. For this channel model a simple decoder has been proposed. It has been shown that the problem of finding the upper bound on the incorrect decision probability can be interpreted in terms of the well known problem of finding an upper bound on the probability of the fact that the sum of independent random values deviates from the expected value. Using the afore-said techniques explicit upper bounds on the performance characteristics of the communication system under consideration were obtained.

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