A Passenger-centric Multi-agent System Model for Multimodal Public Transportation

Stefan Haar, Simon Theissing

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MExICo team, INRIA and LSV, CNRS & ENS de Cachan,
Cachan, France

Abstract. If we want to understand how perturbations spread across a multi-modal public transportation system, we have to include passenger flows into the model and the analysis. Indeed, in general no two different lines in such a system are physically connected directly, or share tracks or other resources. Rather, they are connected by passengers changing lines and thus transmit perturbations from one line or mode to another. We present a formal passenger-centric multi-agent system model that can capture (i) individual and possibly multi-modal trip profiles with branches resulting from different decision outcomes, (ii) the movement of fixed-route operated transportation means, and (iii) in-vehicle and in-station capacity constraints. The model is based on a nets-within-nets approach with Petri nets as the basic building entities. Thus, it has a convenient graphical representation, and the possibility of execution.

Keywords: Nets-Within-Nets, Transportation Networks

1 Introduction

A multimodal public transportation system is a web of services, and - apart from a few exceptions such as the street network that might be shared between the tram and private cars - it is mainly the passenger who connects the different lines or modes. Thus, if we want to understand the spread of perturbations across the respective network, we have to account for the passenger transfers, i.e. we have to include them in our model and in the analysis.

In this paper we are going to present a passenger-centric multi-agent system model for multimodal public transportation that accounts for the interconnection of the transportation services by the passenger transfers, uncertainty, and capacity constraints. It is based on a nets-within-nets approach with Petri nets as the basic modelling entities [11], which can capture mobility in a discrete-event dynamic environment [5], [1]. Moreover, it has a formal specification with a convenient graphical representation, the possibility of execution [6], and can serve as a starting point for the development of a performance model. Compared to traffic assignment models [4] we explicitly account for the transportation means in form of vehicle agents that execute missions and thereby provide fixed-route transportation services to passenger agents. We divide the passenger agents, on the other hand, into a finite set of trip profiles that define pre-chosen but not necessarily efficient paths in the considered infrastructure. Thereby, our focus is neither on the detailed interaction of the passenger agents with their infrastructure as can be studied e.g. with sophisticated microsimulation platforms [10, 2], nor on the synchronization of individual vehicle runs at local points in the network as can be studied e.g. with models employing the Max Plus-Algebra [7].

The rest of the report is structured as follows. Frequently used notations are introduced in section 2. The basic modelling entities of the multi-agent system, i.e., Petri nets are introduced in section 3. The multi-agent system with nets nested within nets is then elaborated in section 4. Section 5 completes the report with a brief summary and outlook.

2 Notations

$\mathbb{N} := \{0, 1, 2, \dots\}$ denotes the non-negative integers, $\mathbb{N}^+ := \{1, 2, \dots\}$ the positive integers, and $\emptyset := \{\}$ the empty set. $A \times B := \{(a, b) : a \in A \wedge b \in B\}$ denotes the Cartesian product of two sets A and B.

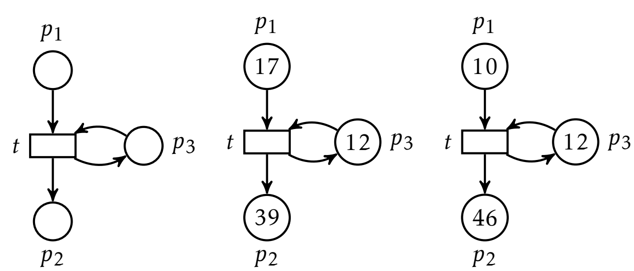
3 Basic Modelling Entities: Petri Nets

A Petri net is a mathematical model of a distributed dynamic system with a convenient graphical representation [8]. Its structure is defined by a bipartite graph called place transition net.

Definition 1. *A* place transition net *is a 4-tuple* $N := (P, T, PRE, POST)$, *with*

- *the finite set of places* P,
- *the finite set of transitions* T, *in which* $P \cap T = \emptyset$,
- *the pre-incidence function* $PRE : P \times T \to \{0, 1\}$, *and*
- *the post-incidence function* $POST : P \times T \to \{0, 1\}$.

Fig. 1a depicts a place transition net comprising the three places p_1 to p_3, and the single transition t.



(a) Unmarked (b) Before firing (c) After firing

Fig. 1: A reference place transition net

In general, we depict places as circles and transitions as boxes. We connect (i) place $p \in P$ to transition $t \in T$ by an arc if and only if $\mathrm{PRE}(p,t) = 1$, and (ii) transition t to place p by an arc if and only if $\mathrm{POST}(p,t) = 1$. The preset and the postset of a place or a transition characterize its *neighbourhood* in the graph.

Definition 2. $^\bullet p := \{t \in T : POST(p,t) = 1\}$ *is the* preset *of place* $p \in P$*, and* $^\bullet t := \{p \in P : PRE(p,t) = 1\}$ *is the preset of transition* $t \in T$.

Definition 3. $p^\bullet := \{t \in T : PRE(p,t) = 1\}$ *is the* postset *of place* $p \in P$*, and* $t^\bullet := \{p \in P : POST(p,t) = 1\}$ *is the postset of transition* $t \in T$.

In Fig. 1a, $^\bullet t = \{p_1, p_3\}$ and $t^\bullet = \{p_2, p_3\}$. The state of the Petri net is defined by the marking of its net.

Definition 4. *A* marking *m of a place transition net is a function* $m : P \to \mathbb{N}$ *that maps each place* $p \in P$ *to a non-negative integer.*

Fig. 1b and Fig. 1c depict two different markings of the net from Fig. 1a. The enabling and firing rules define how the marking, i.e. the state of the system changes w.r.t. transition firings.

Definition 5. *The* enabling degree *of transition* $t \in T$ *of the place transition net* N *in marking* m *is defined as* $ED[t,(N,m)] := \min_{p \in {}^\bullet t} m(p)$.

Definition 6. *A transition is* enabled *in a marking of a place transition net if its enabling degree in that marking is positive.*

Definition 7. *If transition* $t \in T$ *of the place transition net* N *is enabled in marking* m*, then it can* fire*. An i-fold firing of transition t produces the marking* $m'(p) := m(p) + i\,[POST(p,t) - PRE(p,t)]$, $\forall p \in P$ *with*

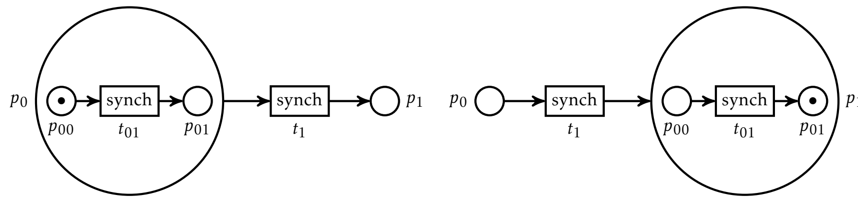
$$i \in \{1, 2, \ldots, ED[t,(N,m)]\}$$

if $ED[t,(N,m)] < \infty$*, and* $i \in \mathbb{N}^+$ *otherwise.*

The enabling and firing rules regard only the immediate *neighbourhood* of a transition in a net, i.e. its preset and postset. No firing order is specified in case two or more transitions are enabled at the same time. The enabling degree of transition t from Fig. 1b is 12. A 7-fold firing of transition t produces the marking from Fig. 1c. Note that the net imposes an invariant marking of place p_3.

4 The Multi-Agent System Model

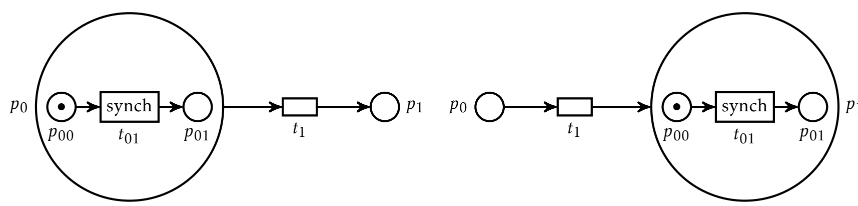
Before we provide a definition of the multi-agent system and proceed with its elaboration, we give some intuition of its nets-within-nets model. In contrast to an ordinary Petri net, the marking of a nets-within-nets model's system net is not restricted to simple tokens, but may comprise net tokens with internal structure and marking of their own. Fig. 2a depicts an example net with a 2-level nested marking.



(a) Before the synchronous firing (b) After the synchronous firing

Fig. 2: The synchronous firing of a net with a 2-level nested marking

Place p_0 of the system net holds one net token. The net token's marking is such that p_{00} holds one simple token. The common inscription "synch" of the two transitions t_1 and t_{01} indicates that the firing of both transitions is synchronized. Thus, both transition firings can only occur together. Fig. 2b depicts the nested marking of the system net after a synchronous firing of transition t_1 and t_{01}. In general, the nested marking of a nets-within-nets model is not necessarily restricted to two levels. A marking of a net token can again comprise net tokens with internal structure and marking of their own. Beside synchronous transition firings, autonomous transition firings can occur. A missing transition inscription then indicates that the respective transition can fire autonomously as depicted in Fig. 3a.



(a) Before the system-autonomous firing (b) After the synchronous firing

Fig. 3: The system-autonomous firing of a net with a 2-level nested marking

Whereas transition t_{01} of the net token in place p_0 of the system net is inscribed, transition t_1 is not. Thus, transition t_1 can fire autonomously. However, transition t_{01} cannot fire since it is not in a place of the preset of a firing enabled transition with the same inscription. Fig. 3b depicts the nested marking of the system net after the autonomous firing of transition t_1. The marking of the net token is unaffected. In general, autonomous firings can occur at all net levels. Here, we have regarded the autonomous firing of transition t_1 at the system net level. Another important aspect of a nets-within-nets model is that transition firings might involve simple tokens as well as net tokens of different types at the same time. Moreover, the transition firings might be synchronized with (i) transition firings of some net tokens, but not with all of them, and (ii) transition firings at different net levels. The enabling and firing rules must then explicitly specify how the different token types, the partial synchronization, and the synchronization across several levels are handled. That way, we can map complex synchronization processes to a nets-within-nets model such as the movement of vehicles and passengers, and the boarding of passengers to/from the vehicles. We now turn toward the multi-agent system.

The framework of the multi-agent system is an infrastructure that hosts two sorts of agents, namely passenger agents and vehicle agents. Vehicle agents are executing missions and they are thereby providing fixed-route transportation services to passenger agents. Passenger agents, on the other hand, are travelling according to trip profiles with the goal to perform activities at trip destinations outside the multi-agent system. They connect the fixed-route transportation services by transfers. Activities that constitute the multi-agent system are as follows.

- A1: A passenger agent arrives at an access point to a station from the outside world.
- A2: A vehicle is dispatched from a depot.
- A3: A passenger agent transfers between an access point to and a platform of a station, or between two platforms.
- A4: A passenger agent leaves a station from an access point to the outside world.
- A5: A vehicle agent moves from one waypoint in a transportation grid to another.
- A6: A vehicle agent parks in a depot.
- A7: A passenger agent boards a vehicle agent from a platform in a station.
- A8: A passenger agent alights a vehicle agent to a platform in a station.

We integrate all activities into a nets-within-nets model and call it the public transportation system.

Definition 8. *A* public transportation system *is a 6-tuple* $PTS := (N_I, \mathcal{N}_T, \mathcal{N}_R, d, \Theta, m)$, *with*

- *the place transition net called the infrastructure* N_I,
- *the finite set of place transition nets called trip profiles* $\mathcal{N}_T$,
- *the finite set of place transition nets called ride profiles* $\mathcal{N}_R$,
- *the place typing function* d,
- *the synchronization structure* Θ, *and*
- *the nested marking of the infrastructure* m.

We will describe all synchronization processes of the public transportation system and manipulations performed on its nested marking of the infrastructure in words instead of algebraic equations. Each place in a net is typed. It is either dedicated to hold simple tokens (number or bullet in a simple circle), passenger agents (passenger icon in a double circle), or vehicle agents (vehicle icon in a dashed circle) as depicted in Fig. 4.

(a) A simple token (b) A passenger agent (c) A vehicle agent

Fig. 4: The different place types of a public transportation system

We now look at the infrastructure, the structure of a vehicle agent, and the structure of a passenger agent in the respective order, before we elaborate all activities separately. Finally, we show how events degrading the performance of the infrastructure can be modelled.

Infrastructure: The infrastructure can be decomposed into stations and transportation grids. Stations are made up of access points, platforms, and corridors connecting them. Waypoints, route segments connecting waypoints, and depots form the transportation grids. Fig. 5 depicts a reference station of the infrastructure. It comprises an access point, a platform, and a corridor connecting the access point to the platform. Another corridor connecting the platform to the access point is not depicted. A single passenger agent is staying at the access point, and two passenger agents are staying at the platform. The latter can host 98 more passenger agents. At most 12 passenger agents can simultaneously cross the corridor.

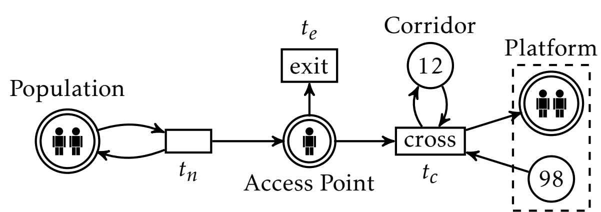


Fig. 5: A reference station

Fig. 6 depicts a reference transportation grid. It comprises three waypoints, a depot, and a route segment connecting waypoint 1 to waypoint 2. One vehicle agent is staying at waypoint 1, another vehicle agent at waypoint 2, and two vehicles are parked in the depot. At maximum one vehicle agent can traverse the route segment (transition t_r) at a time.

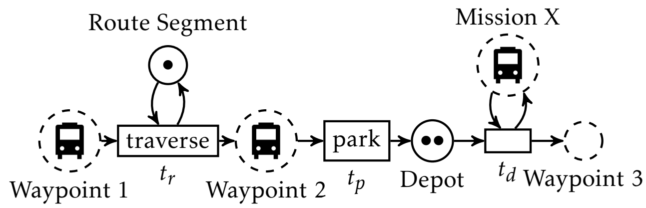
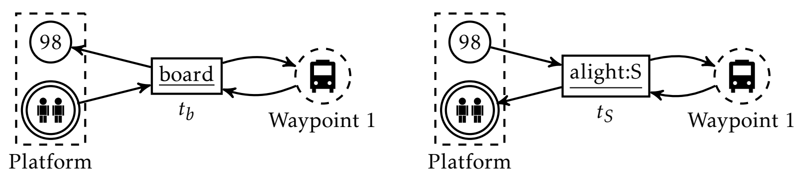


Fig. 6: A reference transportation grid

Fig. 7a shows how the platform of the reference station from Fig. 5 has to be connected to waypoint 1 if the platform is dedicated to the boarding of a vehicle agent at that waypoint. Fig. 7b does likewise for the alighting from a vehicle agent at waypoint 1.



(a) Boarding

(b) Alighting

Fig. 7: Connecting a station and a transportation grid

Vehicle agents: Vehicle agents are marked instances of ride profiles. Their ride profiles are composed of two unconnected subnets, the mission execution and the passenger compartment. The mission execution defines the navigation of the vehicle agent (activities A5 and A6) in a transportation grid (cf. Fig. 8).

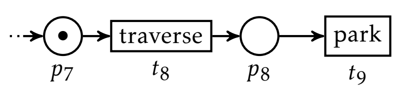


Fig. 8: The mission execution of a vehicle agent

In general, it has an out-tree structure with branches resulting from different decision outcomes such as the direction to take at a crossroad. A single token in a place of the mission execution marks the position of the vehicle agent at a waypoint. Assume that the token in place p_7 refers to the vehicle agent staying at waypoint 1 from Fig. 6. Then, the vehicle agent wants to traverse the route segment connecting waypoint 1 to waypoint 2 (transition t_8). Arrived at waypoint 2 (token in place p_8), the vehicle agent wants to park in the depot (transition t_9). The passenger compartment (cf. Fig. 9), on the other hand, hosts two passenger agents, and it can host 68 more. One passenger agent can board the vehicle agent at a time (transition t_X), and up to two passenger agents can simultaneously alight from it (transition t_a).

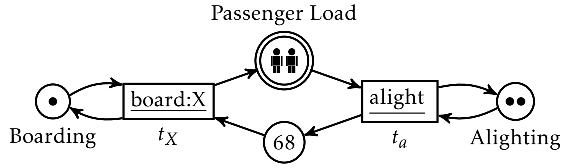


Fig. 9: The passenger compartment of a vehicle agent

Passenger agents: Passenger agents are marked instances of trip profiles. Fig. 10 depicts a reference passenger agent. Its trip profile is similar to the mission execution of the vehicle agent from Fig. 8. It defines the navigation of a group of passenger agents (activities A3, A4, A7, and A8) in the public transportation system. In general, it has an out-tree structure with branches resulting from different decision outcomes such as the choice of a passenger agent at a platform to board a vehicle agent executing mission X or mission Y. A single token in a place marks the internal state of the passenger agent. Assume that the token in place p_0 refers to the passenger agent staying at the access point of the reference station from Fig. 5. Then, the passenger agent wants to cross the corridor (transition t_1) to go to the platform, where the passenger agent waits for a vehicle agent executing mission X to arrive (token in place p_1). At one point the passenger agent boards a vehicle agent (transition t_2), and then waits for this vehicle agent to arrive at the target station S (token in place p_2), where the passenger agent wants to alight from it (transition t_3). Arrived at a platform of the target station S (token in place p_3), the passenger agent has to cross another corridor (transition t_4) in order to (token in place p_4) leave the public transportation system to the outside world (transition t_5).

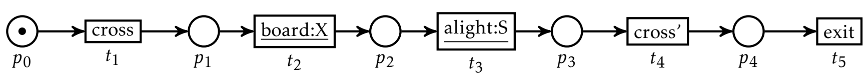


Fig. 10: A passenger agent

We proceed with the modelling of the activities A1 to A8:

Activity A1: Passenger agents enter the public transportation system from the outside world through access points to the stations. Look at transition t_n of the reference station from Fig. 5. A missing inscription indicates, that transition t_n can fire autonomously (cf. Fig. 3). We replace each passenger agent in the preset of transition t_n (cf. definition 2) by a simple token. We then compute the enabling degree of transition t_n according to definition 5, which results to two. Thus, a single and a 2-fold firing of transtion t_n are possible. A single firing copies one out of the two passenger agents from the population of passenger agents in the outside world to the access point, and a 2-fold firing both.

Activity A2: Look at the reference transportation grid from Fig. 6. The depot holds two parked vehicles. Transition t_d is not inscribed and thus can fire autonomously (cf. Fig. 3). Its enabling degree (cf. definition 5) is, independent of the number of parked vehicles, not bigger than one if we replace the sample vehicle agent for mission X in its preset (cf. definition 2) by a simple token. Here, transition t_d can fire, which removes a single token from the depot and copies the sample vehicle agent to waypoint 3.

Activity A3: Passenger agents have to cross a corridor with a limited throughput to transfer between platforms and access points. Look at the reference station from Fig. 5 and assume that 30 passenger agents are staying at the access point. Each passenger agent resembles the reference passenger agent from Fig. 10. Then, 30 passenger agents want to cross the corridor (transition t_1). Transition t_1 and transition t_c have the same inscription. Thus, the firing of both transitions is synchronized (cf. Fig. 2). We replace each passenger agent by a simple token and compute the enabling degree of transition t_c according to definition 5, which results to 12. Thus, an i-fold firing, $i \in \{1, 2, \ldots, 12\}$, of transition t_c together with transition t_1 is possible, which updates the internal states of i out of the 30 passenger agents at the access point (token in place p_1 w.r.t. a firing of transition t_1), and moves them to the platform.

Activity A4: The intention of a passenger agent to leave a station from an access point to the outside world is stored in its trip profile in form of a transition. The firing of this transition is synchronized with the firing of another transition in the respective station via a common inscription (cf. Fig. 2). Look at the reference station from Fig. 5. Assume that the trip profile of the single passenger agent at the access point resembles the trip profile of the reference passenger agent from Fig. 10, and its internal state is such that place p_4 is marked. The firing of transition t_5 is synchronized with the firing of transition t_e of the station. A synchronous firing of both is possible, since transition t_5 is enabled. The synchronous firing of transition t_e and t_5 destroys the passenger agent at the access point. The major difference compared to the modelling of activity A3 is that (i) passenger agents are not moved, but destroyed, and (ii) the enabling degree of transition t_e (cf. definition 6) is limited only by the number of passenger agents at the access point if we replace each passenger agent by a simple token. Thus, the throughput of simultaneously leaving passenger agents to the outside world is unlimited.

Activity A5: The movement of vehicle agents between waypoints is modelled similar to activity A3. Instead of passenger agents and corridors, we are regarding vehicle agents and route segments. Look at the reference transportation grid from Fig. 6. Assume that the mission execution of the single vehicle agent at waypoint 1 resembles the that from Fig. 8. Transition t_8 and transition

t_r have the same inscription. Thus, the firing of both transitions is synchronized (cf. Fig. 2). The enabling degree of transition t_r according to definition 5 is one if we replace the vehicle agent by a simple token. Thus, a synchronous firing of transition t_r together with transition t_1 is possible, which updates the marking of the vehicle agent's mission execution (transition t_8), and moves the updated vehicle agent (token in place p_8) to waypoint 2.

Activity A6: The parking of a vehicle agent in a depot is modelled similar to activity A4. Look at the reference transportation grid from Fig. 6. Assume that the mission execution of the single vehicle agent at waypoint 2 resembles the mission execution of the reference vehicle agent from Fig. 8, and the state of its mission execution is such that place p_8 is marked. The firing of transition t_9 is synchronized with the firing of transition t_p (cf. Fig. 2). It destroys the vehicle agent at waypoint 2, and creates adds a token to the depot.

Activity A7: Look at Fig. 7a, where the platform of the reference station from Fig. 5 is connected to waypoint 1 of the reference transportation grid from Fig. 6 for the purpose of boarding. Assume that (i) the trip profiles of the two passenger agents at the platform resemble the trip profile of the reference passenger agent from Fig. 10, and their internal states are such that place p_1 is marked, and (ii) the passenger compartment of the vehicle agent resembles that from Fig. 9. Thus, both passenger agents want to board a vehicle agent executing mission X (transition t_2). Note that the common inscription "board:X" of transition t_X of the ride profile and transition t_2 of the trip profile is underlined. The common inscription indicates that the firings of both transitions are synchronized. The underline on the other hand indicates the need for another synchronization partner in the infrastructure that might be every transition implementing the boarding of passenger agents from a platform to a vehicle agent at a waypoint such as transition t_b from Fig. 7a. Here, the computation of the enabling degree of transition t_X of the vehicle agent results to 1. Thus, one passenger agent can board the vehicle agent. A synchronous firing of transition t_X together with the two transitions t_b and t_2 (i) consumes a token of the vehicles free capacity, (ii) updates the internal state of one passenger agent (token in place p_2 w.r.t. firing of transition t_2), and (iii) moves the passenger agent with the updated internal state from the platform to the passenger compartment on-board the vehicle agent. The passenger compartment is not full and the procedure can be repeated for the remaining passenger agent.

Activity A8: Look at Fig. 7b, where the platform of the reference station from Fig. 5 is connected to waypoint 1 of the reference transportation grid from Fig. 6 for the purpose of alighting. Assume that (i) the passenger compartment of the single vehicle agent at waypoint 1 resembles that from Fig. 9, and (ii) the trip profiles of the two passenger agents on-board the vehicle agent resemble the trip profile of the reference passenger agent from Fig. 10, and their internal states are such that place p_2 is marked. Thus, both passenger agents want to alight from the vehicle agent at station S (transition t_3). Note that the common inscription "alight:S" of transition t_S of the infrastructure and transition t_3 of the trip profile is underlined. The common inscription indicates that the firings of both transitions are synchronized. The underline, on the other hand, indicates the need for a synchronization partner in the ride profile of any vehicle agent, namely the transition that is dedicated to the alighting of the passenger agents such as transition t_A of the reference ride profile. Here, the computation of the enabling degree of transition t_A of the vehicle agent results to 2 (cf. definition 5) if each on-board passenger agent is replaced by a simple token. Thus, a single or a 2-fold synchronous firing of transition t_A of the vehicle agent, transition t_S of the infrastructure, and transition t_3 of the trip profile can occur. A single firing (i) adds a token to the vehicle agent's free capacity, (ii) updates the internal state of one on-board passenger agent (token in place p_3 w.r.t. the firing of transition t_3), and (iii) moves the passenger agent with the updated internal state to the platform. A 2-fold firing moves both passenger agents at the same time.

So far we have looked at the infrastructure, the vehicle agents, the passenger agents, and their interactions. We now like to provide a brief idea about how events influencing the performance of the infrastructure can be modelled and thus integrated in the public transportation system.

Performance of the infrastructure: In Fig. 11 we have extended the reference transportation grid from Fig. 6 to account for a temporary closure of the route segment that connects waypoint 1 to waypoint 2.

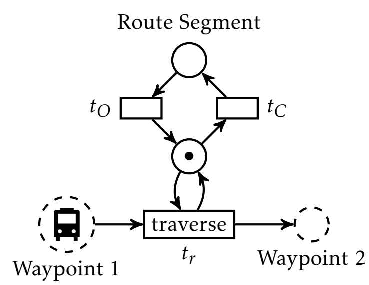


Fig. 11: Closure of a route segment

Two events characterize the temporary closure of the route segment, namely the actual closure of the route segment (transition t_C) and its subsequent opening (transition t_O). Both transitions t_C and t_O are not inscribed. Thus, they can fire autonomously. In Fig. 11 the route segment is open (cf. activity A5). A firing of transition t_C moves the token from the place in its preset to the place in its postset, in which transition t_r cannot fire and thus the route segment is closed.

5 Summary & Outlook

In this article we have presented a formal, passenger-centric model of a multi-agent system for multimodal public transportation with a convenient graphical representation, and the possibility of execution. We intend to use it as a starting point for a performance model suited to the development of multimodal supervision algorithms. In achieving so, we have to face the famous state space explosion problem w.r.t. the discrete state space of the multi-agent system. We think that the latter can be overcome by a fluidification of the passenger movements [9]. The result would be a form of a hybrid-dynamical Petri net model [3]: The transportation means are still represented by indivisible tokens with internal structure and state of their own that are moved upon the occurrence of discrete events. The passenger movements, on the other hand, are captured by fluids that flow in the infrastructure as a continuous-function of time.

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