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Department of Information security, Royal Holloway College, University of London, UK
Mwawi.NyirendaKayuni.2011@live.rhul.ac.uk,
{S.Ng, keith.martin}@rhul.ac.uk,

Abstract. In a frequency hopping multiple access (FHMA) system a set of users communicates simultaneously using frequency hopping sequences defined on the same set of frequency channels. A frequency hopping sequence is comprised of frequency channels used to switch channels as communication progresses. Much of the research on the performance of FHMA systems is based on either pairwise mutual interference or adversarial interference but not both. In this paper, we evaluate the performance of an FHMA system with respect to both *group-wise mutual interference* and *adversarial interference* (jamming), bearing in mind that more than two users may be transmitting simultaneously in the presence of an adversary. Our main contributions are as follows. We point out a correspondence between a cover-free code and a frequency hopping scheme. Cover-free codes give a well defined structure on the transmission capacity of a frequency hopping multiple access system. Further, we specify a jammer model for an FHMA. Considering the resources and knowledge of a jammer, we look at the possible mitigation strategies to be employed by a frequency hopping scheme against the jammer, and determine necessary and desirable additional properties of cover-free codes such that they may be used in the presence of adversarial interference.

1 Introduction

Frequency hopping (FH) sequences are widely used in signal transmission such as WI-FI, Bluetooth, ultrawideband (UWB) communications [8, 19, 22]. A frequency hopping sequence specifies which channel to switch to as communication progresses. Compared to stationary transmission techniques, this technique has advantages such as improved performance in the presence of interference. In a frequency hopping multiple access (FHMA) system a set of users communicate simultaneously using frequency hopping sequences defined on the same set of frequency channels. We are interested in investigating properties of frequency hopping schemes for FHMA systems used in the presence of adversarial interference.

In literature the well-known Hamming correlation has been used as a criteria to measure performance of FH sequences. Numerous constructions give FH se-

quences which meet either the Lempel-Greenberger bound [12] or the Peng-Fang bound [15]. For example, sets of FH sequences are taken as a set of representative codewords from each cyclic equivalence class of a code [17], a coset of a Reed-Solomon code [20], and a transformation of m-sequences [12]. Further, FH sequences have been constructed based on cyclotomies [2–6, 18, 24]. Much of this research uses Hamming correlation as a measure of the performance of FH sequences and do not consider adversarial interference.

On the other hand, in [1] and [7], the authors considered adversarial interference only and not Hamming correlation. In both constructions, an adversary can eavesdrop and jam a number of frequency channels over a given period of time. In [7] the authors consider only a single pairwise communication while in [1] the authors consider multiple pairwise communication over a given period of time. Unfortunately, it was shown in [13] that the set of FH sequences in the latter [1], is insecure.

In this paper, we evaluate the performance of an FHMA system with respect to both *group-wise mutual interference* and *adversarial interference*. This framework was introduced in [13], bearing in mind that more than two users may be transmitting simultaneously in the presence of an adversary therefore rending the pairwise mutual interference criterion inadequate.

An overview of our contributions are as follows. We point out a correspondence between a cover-free code and a frequency hopping scheme. We note that when a cover-free code is considered as a frequency hopping scheme, then a user can successfully transmit in at least a specified fraction of time in the presence of a given number of interfering FH sequences. We specify a jammer model for an FHMA. Considering the resources and knowledge of a jammer, we look at how a frequency hopping scheme may mitigate against the jammer. We examine necessary and desirable additional properties of cover-free codes such that they may be used in the presence of adversarial interference. A cover-free code will enable us to determine the number of places a FH sequence can be successfully used in the presence of other interfering FH sequences. However, it provides no additional information for use of the FH sequences in the presence of adversarial interference. Therefore, we seek to determine these additional properties of cover-free codes that mitigate adversarial interference activities.

The rest of the paper is organised as follows. In Section 2 we introduce the system model and the necessary notation. In Section 3 we introduce cover-free codes and show their equivalence to frequency hopping schemes. In Section 4 we introduce the attacker model and explore further properties of cover-free codes being used in the presence of adversarial interference.

2 System model

We consider a frequency hopping multiple access system. Users communicate pairwise and pre-agree on a FH sequence to be used in a session. A session is a number of pre-defined time slots. We take a time slot as a unit of time.

Let $\mathcal{F} = \{f_0, f_1, \ldots, f_{m-1}\}$[1] be a finite alphabet of m frequency channels; $\mathcal{F}$ is called a *frequency library*.

Definition 1. *A **frequency hopping** (**FH**) **sequence** is a sequence $X = (x_t)_{t=0}^{v-1}$ of length v over a frequency library $\mathcal{F}$.*

Definition 2. *A (v, m, k)-**frequency hopping scheme** ((v, m, k)-FHS), is a set $\mathcal{S} = \{X_i : 0 \leq i \leq k-1\}$ of size k where X_i is an FH sequence of length v over a frequency library $\mathcal{F}$ of size m.*

Definition 3. *The **Hamming group correlation** $G(X,\mathcal{U})$ between a FH sequence $X \in \mathcal{S}$ and the FH sequences in $\mathcal{U} \subseteq \mathcal{S}, |\mathcal{U}| = w, 1 \leq w \leq k$, is defined as the number of coordinates in X that contain the same symbols as the corresponding coordinates of some FH sequence in $\mathcal{U}$,*

$$G(X,\mathcal{U}) = |\{x_t | \exists Y \in \mathcal{U} \text{ such that } x_t = y_t, t = 0, \ldots, v-1\}|. \tag{1}$$

The Hamming group correlation $G(X,\mathcal{U})$ gives the number of coordinates of an FH sequence X that are blocked by the FH sequences in the w-subset $\mathcal{U}$ of $\mathcal{S}$.

Definition 4. *Let $\mathcal{S}$ be a (v, m, k)-FHS. Let $\mathcal{U} \subseteq \mathcal{S}$, $|\mathcal{U}| = w, 1 \leq w \leq k$ and $X \in \mathcal{S} \setminus \mathcal{U}$. Then the w-**throughput** of X is the rate of successful transmission in a session, in the presence of FH sequences in $\mathcal{U}$,*

$$\rho_w(X,\mathcal{U}) = 1 - \frac{G(X,\mathcal{U})}{v}. \tag{2}$$

It is desirable that $\rho_w(X,\mathcal{U})$ be large, so a FH sequence transmits in many time slots.

Given a (v, m, k)-FHS, $\mathcal{S}$, let $\mathcal{V} \subseteq \mathcal{S}$, $|\mathcal{V}| = w + 1$. Then the worst-case w-throughput of a (v, m, k)-FHS, $\mathcal{S}$, is the minimum of the values $\rho_w(X, \mathcal{V} \setminus \{X\})$s for each possible FH sequence X and w-set $\mathcal{V} \setminus \{X\}$ in $\mathcal{S}$ not containing X,

$$\hat{\rho}_w(\mathcal{S}) = \min_{\substack{\mathcal{V} \subseteq \mathcal{C} \\ |\mathcal{V}| = w+1}} \left\{ \min_{X \in \mathcal{V}} \{\rho_w(X, \mathcal{V} \setminus \{X\})\} \right\} \tag{3}$$

A (v, m, k)-FHS, $\mathcal{S}$, with worst-case w-throughput $\hat{\rho}_w(\mathcal{S})$ will be denoted a $(v, m, k; \hat{\rho}_w(\mathcal{S}))$-FHS.

[1] For simplicity, we take a one-to-one mapping between the frequency channels in $\mathcal{F}$ and a set of m elements, that is $f_i = i, 0 \leq i \leq m-1$.

3 Cover-free Codes as Frequency Hopping Schemes

3.1 Cover-free codes

The notion of cover-free codes has been used in [10, 11, 21] for blacklisting and traitor tracing schemes. In this paper we use the definition of [21] in the setting of frequency hopping schemes.

Let $\mathcal{F}^v$ be the universal set of m-ary words of length v. A code $\mathcal{C} \subseteq \mathcal{F}^v$ with k codewords and minimum Hamming distance d is denoted as a $(v, k, m; d)$-code or as (v, k, m)-code when d is unspecified.

Definition 5 (Staddon, Stinson and Wei, [21]). *Suppose that $\mathcal{C}$ is a (v, k, m)-code. For any subset $\mathcal{C}' \subseteq \mathcal{C}$ and any $\in \mathcal{F}^v$, define*

$$I(X, \mathcal{C}') = \{i : x_i = y_i \text{ for some } Y \in \mathcal{C}'\}. \tag{4}$$

*Then $\mathcal{C}$ is called (w, α)-**cover-free code**, denoted (w, α)-CFC, if $|I(Z, \mathcal{C}')| < (1 - \alpha)v$ for any $\mathcal{C}' \subseteq \mathcal{C}$, $|\mathcal{C}'| = w$ and any $Z \in \mathcal{C} \setminus \mathcal{C}'$.*

3.2 Equivalence of Cover Free Codes and Frequency Hopping Schemes

There is a direct correspondence between a frequency hopping scheme with a given Hamming group correlation and a cover free code.

Theorem 1. *Suppose $\mathcal{C}$ is a (v, k, m)-code over $\mathcal{F}$, $|\mathcal{F}| = m$. Then $\mathcal{C}$ is a $(w; \alpha)$-CFC if and only if $\mathcal{C}$ is a (v, m, k)-FHS with worst-case w-throughput at least α.*

Proof. The proof is straightforward and will be included in the full paper.

It was proved in [21] (Theorem 4.3) that codes with large minimum distance are cover-free codes.

Example 1. A $(v, m, m; v)$-repetition code is a $(m-1, 1)$-CFC, so it is a $(v, m, m; 1)$-FHS. The $(m-1)$-throughput of any FH sequence is 100%, so the worst-case $(m-1)$-throughput is 100%.

Theorem 2. *([21]) Suppose that $\mathcal{C}$ is a $(v, k, m; d)$-code such that $d > v(1 - \frac{1}{w^2})$. Then $\mathcal{C}$ is a $(w, 1 - \frac{1}{w})$-CFC.*

The following theorem was proved in [10] (Corollary 2.1), it provides a correspondence between MDS codes and frequency hopping schemes.

Theorem 3. *([10]) Let $\mathcal{C}$ be a $(v, k, m; d)$-linear MDS code. If $\mathcal{C}$ is a $(w, 1-\frac{1}{w})$-CFC, then $d > v(1-\frac{1}{w^2})$ and vice versa.*

From Theorems 2 and 3 we conclude that a $(v, k, m; d)$-linear MDS code where $d > v(1-\frac{1}{w^2})$ is also a $(w, 1-1/w)$-CFC which give us a (v, m, k)-FHS with worst-case w-throughput at least $1-1/w$.

4 Attacker Model

We consider the presence of an adversary that send noisy signals on frequency channels to block the signal transmissions of legitimate users; we call this adversary a *jammer*. It knows $\mathcal{F}$, (v, m, k)-FHS, $\mathcal{C}$, and $w+1$ $(0 < w < k)$ the number of FH sequences to be used in a session. However, it has no knowledge of the actual FH sequences to be used. Its strategy is to eavesdrop and jam. At each time slot it has enough resources to eavesdrop on $\theta_1 m$ channels, $0 \leq \theta_1 \leq 1$, and jam on $\theta_2 m$ channels, $0 \leq \theta_2 < 1$. We assume that it cannot jam all the frequency channels at each time slot. It may use the information it acquires while eavesdropping to jam. This adversary is denoted (θ_1, θ_2)-*adaptive jammer*. When a signal is jammed, legitimate users hear noise and acknowledge failure of transmission. So we treat a jamming signal as an erasure. In this paper, the goal of a jammer is to reduce the worst-case w-throughput of a (v, m, k)-FHS, $\mathcal{C}$.

We model a jammer's channel selection strategy for jamming as a set of FH sequences $\mathcal{J} = \{Y_i | i = 0, \ldots, \theta_2 m - 1\}$, where Y_i is an FH sequence of length v over $\mathcal{F}$. The $(w, \mathcal{J})$-throughput of a FH sequence X in the presence of *both* other legitimate FH sequences of $\mathcal{C}' \subseteq \mathcal{C}$, $|\mathcal{C}'| = w$ and jamming FH sequences of $\mathcal{J}$ is,

$$\rho_{w,\mathcal{J}}(X, \{\mathcal{C}' \cup \mathcal{J}\}) = 1 - \frac{G(X, \{\mathcal{C}' \cup \mathcal{J}\})}{v}. \tag{5}$$

The worst-case $(w, \mathcal{J})$-throughput of a (v, m, k)-FHS, $\mathcal{C}$, is the minimum number of time slots every FH sequence in $\mathcal{C}$ can transmit in the presence of *both* some other FH sequences and jamming FH sequences of $\mathcal{J}$,

$$\hat{\rho}_{w,\mathcal{J}}(\mathcal{C}) = \min_{\substack{\mathcal{C}'' \subseteq \mathcal{C} \\ |\mathcal{C}''| = w+1}} \left\{ \min_{X \in \mathcal{C}''} \{\rho_{w,\mathcal{J}}(X, \{\mathcal{C}'' \setminus \{X\} \cup \mathcal{J}\})\} \right\}. \tag{6}$$

In literature, jammers are classified according to their capabilities (broadband or narrowband) and their behaviour (constant, random or reactive) [14, 16, 23]. Our (θ_1, θ_2)-adaptive jammer includes all the jammers that apply to an FHMA system.

4.1 Jamming Resistance Properties for Cover-Free Codes

In this section we look at the performance of cover free codes in the presence of both mutual interference and a jammer. We delve into further properties that cover-free codes should have to mitigate an adaptive (θ_1, θ_2)-*jammer*. For simplicity, we assume $\theta_1 m = \theta_2 m = 1$.

Consider a $(v, m, k; \hat{\rho}_w(\mathcal{C}))$-FHS, $\mathcal{C}$, over $\mathcal{F}$. In any session there are $w+1$ FH sequences that are in use by legitimate users, we call them *active FH sequences*. At any time slot t, $0 \leq t \leq v-1$, in a session, there are at most $w+1$ frequency channels in use which we call *active channels*. So, at any time slot t, the multiset $\mathcal{F}_t = (x_t^0, ..., x_t^{k-1})$ denotes all the channels that appear in all the FH sequences at that time. The vector $M_t = (a_0, ..., a_{m-1})$ denotes the multiplicity of each channel at time slot t where $a_i = |\{j : x_t^j = i\}|$. Clearly a jammer has this information. However, the multiset of active frequency channels is $\mathcal{F}_t^{active} = (x_t^{i_0}, ..., x_t^{i_w})$ with the vector of multiplicities of the active channels as $M_t^{active} = (a'_0, ..., a'_{m-1})$ where $a'_i = |\{j : x_t^{i_j} = i\}|$. Note that $a'_i \leq a_i$ for all i. The jammer does not know $\mathcal{F}_t^{active}$ or M_t^{active}.

A jammer aims to identify an active FH sequence , then it can reduce the worst case w-throughput to 0 or close to 0. The number of time slots it takes a jammer to determine an active FH sequence is denoted γv, $0 < \gamma \leq 1$. It is desirable that γ be large. The aim of the $(v, m, k; \hat{\rho}_w(\mathcal{C}))$-FHS, $\mathcal{C}$, is to make the jammer's advantage not much better than a random guess.

A jammer can trivially reduce the w-throughput to 0 if it knows $\mathcal{F}_t^{active}$ and M_t^{active}. Consider $\mathcal{C}'' = \mathcal{C}$. Clearly a jammer knows $\mathcal{F}_t^{active}$ and M_t^{active} for all $0 \leq t \leq v-1$. As a mitigation strategy against this trivial case, let $\mathcal{C}'' \subset \mathcal{C}$. From henceforth, we will assume $\mathcal{C}'' \subset \mathcal{C}$, only a fraction of the FH sequences in the scheme are active.

If the jammer doesn't know $\mathcal{F}_t^{active}$ or M_t^{active} it can always guess which frequency channel to eavesdrop on. At time 0, there are k FH sequences assumed to be equally likely over m frequency channels, and for each frequency channel i there are a_i FH sequences of that frequency channel. The probability that frequency channel i is active is,

$$Prob(i \text{ is active}) = 1 - \binom{k - a_i}{w+1} / \binom{k}{w+1}.$$

The probability above is maximum when the jammer selects a frequency channel i such that $a_i \geq a_j$ for all $i \neq j$. Therefore, if there exists some i such that $a_i \geq a_j$ for all $i \neq j$, then a jammer would choose such frequency channel i. A jammer does the same strategy at any time slot $0 \leq t \leq v-1$. A mitigation strategy against a $(\frac{1}{m}, \frac{1}{m})$-jammer is that a (v, m, k)-FHS should have the property that all frequency channels used at any time slot t are uniformly distributed. Recall, for an adaptive jammer what happens at time t informs its next action on $t+$

1. Therefore a (v, m, k)-FHS should further be that for all FH sequences with frequency channel i at time slot t, all frequency channels on the next time slot $t+1$ should be again uniformly distributed. This forces a jammer to guess randomly at any time slot.

The desirable properties of a (v, m, k)-FHS described above are those of orthogonal arrays.

Definition 6 (Hedayat, Sloane and Stufken, [9]). *A $k \times v$ array A with entries from $\mathcal{F}$ is said to be an **orthogonal array** with m levels, strength t', $0 \leq t' \leq v-1$, and index λ if every $k \times t'$ subarray of A contains each t' tuple based on $\mathcal{F}$ exactly λ times as a row and is denoted $OA_\lambda(m^{t'}, v, m, t')$.*

An $\mathrm{OA}_1(m^{t'}, v, m, t')$, A, is a $(v, m^{t'}, m; v-t'+1)$ MDS code, $\mathcal{C}$. Suppose we treat our $(v, m^{t'}, m; v-t'+1)$ MDS code as a $(v, m, m^{t'})$-FHS. Then the properties of the $(v, m, m^{t'})$-FHS are as follows. Consider any t' consecutive time slots, for simplicity, $0, \ldots, t'-1$. Any frequency channel in $\mathcal{F}$ appears $m^{t'-1}$ number of times in time slot 0. Next consider any $m^{t'-1}$ FH sequences with a frequency channel in $\mathcal{F}$ that appeared in the previous time slot. Then in time slot 1, any frequency channel in $\mathcal{F}$ appears $m^{t'-2}$ number of times. In time slot $t'-1$ any frequency channel in $\mathcal{F}$ appear once on FH sequences with a particular frequency channel on time slot $t'-2$.

Now we introduce a $(\frac{1}{m}, \frac{1}{m})$-jammer in our $(v, m, m^{t'})$-FHS. Since at any time slot t, $0 \leq t \leq v-1$, the number of times any frequency channel in $\mathcal{F}$ is used is uniform, a jammer randomly guesses a frequency channel to eavesdrop on. For any active frequency channel in $\mathcal{F}$ that it eavesdrops on, its search is done if the multiplicity of that frequency channel is 1, that is one of the $w+1$ active FH sequence is discovered. Otherwise at time slot $t+1$ its search is concentrated on the FH sequences with that particular channel that appeared in the previous time slot. However, for any inactive frequency channel in $\mathcal{F}$ it eavesdrops on at any time slot t, it discards the FH sequences with that specific channel and on $t+1$ continues its search on the remaining possible active FH sequences at time t. The jammer continues this action until one active frequency channel is found or until the end of a session.

If a jammer is really lucky such that it eavesdrops on an active frequency channel from the first time slot onwards, then by the properties of frequency channels in the first t' consecutive time slots of the $(v, m, m^{t'}; \hat{\rho}(\mathcal{C}))$-FHS, it restricts its search to $m^{t'-(t+1)}$ FH sequences at each time slot $0 \leq t \leq t'-1$. So one of the active FH sequence is discovered on the $t'-1$ time slot since only one active channel remain meaning one active FH sequence . Therefore we have $\gamma v = t'$ in this case.

We conclude on γv. A $(v, k, m; d)$-linear MDS code, $\mathcal{C}$, of size k over an alphabet of size m with distance $d > v(1-\frac{1}{w^2}), 1 \leq w \leq k$ is also a $(w, 1-\frac{1}{w})$-CFC and will give a (v, m, k)-FHS with $\hat{\rho}_w(\mathcal{C}) > 1-\frac{1}{w}$ if $w+1$ sequences are used in a

session and will resist a $(\frac{1}{m}, \frac{1}{m})$-adaptive jammer for γv number of time slots, $\gamma v \geq v - d + 1$, if $w + 1 \leq (m-1)^{v-d+1}$.

Example 2. A $(v, m, m; v)$-repetition code can withstand a jammer up to $1 \leq \gamma v \leq m - (w + 1)$, $w < m - 1$.

5 Conclusion

We have considered cover-free codes in the context of frequency hopping multiple access systems in the presence of an active adversary. We looked at linear MDS codes which are cover-free codes of certain parameters as frequency hopping schemes and we were able to determine the worst-case throughput of the frequency hopping scheme. However, when they are used in the presence of an adaptive jammer, then they do not withstand it for long. Therefore we conclude as it was noted in the strongly resilient Bag-Ruj-Roy (sR-BRR) and strongly resilient Latin square (sR-LS) schemes in [13] that there is a good indication that a cover-free code used in conjunction with pseudo random number generator may provide a frequency hopping scheme that can be used in the presence of an adversary.

As a possible future work, we will investigate combining pseudo-randomness to the MDS codes and analyse their performance.

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