Simplified theory of an active lift turbine with controlled displacement

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Abstract

It is presented in this article, a simplified theory of the active lift turbine which has been the subject of several patent[4, 5, 11].
A simplified theory is proposed to extend the Betz limit of the yield on vertical axis wind turbine. This work can be extended either on wind driven or marine current turbine. Based on kinetic energy calculation, that theory demonstrates that the radial force acting on the blade can be used to extend the maximum recoverable power, mainly by transforming a linear motion into a rotating motion. The geometry of this new type of vertical axis turbine is described. Finally the driving power is calculated through the tangential power coefficients. Consequences on the velocity variation generation and induced vibrations are exposed.
Two parts is treated in this article : The calculation of power coefficient and the calculation natural modes of vibration

Keywords
Darrieus, betz limit, VAWT, Vertical axis turbine, Alternative linear motion

*Inventor of the active lift turbine concept

1 prelude

Vertical axis turbines, driven by wind or water, have been intensively developed after the 70's (South and Rangi [12] from the National Research Council of Canada). The Darrieus wind turbine type was initialy patented in the USA in 1931 (Darrieus [7]) in which both curved blade shape (Troposkien) and straight bladed rotor were designed. Continuous technical developments have been pursued thanks to experimental works in wind tunnels (Sheldahl et al. [9], Blackwell et al. [8]). H-type rotor including straight blades and speed control have been studied by Musgrove [10] in the mid 80's.

2 The calculation of power coefficient

2.1 Introduction :

German physicist Albert Betz[1] has proven that the recoverable maximum power for a wind turbine is closed to 60% (16/27) of the power of the wind that runs through the area swept by the sensor. That theory is based on the calculation of the kinetic energy. Modern wind turbines are reaching aeronautical yields close to the Betz limit. These wind turbines are subjected to very substantial structural stresses. When the wind speed becomes too high, in the order of 40 m/s, it is necessary to force these large wind turbines to stall and then to stop. The main purpose presented in this paper is to transform these constraints into additional energy recovery. The wind turbines with horizontal axis are facing constant structural stresses without a complementary system, this type of turbine can not transform directly these constant stresses into energy.
The (vertical or horizontal axis) Darrieus [7] turbines are facing stresses alternatively in each arm during each rotation. These structural stresses in the arms are of compression or extension type. the forces causing alternative constraints can be transformed to recover the extra energy.
The blades of the Darrieus-type turbines have an incidence angle which varies during the rotation. The induced force of the fluid velocity on the profile of the blade depends essentially on the fluid velocity and the angle of incidence.
This induced force can be decomposed into a tangential force and a radial force. The tangential force associated to the arm radius creates a torque. The radial force causes alternative constraints on the structure of turbine arm. Much research is performed on the angle of incidence in order to maximize the value of the tangential force or whether to try to keep that angle

at a constant value for each half turn of revolution.
With aircraft type profiles (NACA, etc.), the radial force is substantially greater than the tangential force and thus creates alternative structural stresses on the arms.
The first version of the active lift turbine [4, 3] was to put linear stiff generators directly inside the arms. The rotation frequency of the turbines is low thus the yields of these generators are very low. The second version of the active lift turbine is to transform the alternative linear motion into a rotating motion. This solution allows the energy recovery directly on the same rotation axis due to the tangential force, associated to the arm radius, but also extra energy due to alternative constraints Induced force created by the fluid on the blade profile is due to the relative speed which is the combination of the fluid velocity and of the rotation speed of the profile. With the active lift turbine, the relative velocity is the combination of the fluid velocity, the rotational velocity of the profile and the linear radial velocity.

2.2 Geometry of the Active Lift Turbine :

The point O is the main axis of the turbine. The gear F with its center A is not coincident with the main axis O.The gear F is fixed relative to the direction of flow. The satellite gear S (center B) is turning without sliding around the fixed gear F (center A). The slide support is one of the turbine's arm and turns around the axis O. The profile (blade) is fixed with the slide bar in E. The slide bar has a reciprocating translatory motion with a low speed of movement. The rod is connected to the slide bar by a pivot connection in D and is connected to the satellite gear S by a pivot connection in C. The slide bar is driven in rotation by the slide support.

$$\bar{OA} = e \quad \bar{AB} = 2r \quad \bar{BC} = r_b \quad \bar{CD} = L \quad \bar{DE} = d \quad \vec{OE} = R(\beta)\vec{\imath}$$

$$R : radius \qquad \bar{R} : mean\, radius \qquad \dot{R} = \frac{\partial R}{\partial t}$$

Standard Darrieus Turbine :

$$R = 2r + d + L \qquad \bar{R} = R \qquad \dot{R} = 0$$

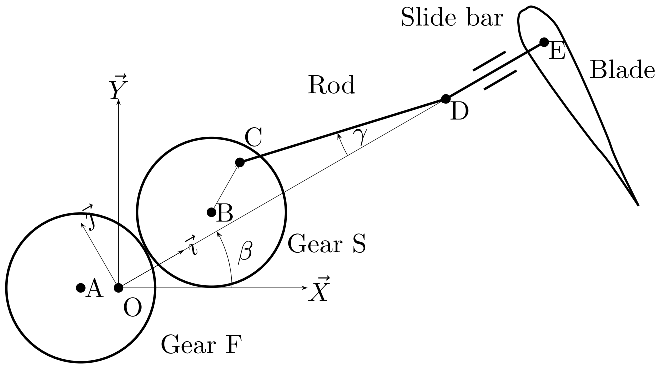


Figure 1: geometry

Active lift turbine :

$$\gamma = angle\,\widehat{ODC} \qquad \cos\gamma \approx 1 \qquad avec \qquad L > 2r$$

$$R \approx 2r + d + L + (r_b - e)\cos\beta \qquad \bar{R} \approx 2r + d + L$$

$$R \approx \bar{R} + (r_b - e)\cos\beta \tag{1}$$

$$\dot{R} = (-r_b + e)\dot{\beta}\sin\beta \tag{2}$$

2.3 Theory on Power Coefficient :

2.3.1 Transformation of the slide force into a torque :

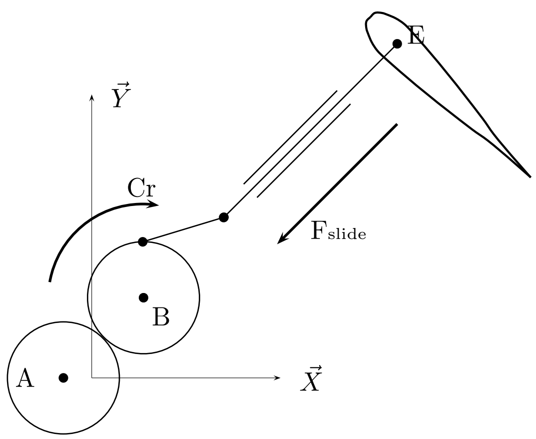


Figure 2: Actions on the rod

$$\bar{AE} \approx 2r + d + L + (r_b - e)\cos\beta \qquad \bar{AB} = 2r - e\cos\beta$$

The variation of the length AE is negligible compared to the variation of AB

$$r_b \sim e \quad \Delta AE_{max} = |r_b - e| \quad \Delta AB_{max} = e \quad \Delta AE \ll \Delta AB \quad \Delta AE \approx 0$$

The system is equivalent to a crank rod.

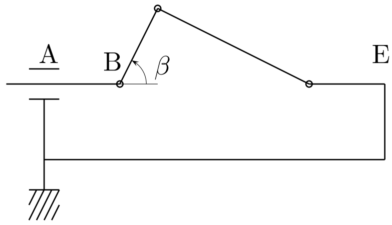


Figure 3: crank rod

Equality of powers provides the couple

$$C_r\dot{\beta} = F_{slide-bar}\frac{d(\bar{AB})}{dt} = F_{slide-bar}\dot{R}$$

By using (2), the torque is equal to:

$$C_r = \{e\,\sin\beta\}F_{slide-bar} \tag{3}$$

2.3.2 Power coefficient calculation

2.3.2.1 The Bernoulli equations

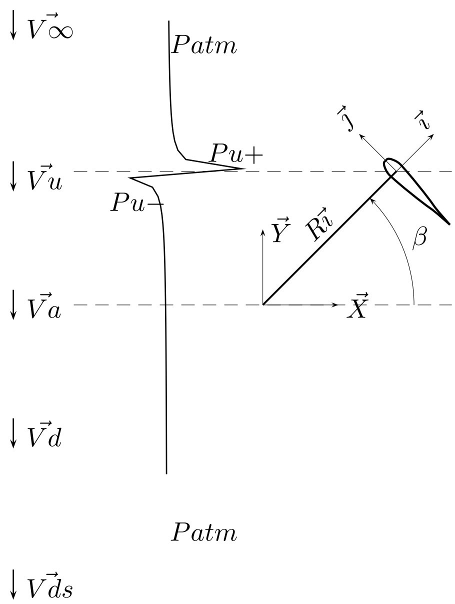


Figure 4: Pressure variations

$$\vec{V_\infty} = V_\infty \vec{Y} \qquad with \qquad V_\infty \leq 0$$

$$P_{atm} + \frac{1}{2}\rho V_\infty^2 = P_{u+} + \frac{1}{2}\rho V_u^2$$
$$P_{u-} + \frac{1}{2}\rho V_u^2 = P_{atm} + \frac{1}{2}\rho V_a^2 = P_{d+} + \frac{1}{2}\rho V_d^2$$
$$P_{d-} + \frac{1}{2}\rho V_d^2 = P_{atm} + \frac{1}{2}\rho V_{ds}^2$$

From these equations, we can defined these pressure differences

$$P_{u+} - P_{u-} = \frac{1}{2}\rho(V_\infty^2 - V_a^2) \qquad P_{d+} - P_{d-} = \frac{1}{2}\rho(V_a^2 - V_{ds}^2) \tag{4}$$

2.3.2.2 The continuity equation along a streamtube

$$dAx_u V_u = dAx_d V_d \quad dAx_u = R_u d\beta_u |\sin\beta_u| dH \quad dAx_d = R_d d\beta_d |\sin\beta_d| dH$$

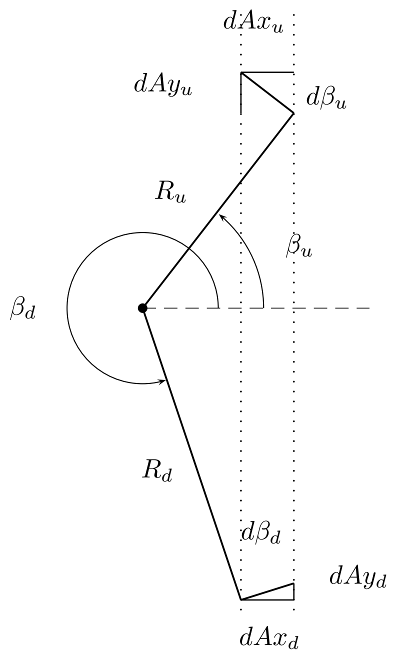


Figure 5: streamtube

$$dAy_u = R_u d\beta_u |\cos\beta_u| dH \qquad dAy_d = R_d d\beta_d |\cos\beta_d| dH$$
$$dH : elementary\, height\, of\, the\, blade$$

We can define the relation between these two elementary surfaces

$$dAx_d = dAx_u \frac{V_u}{V_d} \tag{5}$$

2.3.2.3 Forces on a blade crossing a streamtube

$$0 \leq \beta \leq \pi \quad dFy_u = (P_{u+} - P_{u-}) dAx_u \quad dFx_u = (P_{u+} - P_{u-}) dAx_u \frac{1}{tan\beta}$$

$$\pi \leq \beta \leq 2\pi \quad dFy_d = (P_{d+} - P_{d-}) dAx_u \frac{V_u}{V_d} \quad dFx_d = (P_{d+} - P_{d-}) dAx_u \frac{V_u}{V_d} \frac{1}{tan\beta}$$

By using(5), these equations become

$$dFy_u = \frac{1}{2}\rho(V_\infty^2 - V_a^2) dAx_u \quad dFy_d = \frac{1}{2}\rho(V_a^2 - V_{ds}^2) dAx_u \frac{V_u}{V_d} \tag{6}$$

$$dFx_u = \frac{1}{2}\rho(V_\infty^2 - V_a^2) dAx_u \frac{1}{tan\beta} \quad dFx_d = \frac{1}{2}\rho(V_a^2 - V_{ds}^2) dAx_u \frac{V_u}{V_d} \frac{1}{tan\beta} \tag{7}$$

2.3.2.4 Euler Equations

$$dFy_u = (V_\infty - V_a) dm_u \quad dm_u = \rho V_u dAx_u$$
$$dFy_d = (V_a - V_{ds}) dm_d \quad dm_d = \rho V_u dAx_u \tag{8}$$

By using(6), the previous equations, we determine the following relations between the velocities

$$V_a = 2V_u - V_\infty \qquad V_{ds} = 2(V_d - V_u) + V_\infty$$

Finally, the Euler equations give

$$dFy_u = 2(V_\infty - V_u)\rho V_u dA_u \quad dFy_d = 2(2V_u - V_d - V_\infty)\rho V_u dA_u \tag{9}$$

2.3.2.5 Projection of the aerodynamic force Induced velocity creates an aerodynamic force on the profile which can be decomposed into a lift and a drag force.

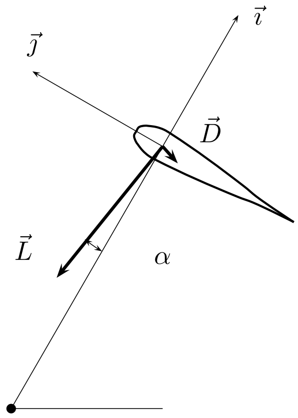
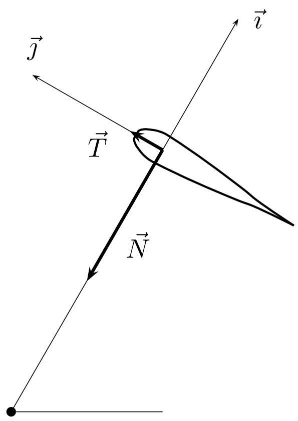


Figure 6: Lift and drag force on a blade

At the turbine level, it is important to know the decomposition of this force into a normal force N, and a tangential force T.



Forces are set according to X and Y axis from the normal force N and the tangential force T

Figure 7: Axial and normal force on a blade

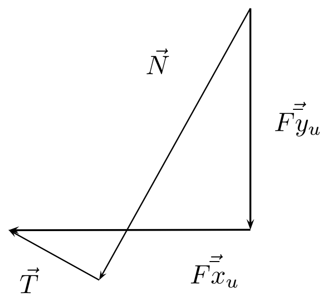


Figure 8: Resolution of forces

$$Fy_u = N\sin\beta - T\cos\beta \qquad Fx_u = N\cos\beta + T\sin\beta$$

$$T = L\sin\alpha - D\cos\alpha \qquad N = L\cos\alpha + D\sin\alpha$$

$$L = C_L\frac{1}{2}\rho W_u^2 cdH \qquad D = C_D\frac{1}{2}\rho W_u^2 cdH \qquad c : blade\,chord$$

$$T = C_T\frac{1}{2}\rho W_u^2 cdH \qquad N = C_N\frac{1}{2}\rho W_u^2 cdH$$

Thanks to mathematical approximation, drag and lift coefficients can be defined for a symmetrical profile of a Naca0012 type and for low incidence values.

$$C_L = 2\pi\sin\alpha \qquad C_D = constant$$

$$C_D\, has\, a\, value\, far\, below\, C_L\, in\, the\, order\, of\, C_D \;<\; 10\,C_L \qquad C_D \;\approx\; 0$$

$$C_T = 2\pi\sin^2\alpha - C_D \qquad C_N = 2\pi\sin\alpha\cos\alpha \tag{10}$$

$$Fy_u = \frac{1}{2}\rho W_u^2 cdH\sin\beta(C_N - C_T\frac{\cos\beta}{\sin\beta})$$
$$and \quad Fy_d = \frac{1}{2}\rho W_d^2 cdH\sin\beta(C_N - C_T\frac{\cos\beta}{\sin\beta}) \tag{11}$$

2.3.2.6 Calculation of the value of Fy_u

For a two blades turbine, only one blade follows a half-turn.[2]

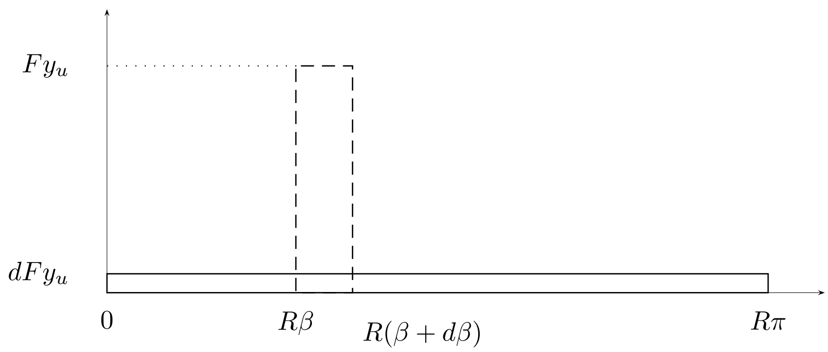


Figure 9: Equality of surfaces

The following equation is given for a turbine with Np blades and by integration :

$$dFy_u R\pi = Fy_u R d\beta \frac{N_p}{2} \; Fy_u = dFy_u \frac{2\pi}{N_p d\beta} \qquad Fy_d = dFy_d \frac{2\pi}{N_p d\beta}$$

by using the equations (9) and (11) and previous equations

$$\frac{1}{8\pi}\frac{N_p c}{R} W_u^2 (C_N - C_T \frac{\cos\beta}{\sin\beta}) = (V_\infty - V_u) V_u$$

$$\frac{1}{8\pi}\frac{N_p c}{R} W_d^2 (C_N - C_T \frac{\cos\beta}{\sin\beta}) = (2V_u - V_d - V_\infty) V_u \quad (12)$$

2.3.2.7 velocities triangle

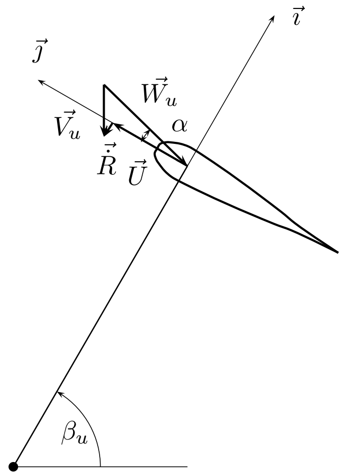


Figure 10: velocities triangle

$$\vec{V}_u = \vec{W}_u + \vec{U} + \dot{R}\vec{\imath} \qquad \vec{U} = R\dot{\beta}\vec{\jmath} \qquad W_u : vitesse\,relative$$

$$\vec{V}_u = -V_u\vec{Y} \qquad \vec{U} = U\vec{\jmath} \qquad U \geq 0$$

$$U = R\dot{\beta} \qquad \dot{R} = (-r_b + e)\dot{\beta}\sin\beta \qquad R \gg (-r_b + e) \qquad \vec{V}_u \approx \vec{W}_u + \vec{U}$$

The Naca profile is following pretty much a circle trajectory during one rotation turn. $\quad \dot{R} \approx 0)$

$$V_u \sin\beta = -\cancel{\dot{R}} + W_u \sin\alpha \qquad V_u \cos\beta = W_u \cos\alpha - R\dot{\beta}$$

$$\bar{\sigma} : \, stiffness\ coefficient \qquad \bar{\sigma} = \frac{N_p c}{\bar{R}}$$

$$\bar{\lambda} : \, velocity\ coefficient \qquad \bar{\lambda} = \frac{\bar{R}\dot{\beta}}{V_\infty}$$

$$b \, = \, \bar{\sigma}\,\bar{\lambda}$$

$$\sin\alpha = \frac{V_\infty}{W_u}\frac{V_u}{V_\infty}\sin\beta \qquad \cos\alpha = \frac{V_\infty}{W_u}\{\frac{V_u}{V_\infty} + \frac{\bar{\lambda}}{\cos\beta} + \bar{\lambda}\frac{r_b - e}{\bar{R}}\}\cos\beta \quad (13)$$

by using equations (2),(10),(12) and also the above equations.

$$C_{Tu} = 2\pi \frac{V_\infty^2}{W_u^2} \sin^2\beta \, \{\frac{V_u}{V_\infty}\}^2 - C_D$$

$$C_{Nu} = 2\pi \frac{V_\infty^2}{W_u^2} \sin\beta \cos\beta \frac{V_u}{V_\infty} \{\frac{V_u}{V_\infty} + \frac{\bar{\lambda}}{\cos\beta} + \bar{\lambda}\frac{r_b - e}{\bar{R}}\}$$

$$C_{Nu} - C_{Tu}\frac{\cos\beta}{\sin\beta} = 2\pi \frac{V_\infty^2}{W_u^2} \sin\beta \bar{\lambda} \frac{V_u}{V_\infty} \{1 + \frac{r_b - e}{\bar{R}} \cos\beta\} + C_D \frac{\cos\beta}{\sin\beta} \quad with \quad C_D \approx 0$$

$$A_u = \frac{1}{V_\infty}\{V_\infty - V_u\} \qquad A_d = \frac{1}{V_\infty}\{2V_u - V_d - V_\infty\}$$

$$A(V,\beta,W) = \frac{1}{V}\frac{1}{V_\infty} \{ \frac{1}{8\pi}\bar{\sigma} W_u^2 (C_N - C_T \frac{\cos\beta}{\sin\beta}) \}$$

$$A(V,\beta,W) = \frac{b}{4} \sin\beta (1 + ...) \qquad (14)$$

$$A_u = A(V_u, \beta, W_u) \qquad A_d = A(V_d, \beta, W_d)$$

$$A_m = \frac{b}{4} \sin\beta \qquad (15)$$

$$In\, order\, to\, simplify\, the\, calculations,\; we\, will\, use\, A_m\, instead\, of\, A_u\, or\, A_d$$

$$in\, the\, calculation\, of\, V_u\, or\, V_d$$

$$A_m = \frac{1}{V_\infty}(V_\infty - V_u) \quad A_m = \frac{1}{V_\infty}(2V_u - V_d - V_\infty)$$

$$\frac{V_u}{V_\infty} = 1 - A_m \quad \frac{V_d}{V_\infty} = 1 - 3\,A_m \qquad (16)$$

2.3.2.8 Calculation of the mean dimensional values of Vu et Vd

$$\bar{\sigma} : \, stiffness \; coefficient \qquad \bar{\sigma} = \frac{N_p c}{\bar{R}}$$

$$\bar{\lambda} : \, velocity \; coefficient \qquad \bar{\lambda} = |\frac{\bar{R}\dot{\beta}}{V_\infty}|$$

$$b \, = \, |\bar{\sigma} \, \bar{\lambda}|$$

$$A_{max} = -\frac{1}{4} \, b \, | \sin \beta|$$

$$\bar{v_u} = (\frac{V_u}{V_\infty})_{moyen} = \frac{1}{\pi} \int_0^\pi (1 - \frac{1}{4} \, b \, \sin \beta) d\beta = 1 - \frac{1}{2\pi} \, b$$

$$with \quad 1 - \frac{1}{4} \, b \, <= 1$$

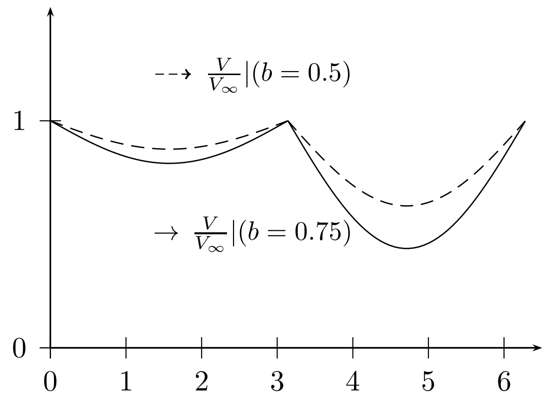
$$\bar{v_d} = (\frac{V_d}{V_\infty})_{moyen} = \frac{1}{\pi} \int_\pi^{2\pi} (1 - 3 \, \frac{1}{4} \, b \, \sin \beta) d\beta = 1 - \frac{3}{2\pi} \, b$$

$$with \quad 1 - \frac{3}{4} \, b \, <= 1 \, \rightarrow \, b \, <= \, \frac{4}{3}$$

Defining

$$a = 1 \, for \, 0 \leq \beta \leq \pi \qquad a = -3 \, for \, \pi \leq \beta \leq 2\pi$$

$$\frac{V}{V_\infty} = 1 - a \, \frac{b}{4} \sin \beta$$



One can define the mean value of $\frac{V}{V_\infty}$

$$(\frac{\bar{V_x}}{V_\infty}) = \frac{1}{\pi}(\pi - a \, \frac{b}{4}) \qquad x = u \, or \, v$$

Figure 11: curve of Vu and Vd

2.3.2.9 Tangential velocity, angular velocity and rotational velocity

$$In\,order\,to\,follow\,the\,maximum\,angle\,\alpha_{max}\,of\,the\,profil,\ with\,\dot{R}=0\ \ \beta\ =\ \frac{\pi}{2}$$

$$\alpha_{max}\,maximun\,incidence\,angle\,of\,the\,profil \qquad (\beta\ =\ \frac{\pi}{2})$$

$$U_{min} = \frac{(1-\frac{b}{4}\,\sin\frac{\pi}{2})\,V_\infty}{\tan\alpha_{max}}$$

$$The\,angular\,velocity\,is\,following\ : \qquad \omega\ =\ \dot{\beta}\ =\ \frac{U_{min}}{\bar{R}}$$

$$Rotational\,velocity\,is \qquad N_{rpm}\ =\ \frac{\omega}{2\,\pi}\,60$$

$$U = (\bar{R} - (r_b - e)\,\cos\beta)\omega \qquad \lambda = \frac{\omega\,\bar{R}}{V_\infty} \qquad \sigma = \frac{b}{\lambda} \qquad c = \frac{\sigma\,\bar{R}}{N_p} \qquad b\ <=\ \frac{4}{3}$$

2.3.2.10 Incidence angle

By using (1)(2) and (13)

$$\tan\alpha\ = \frac{\sin\beta}{\cos\beta}\frac{[(1-a\frac{b}{4})+\frac{(-r_b+e)}{\bar{R}}\lambda]}{[(1-a\frac{b}{4})+\frac{\lambda}{\bar{R}}+\frac{(r_b-e)}{\bar{R}}\lambda]} \tag{17}$$

$$\tan\alpha\ = \tan\beta\,(\frac{1}{1+\frac{\lambda}{(1-a\frac{b}{4}\sin\beta)}(\frac{1}{\cos\beta}+\frac{r_b-e}{\bar{R}})}) \tag{18}$$

$$Incidence\,angle\,having\,a\,low\,value, \qquad \alpha\approx\tan\alpha\approx\sin\alpha \qquad \cos\alpha\approx 1-\frac{\alpha^2}{2}\approx 1$$

The incidence angle must be below the maximum incidence angle of the profile.

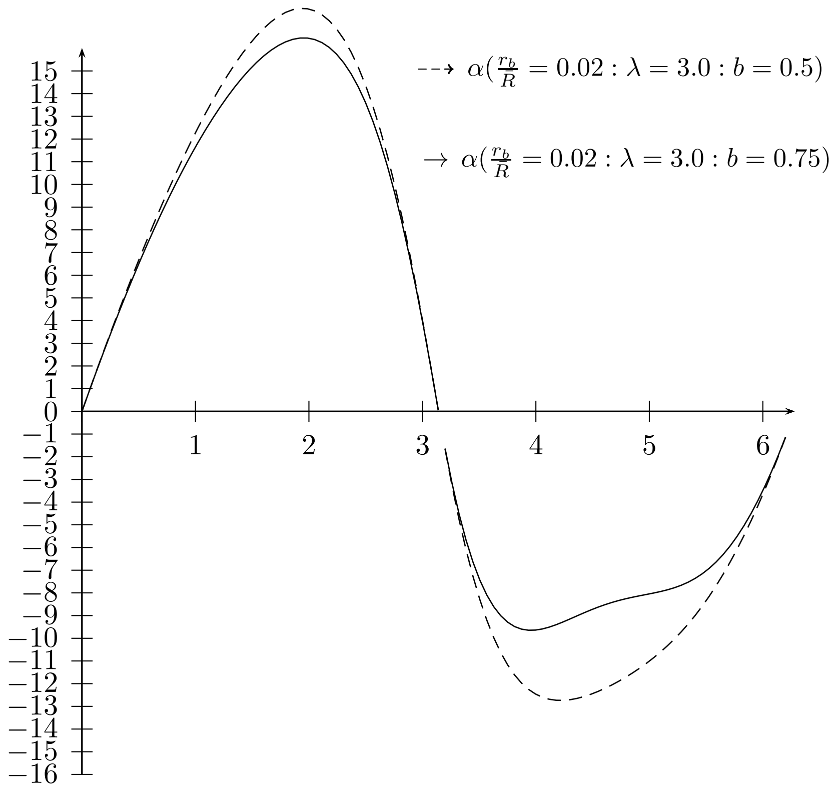


Figure 12: incidence angle variations

2.3.2.11 Calculations of the tangential and normal coefficients

By using (10), the following incidence angle definition below

$$avec\, C_D \,=\, 0 \qquad C_T = 2\pi \sin^2 \alpha \qquad C_N = 2\pi \sin \alpha \cos \alpha$$

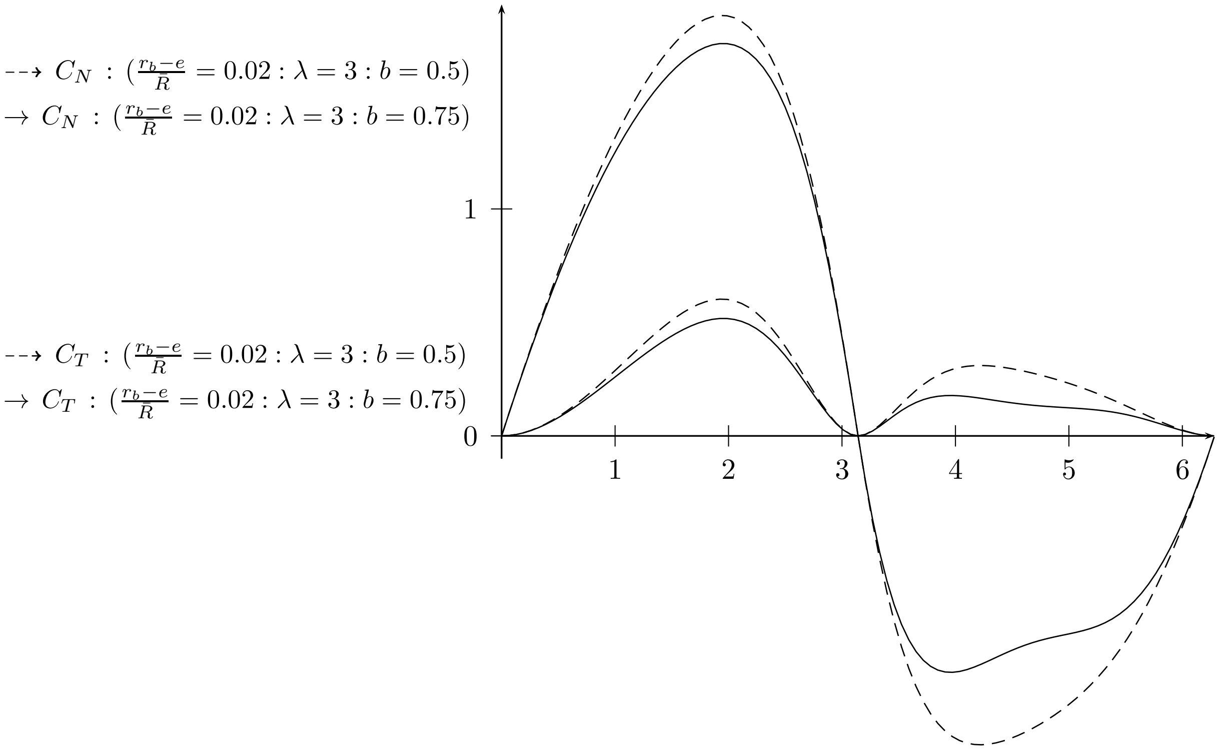


Figure 13: tangential and normal coefficient

2.3.2.12 Calculation of the relative velocity :

by using (13) dimensional relative velocity is

$$\frac{W^2}{V_\infty^2} = \frac{V^2}{V_\infty^2}(1 + \lambda^2 + 2\lambda \cos\beta)$$

$$\frac{V}{V_\infty} = 1 - a\,\frac{b}{4}\sin\beta$$

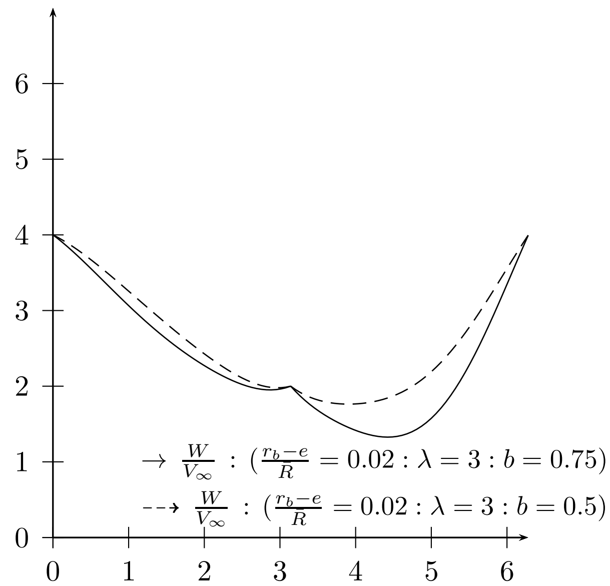


Figure 14: relative velocity

The dimensional relative velocity $\frac{W}{V_\infty}$ can be approximated by

$$(\frac{\bar{W_x}}{V_\infty}) = \lambda\,(\frac{\bar{V_x}}{V_\infty}) = \lambda\,\frac{1}{\pi}(\pi - a\,\frac{b}{4}) \qquad x = u\,or\,v$$

2.3.2.13 Torque and power calculations Following the document [6] , a mean deviation angle can be defined ϕ like being the mean value of $\phi = \frac{\beta_u + \beta_d}{2}$ and the section of the flow area dAx_m of a streamtube ruled by the velocity V_m. The mean flow area is defined by $dAx_m = \bar{R} d\phi \sin\phi dH$.

$$dAx_u V_u = dAx_d V_d = dAx_m V_m \qquad dAx_m = \frac{dAx_u + dAx_d}{2}$$

$$dAx_u = R_u d\beta_u \sin\beta_u dH = R_u \sin\beta_u \frac{d\beta_u}{d\phi} d\phi dH$$

$$dAx_d = R_d d\beta_d \sin\beta_d dH = R_d \sin\beta_d \frac{d\beta_d}{d\phi} d\phi dH$$

We infer

$$\frac{dAx_u}{dAx_m} = \frac{V_m}{V_u} = \frac{2V_d}{V_u + V_d} \qquad \frac{dAx_d}{dAx_m} = \frac{V_m}{V_d} = \frac{2V_u}{V_u + V_d}$$

$$\frac{dAx_u}{dAx_m} = \frac{R_u d\beta_u \sin\beta_u}{\bar{R} d\phi \sin\phi} \qquad R_u \approx \bar{R} \qquad \frac{d\beta}{d\phi} = \frac{\sin\phi}{\sin\beta_u}\frac{dAx_u}{dAx_m} = \frac{\sin\phi}{\sin\beta_u}\frac{2V_d}{V_u + V_d}$$

$$dAx_u = R_u \sin\beta_u \frac{d\beta_u}{d\phi} d\phi dH \quad R_u \approx \bar{R}$$

$$\frac{dAx_u}{\sin\beta_u dH} = \bar{R}\frac{d\beta_u}{d\phi}d\phi = \bar{R}\frac{\sin\phi}{\sin\beta_u}\frac{2V_d}{V_u + V_d}d\phi$$

The angle ϕ is of the same order of magnitude than β.

$$\frac{dAx_u}{\sin\beta_u dH} = \bar{R}\frac{2V_d}{V_u + V_d}d\beta_u$$

likewise

$$\frac{dAx_d}{\sin\beta_d dH} = \bar{R}\frac{2V_u}{V_u + V_d}d\beta_d$$

The torque due to the tangential force dT_u is defined by :

$$T_u = C_T \frac{1}{2}\rho W_u^2 c dH \qquad T_u = dT_u \frac{2\pi}{N_p d\beta}$$

$$dCT_u = R_u \, dT_u \qquad dAx_u = R_u d\beta_u \sin\beta_u dH$$

$$dCT_u = \frac{1}{4\pi}\sigma\bar{R}C_T\rho W_u^2 \frac{dAx_u}{\sin\beta_u} = \frac{1}{4\pi}\sigma C_T \rho W_u^2 \bar{R}^2 \frac{2V_d}{V_u + V_d}d\beta_u$$

likewise

$$dCT_d = \frac{1}{4\pi}\sigma\bar{R}C_T\rho W_d^2 \frac{dAx_d}{\sin\beta_d} = \frac{1}{4\pi}\sigma C_T \rho W_d^2 \bar{R}^2 \frac{2V_u}{V_u + V_d}d\beta_d$$

The wind power is

$$P_{wind} = \frac{1}{2}\rho S_{wept-area} V_\infty^3 = \frac{1}{2}\rho \, 2\bar{R} \, dH \, V_\infty^3$$

The power coefficient is

$$C_p = \frac{Power_{turbine}}{P_{wind}} = \frac{Torque_{turbine}\,\dot{\beta}}{P_{wind}} \qquad \dot{\beta} = \frac{\lambda V_\infty}{\bar{R}}$$

$$C_p = \lambda \frac{Torque_{turbine}}{\rho\bar{R}^2 H V_\infty^2} = \lambda \, Cp_{dim_less} \qquad Cp_{dim_less} = \frac{Torque_{turbine}}{\rho\bar{R}^2 H V_\infty^2}$$

the torque due to the tangential forces is equal to

$$Torque_{ForcesT} = (\int_0^{\pi} dCT_u d\beta + \int_{\pi}^{2\pi} dCT_b d\beta)\, H$$

$$CpT_{dim_less} = \frac{\sigma}{2\pi}\{\int_0^{\pi} \frac{V_d}{V_u + V_d}(\frac{W_u}{V_\infty})^2 d\beta_u + \int_{\pi}^{2\pi} \frac{V_u}{V_u + V_d}(\frac{W_d}{V_\infty})^2 d\beta_d\}$$

By using (3) $Cr = \{e\,\sin\beta\} F_{slide-bar}$

$$F_{slide-bar} = -C_N \frac{1}{2}\rho W_u^2 c\, dH = -N \qquad C_T = 2\pi\sin^2\alpha - C_D \qquad C_N = 2\pi\sin\alpha\cos\alpha$$

$$C_D \approx 0 \qquad \cos\alpha \approx 1 \qquad \sin\beta = \frac{W}{V}\sin\alpha \qquad (\frac{\bar{W}}{V_\infty}) \approx \lambda\,(\frac{\bar{V}}{V_\infty})$$

$$CpN_{dim_less} \approx \frac{e}{\bar{R}}\,\lambda\, CpT_{dim_less}$$

The dimensionnal power coefficient has a value of

$$Cp_{dim_less} = (1 + \frac{e}{\bar{R}}\lambda) CpN_{dim_less}$$

Thanks to an analytical calculation, the analytical power coefficient can be defined by

$$Cp_{analytical} = (1 + \frac{e}{\bar{R}}\lambda)\,(\frac{9\pi}{2^7}b^3 - \frac{2^2}{3}b^2 + \frac{\pi}{2}b) \qquad b = \sigma\,\lambda \qquad with \qquad Cp_{analytical} < 1.$$

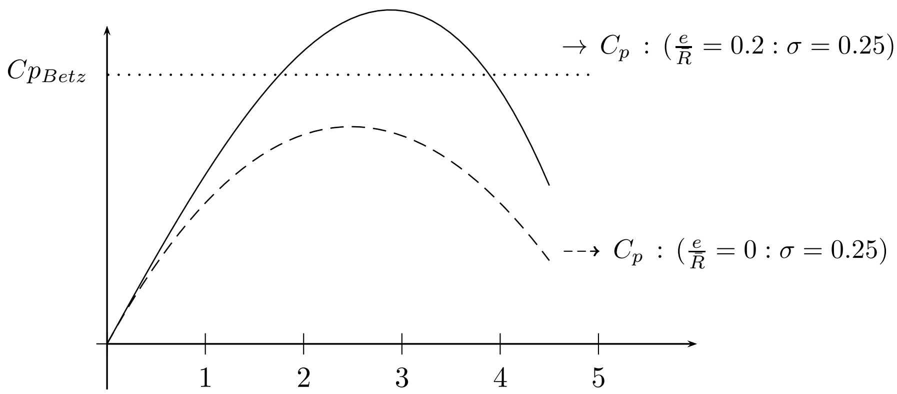


Figure 15: power coefficient

2.4 Conclusion about Power Coefficient

Mathematician Betz determined, thanks to the kinetic energy (with appropriate parameters), that it is possible to collect 60% of the wind energy. The remaining energy (40%) is not lost and generates forces on the turbine.
For a standard Darrieus turbine, the normal force creates compressive stress and extension of the arms. The active lift turb ine is partially transferring these normal forces thanks to the crank rod system into kinetic energy.
Concerning standard Darrieus turbine, the driving power is calculated through the tangential power coefficients C_{pT}.
Concerning active lift turbine, the driving power is calculated through the tangential and normal power coefficients $C_{pT}\, and\, C_{pN}$.

3 Theory on vibrations :

3.1 Resonant modes

The relative velocity is not uniform during each turn. This generates velocity variations of the rotational velocity and induced vibrations. By using Lagrange equations (virtual powers), we will estimate these vibrations

The velocity variations are
$\dot\beta$ has been considered as constant $\dot\beta = \omega = constant$
The rotational angular velocity ω follows the variations.
In order to take into account these variations, the angular velocity can be defined as

$$\dot\beta = \dot{\bar\beta} + \dot{\acute\beta} \qquad \dot{\bar\beta} = \omega = Mean\,constant\,velocity$$

The velocity variations compared to the mean velocity are corresponding to the terms $\dot{\acute\beta}$
First approximation is giving $\dot{\acute\beta}$ by $a\,\ddot\beta$ a being a constant

$$\dot\beta = \dot{\bar\beta} + a\,\ddot\beta = \omega + a\,\ddot\beta$$

The kinetic energy is

$$T_e = \frac{1}{2}m\{(R\dot\beta)^2 + \dot R^2\} = \frac{1}{2}m\dot\beta^2\{\bar R^2 + (r_b - e)^2 - 2(r_b - e)\bar R\cos\beta\}$$

$Considering\,the\,profil\,mass\,being\,in\,one\,point\,m\,for\,simplify\,calculations$

The Lagrange equations terms at the kinetic energy level are

$$\frac{\partial T_e}{\partial\dot\beta} = m\dot\beta\{\bar R^2 + (r_b - e)^2 - 2(r_b - e)\bar R\cos\beta\}$$

$$\frac{d}{dt}\{\frac{\partial T_e}{\partial\dot\beta}\} = m\ddot\beta\{\bar R^2 + (r_b - e)^2 - 2(r_b - e)\bar R\cos\beta\} - 2(r_b - e)\bar R m\dot\beta^2\sin\beta$$

$$\frac{\partial T_e}{\partial\beta} = -m(r_b - e)\bar R\dot\beta^2\sin\beta$$

$$\frac{d}{dt}\{\frac{\partial T_e}{\partial\dot\beta}\} - \frac{\partial T_e}{\partial\beta} = m\ddot\beta\{\bar R^2 + (r_b - e)^2 - 2(r_b - e)\bar R\cos\beta\} + m(r_b - e)\bar R\dot\beta^2\sin\beta$$

4 Powers calculation

$$Power_{Driving} = \{N\vec{\imath} + T\vec{\jmath}\} \wedge \{\dot{R}\vec{\imath} + TR\dot{\beta}\vec{\jmath}\} = N\dot{R} + TR\dot{\beta}$$
$$N\dot{R} + TR\dot{\beta} = -(r_b - e)N\dot{\beta}\sin\beta + T(\bar{R} + (r_b - e)\cos\beta)\dot{\beta}$$
$$Power_{Resisting} = -\,C_{Resisting}\,\dot{\beta}$$
$$C_{Resisting}\ :\ resisting\,torque\,due\,electrical\,generator\,,to\,the\,friction,\ etc$$

The lagrange equation terms of the power level are

$$A_\beta = T\bar{R} + (r_b - e)T\cos\beta - (r_b - e)N\sin\beta\ -\ C_{Resisting}$$

The Lagrange equation is

$$\frac{d}{dt}\{\frac{\partial T_e}{\partial\dot{\beta}}\} - \frac{\partial T_e}{\partial\beta} = A_\beta$$
$$m\ddot{\beta}\{\bar{R}^2 + (r_b - e)^2 - 2(r_b - e)\bar{R}\cos\beta\} =$$
$$T\bar{R} + (r_b - e)T\cos\beta - (r_b - e)N\sin\beta - m(r_b - e)\bar{R}\dot{\beta}^2\sin\beta - C_{Resisting}$$

The Lagrange equation for a Darrieus turbine type r=0

$$m\ddot{\beta}\bar{R}^2 = T\bar{R} -\ C_{rsistant} \quad T = C_T\frac{1}{2}\,\rho\,c\,H\,W^2$$
$$\ddot{\beta} = \frac{1}{m\bar{R}^2}\{T\bar{R} -\ C_{Resisting}\}$$

T depends on C_T and on W^2, for a blade, simplified curve $|\ddot{\beta}|$ is

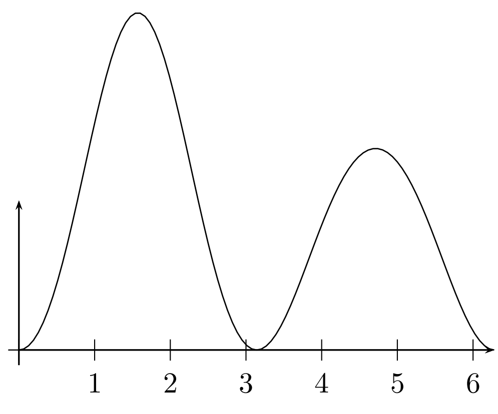


Figure 16: approximately curve of $|\ddot{\beta}|$

For one blade, there will be two pulsations per turn. The blade pace will be equivalent (translation of 120) for a three blades case. The association of the three paces is lowering the vibrations and is increasing the vibration frequency. Six pulsations will be obtained per turn.
The Lagrange equation for an active lift turbine with a controlled displacement is

$$\ddot{\beta} = 1 \times \frac{1}{m\{\bar{R}^2 + (r_b - e)^2 - 2(r_b - e)\bar{R}\cos\beta\}} \times \{T\bar{R} + (r_b - e)T\cos\beta - (r_b - e)N\sin\beta - m(r_b - e)\bar{R}\dot{\beta}^2\sin\beta - C_{Resisting}\} \quad (19)$$

The same kind of equations is obtained for standard Darrieus turbine type.

The attenuation of the amplitudes has almost the value of $\frac{(r_b - e)}{\bar{R}}$.

4.1 Conclusion on vibrations

The same range of vibrations is generated for a Darrieus turbine type or for an active lift turbine with a controlled displacement. Vibrations are reduced slightly.

These frequencies can be a source of fatigue for the structure or of noise generation. For Darrieus type turbines, manufacturers are building helicoidal blades or stepped blades. The yield of the active lift turbine is higher than any other turbine types but the vibrations are not cancelled. These are slightly dampened.

5 Conclusion

cf. conclusions : 2.4 and 4.1

It has been theoretically demonstrated that active lift turbine is delivering a power coefficient greater than defined by Betz [1] for classical vertical axis Darrieus [7] type turbine. The active lift turbine will not solve the generated vibrations problem. It will be necessary to add devices (such as twisted blades for example) in order to limit the vibrations. The first version of this turbine was the subject of a Phd work (numerical and experimental)[3] which validated the concept. The latest version of the active lift turbine provides a real gain on the theoretical power and requires a research plan to validate and optimize this concept in order to achieve an industrial version.

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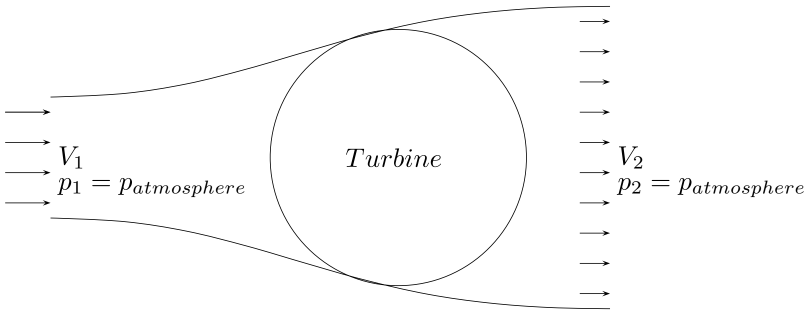
6 Betz power coefficient

Maximum wind power (from kinetic energy)

$$E_c = \frac{1}{2}\, m\, V^2$$

$$\frac{dE_c}{dt} = \frac{1}{2}\,\frac{dm}{dt}\, V^2 + \frac{1}{2}\, m\, \frac{dV^2}{dt} \qquad \frac{dV}{dt} = 0$$

$$\frac{dE_c}{dt} = \frac{1}{2}\,\dot{m}\, V^2 = \frac{1}{2}\rho\, S\, V^3$$



V Wind speed at the turbine level
Force applied by the wind on the rotor

$$F = m\frac{dV}{dt} = \dot{m}\Delta V \ = \ \rho S V\,(V_1 \ - \ V_2)$$

$$P = FV \ = \ \rho S V^2\,(V_1 \ - \ V_2)$$

$$P = \frac{\Delta E}{dt} = \frac{\frac{1}{2}mV_1^2 - \frac{1}{2}mV_2^2}{\Delta t} \ = \ \frac{1}{2}\dot{m}(V_1^2 \ - \ V_2^2) = \ \frac{1}{2}\rho S V(V_1^2 \ - \ V_2^2)$$

From these equalities

$$V = \bar{V} = \frac{V_1 + V_2}{2}$$

$$F = \ \rho S V\,(V_1 \ - \ V_2) \ = \ \frac{1}{2}\rho S(V_1^2 \ - \ V_2^2)$$

$$P = FV \ = \ \rho S V^2\,(V_1 \ - \ V_2) = \frac{1}{4}\rho S(V_1^2 \ - \ V_2^2)(V_1 \ + \ V_2)$$

defining $a = \frac{V_2}{V_1}$

$$F == \frac{1}{2}\rho S V_1^2(1\ -\ a^2) \qquad P = \frac{1}{4}\rho S V_1^3(1 - a^2)(1 + a)$$

The wind power is $P_{wind} = \frac{1}{2}\rho S V_1^3$
The power coefficient is defined by $C_p = \frac{P}{P_{wind}} = \frac{\frac{1}{4}\rho S V_1^3(1-a^2)(1+a)}{\frac{1}{2}\rho S V_1^3} = \frac{1}{2}(1 - a^2)(1 + a)$
Search of the maximum power coefficient

$$\frac{dC_p}{a} = 0 = -2a(1 + a) + 1 - a^2 \qquad a > 0 \qquad a = \frac{1}{3}$$

$$C_p = \frac{16}{27} = 0.593$$

The maximum power coefficient $C_{p_{maxi}}$ **is defined by Betz**
$C_{p_{Betz}} = \frac{16}{27} = 0.593$

References

[1] Betz A. Das maximum der theoretisch moglichen ausnutzung des windesdurch windmotoren. *Zeitschrift fur das gesamte Turbinenwesen*, (26):307–309, 1920.

[2] Gopal b Reddy B.E. *The Darrieus wind turbine : An analytical performance study.* PhD thesis, Texas Tech University, Lubbock, TX 79409USA, 1975.

[3] M. Belhache. *Developpement d'un prototype de recuperation de l'energie des courants [hydrolienne a axe vertical] a l'aide d'une approche couplee [numerique/experimentale].* PhD thesis, Universite de Caen, 2014.

[4] P. Lecanu J. Breard. Turbine telle qu'eolienne, en particulier a axe vertical, notamment de type darrieus. Brevet INPI, June 2009. Numero et date de publication de la demande FR2919686 - 2009-02-06 (BOPI 2009-06).

[5] P. Lecanu J. Breard. Turbine telle qu'eolienne d'axe essentiellement vertical a portance active. Brevet INPI, July 2015. Numero et date de publication de la demande FR3016414 - 2015-07-17 (BOPI 2015-29).

[6] Ir M. Godard Ing J. David Ing G. Genon. Optimisation d'une eolienne darrieus a pales droites, analyse du couple de demarrage et realisation d'un prototype. *ISILF Institus superieurs industriels libres francophones*, 18, 2004. Gramme Liege.

[7] MARIE DARRIEUS GEORGES JEAN. *Turbine a axe de rotation transversal a la direction du courant.* PhD thesis, May 1926. Brevet INPI Numero et date de publication de la demande FR604390 - 1926-05-03.

[8] Blackwell B. Sheldahl R. Feltz L. Wind turbine performance for the darrieus wind turbine with naca0012 blades. Technical report, Sandia National Laboratories, 1976. report No. 76-0130.

[9] SHELDAHL R. KLIMAS P. FELTZ L. Aerodynamic performance of a 5-m diameter darrieus turbine. *Journal of Energy*, 4(5):227–232, 1980.

[10] Musgrove P.J. Wind energy conversion: recent progress and future prospects. *Solar and Wind Technology*, 4(1):37–49, 1987.

[11] P. Lecanu J. Breard O. Sauton. Turbine a portance active a deplacement controle. Brevet INPI, June 2015. demande 1555859 soumission 1000299939.

[12] P. R.S.-Rangi South. A wind tunnel investigation of a 14ft. diameter vertical axis windmill, low speed. Technical report, Aerodynamics Laboratory (Canada) Laboratory Technical, 1972. Report, National Aeronautical Establishment, LTR-LA-105.