Analysis of Two Methods of Systemic Risk Measurement Based on Option

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Abstract. This paper makes a comparison and analysis about those two main systemic risk measurement methods based on option. Through the comparison and analysis in 5 aspects, more knowledge about each method can be discovered including its advantages and disadvantages. Some conclusions about extensions of credit risk measurement methods can also be included in this paper.

Keywords: Systemic risk measurement, contingent claim analysis, option-iPoD

1 Introduction

Data gains its dominance in researches in different fields with the development of efficient accessibility of massive data. As one of the most important fields – financial industry, data from option and future markets offer basis for various theoretical approaches that cannot be applied without high frequency data or sufficient panel data.

The current crisis demonstrates the need for tools to detect systemic risk in financial industry. There are two main systemic risk measurement methods which are based on data from financial market, Contingent Claim Analysis (CCA)[1,2] and option-iPoD[3]. Despite the similar dependence on market data, the different methods show differences in design of methods, in/out parameters, empirical researches, etc. This paper makes a comparison between the two methods in four different aspects, which provides more information about each method that is based on high frequency data. The results show that neither of two methods is suitable for all kinds of analysis while advantages come with disadvantages.

2 Summarization of CCA and option-iPoD

2.1 About two methods

A. Contingent Claim Analysis

Contingent claim are the assets whose value depends on other assets' value, for example, options. Contingent Claim Analysis (CCA) is a widely used analytical method in risk measurement. While contingent claim analysis is applied in credit risk analysis, it is normally called Merton Model. The key to understand CCA is the Risk-Adjusted Balance Sheet, which comes from traditional balance sheet with adding "market risk factor". As for elements in balance sheet, this analysis transfers the book value into market value, which can't be observed directly.

The risk-adjusted balance sheet is:

$$Assets = Equity + RiskyDebt$$
$$A(t) = J(t) + D(t)$$

While

$$RiskyDebt = DefaultFreeDebt - debtguarantee$$
$$D(t) = Be^{-rT} - P(t)$$

Assets A(t) is the market value of assets and the equity J(t) is the market value of equity. Debt guarantee P(t) can be expressed as the expected loss of the debt because the debtor may default at due time. B is the promised payments at time T and r is the risk-free rate. There are two important conclusions as follows:

- Equity is a call option whose underlying assets is Assets and strike price is the promised payments
- Debt guarantee is a put option whose underlying assets is Assets and strike price is the promised payments

According to BSM option pricing model,

$$J(t) = A(t)N(d_1) - Be^{-rT}N(d_2) \qquad (1)$$

with $d_1 = \frac{ln(A/B)+(r+\sigma^2/2)T}{\sigma\sqrt{T}}$, $d_2 = d_1 - \sigma\sqrt{T}$, σequals to the standard deviation of the asset return. N(X) is the cumulative standard normal distribution.

Main Risk exposure indicators

- Risk-neutral default probability: With expect assets return rate equals the risk-free rate r, N(-d_2) is the risk-neutral default probability
- Credit spread: $s = y - r, y = \frac{ln(B/D)}{T}$
- Risk-adjusted balance sheet elements: Assets, Equity, Risk Debt
- Delta, it measures the non-linear change in the value of an option per unit change in the value of the underlying asset

B. Option-iPoD

Buchen and Kelly (1996) propose a numerical method to get probability density distribution function of assets. Capuano (2008) proposes option-iPoD based on former function, which is a systemic financial risk measurement method.

Entropy measures the uncertainty of a variable and maximum entropy shows the most uncertainty of a variable. Principle of Maximum Entropy, that is PME, is ideally suited to estimating the probability distribution of an asset on which a derivative security or contingent claim is written[1].

The problem is following:

$$min_D \left\{ min_{f(V_T)} \int_{V_T=0}^{\infty} f(V_T) \log\left[\frac{f(V_T)}{f^0(V_T)}\right] dV_T \right\} \qquad (2)$$

While $f^0(V_T)$ is the prior probability density distribution function, $f(V_T)$ is the true probability density distribution function.

The constraints are following:

a) Constraints based on Merton Model

$$C_T^K = \max(E_T - K;0) = \max(\max(V - D;0) - K;0) = \max(V_T - D - K;0) \qquad (3)$$

C_T^K: The price of an option whose strike price is K and due time is T.

V: the market price of assets

D: the market price of risky debt

E: the market price of equity

r : the risk-neutral rate

b) Constraints based on option market price

$$C_0^i = e^{-rT} \int_{V_T=0}^{\infty} max(V_T - D - K_i;0)\, f(V_T) dV_T = e^{-rT} \int_{V_T=0}^{\infty} (V_T - D - K_i) f(V_T) dV_T \qquad (4)$$

c) Constraints based on probability density function

$$1 = \int_{V_T=0}^{\infty} f(V_T) dV_T \qquad (5)$$

We treat the problem as solving an optimization problem for $f(V_T)$ with 3 constraints. $f(V_T)$ Will be solved first and given the optimal $f(V_T)$ solve for D. The Larangian is:

$$\begin{aligned} L = & \int_{V_T=0}^{\infty} f(V_T) \log\left[\frac{f(V_T)}{f^0(V_T)}\right] dV_T + \lambda_0 \left[1 - \int_{V_T=0}^{\infty} f(V_T) dV_T\right] \\ & + \lambda_1 \left[E_0 - e^{-rT} \int_{V_T=0}^{\infty} (V_T - D) f(V_T) dV_T\right] \\ & + \sum_{i=1}^{n} \lambda_{2,i} [C_0^i - e^{-rT} \int_{V_T=0}^{\infty} (V_T - D - K_i) f(V_T) dV_T] \end{aligned} \qquad (6)$$

We can solve the function by numerical method and get the probability distribution function of assets by constraints mentioned above.[2] When the function is solved, some risk exposures, such as default probability[3], are measured.

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[1]Peter W. Buchen, Michael Kelly, "The Maximum Entropy Distribution of an Asset Inferred from Option Prices"

[2] Standard Newton numerical method for example. The constraints are constraints in Lagrangian which can be the functions to solve unknown parameters.

[3] Default probability equals $\int_0^D f(V_T)$

2.2 Data source

Not only the algorithm but also the data source are based on financial markets especially option & future market. Data concerning about "level" and "volatile" are required to construct risk detecting indicators, which means a higher frequency data are always better because the second moment of data series are needed. The data required can be divided into following categories:

A. Data from stock market

Market value of equity is required. In most cases, prices of stocks are proxy variables of equity that are unobservable directly. Daily data or even weekly data can be used to construct risk detecting indicators while other frequency are available depending on different scenarios and purposes. When necessary, frequency higher than daily, such as hourly, can be applied to measure a more precise process of changing of risk.

Risk-free interest rate is also needed for both two methods. Government bond or other composite indexes can be used as this indicator.

B. Data from option market

Information concerning about the details of contracts in the option market is used to construct the constraint equations in option-iPoD. Precisely, expiration time of options, prices of options, are required in the algorithm. Analogous to the frequency issue in stock market, higher frequency data are always better.

3 Comparison and analysis

Two methods mentioned above have different features. This part focuses on the difference in 5 fields: thinking of designation, in/out variables, mathematical methods, empirical analysis approaches, and extensions of methods. It is expected that more info about individual method will be discovered through comparison.

3.1 Thinking of designation

Ⅰ) Contingent Claim Approach

Thinking of design of CCA can be divided into 2 parts.

- Risk-adjusted Balance Sheet in theory
- Calibration to risk-adjusted Balance Sheet for different sectors

CCA is an extension of BSM model that assumes a stochastic process of Assets. It is reasonable because of the "Weak Axiom of Efficient Market". CCA assumes that default happens when asset is lower than debt. The core function is following:

$$Assets = Equity + Defaultfree - DebtGuarantee$$

Equity is a call option whose underlying asset is Assets and strike price is the promised payments. Debt guarantee is a put option whose underlying asset is Assets and strike price is the promised payments. This function is a risk-adjusted traditional balance sheet. Scalars in traditional balance sheet are transferred into time variables with "risk factors". Several risk exposure indicators can be calculated including elements in risk-adjusted balance sheet, risk-neutral default probability, Delta[4].

Another innovation is the calibration to risk-adjusted balance sheet for different sectors. For example, some financial institutions are “too-big-to-fall”, so guarantees from government are often existed, which adds the assets of financial sectors. In empirical analysis, some calibrations to equations are made to insure rationality of the methods. Merton et al. (2007) divide the economy into 4 sectors: corporate sector, financial sector, households sector, and sovereign sector, so there are 4 equations for different sectors.

Ⅱ）Option-iPoD

CCA requires many assumptions, such as constant rate, distribution function of assets, which Option-iPoD is designed to release. Too many assumptions are the barrier to suitable empirical solutions. Christian Capuano (2008) proposes option-iPoD, which releases two main assumptions: probability distribution function of Assets and the default barrier[5].

The key to option-iPoD is Maximum Entropy Distribution of Assets, which is the result of Principle of Maximum Entropy (PME). The PME is ideally suited to estimating the probability distribution of an asset which a derivative security or contingent claim is written. With constraints from option market prices, probability distribution of an asset can be calculated.

The unknown coefficient in Lagrangian function and default barrier D can be calculated by means of numerical mathematical methods. When probability distribution of an asset is calculated, some risk exposure indicators can be calculated. “Less assumptions” is a feature of option-iPoD. Exception for prior probability distribution, all constraints are from real option market prices, which do no harm in the freedom of variables.

There are also “CCA factors” in option-iPoD. Market value of assets, partial probability distribution, delta, and gamma can be calculated through option-iPoD.

Table 1． Comparison in design of thinking

	CCA	Option-iPoD
Role of option market	Thinking of BSM option pricing model	Data source of constraints
Focus	Risk-adjusted balance sheet	Probability distribution of an asset
Data	Traditional balance sheet	Option market
similarity	Coming from option market, either thinking or data	

[4]Equation: $debt\ guarantee/Assets$, measures the non-linear change in the value of an option per unit change in the value of the underlying asset

[5]CCA assumes that default happens when asset is lower than promised payments, but promised payments are not a certain volume. In KMV model, default barrier equals 50% long-term debt plus 100% short-term debt

3.2 In/out variables

Comparison between two methods tells attitudes to “variable freedom” from two methods.

Table 2.Comparison in in/out variables

	CCA	Option-iPoD
Input parameters	Market value of equity; Standard variation of equity; Risk-free interest rate; Default barrier; Expect return rate of asset;	Expiration time of options; Stock price; Risk-free interest rate;
Main output variables (relate to risk-adjusted balance sheet)	Default probability; Market value of asset; Expected loss of debt; Equity;	Default probability; Market value of asset; Expected loss of debt; Equity; Default barrier;
Main output variables (relate to risk exposure indicators)	Standard variation of asset; Delta; Gamma; Vega	Standard variation of asset; Probability distribution of asset; Delta; Gamma; Vega

CCA has more input parameters than option-iPoD does while the two methods have nearly same main output variables. Option-iPoD has two distinct outputs: probability distribution of asset and default barriers that are two prior assumptions of CCA. To some degree, CCA has more “constraints” than option-iPoD does, which leads to deviation from real market.

Table 3.Comparison in Assumption & in/out variables

	CCA	Option-iPoD
In/out variables	More input parameters	Less input parameters but two more distinct variables
Similarity	Number of input variables is larger than number of output variables. Fewer assumptions are better for methods.	

3.3 Mathematical methods

Analytic expression and numerical solution are two main mathematical methods used in empirical analysis. CCA applies analytic expression while option-iPoD uses numerical method. Different methods lead to different results that generate different understanding about risk measurement.

Ⅰ）Mathematical methods in CCA

The main mathematical method in CCA is analytic expressions used in BSM option pricing model. While equity and debt guarantee are regarded as two options whose underlying assets are assets. Through mathematical solution, elements in risk-adjusted balance sheet can be solved and relational expression of elements can also be solved.

It is clear that absolute value of elements from risk-adjusted balance sheet and its relationship can be used to do further research through this mathematical method.

Ⅱ）Mathematical methods in option-iPoD

To obtaining the distribution of equation (2), it is necessary to first find the Lagrange parameters, $\lambda_0, \lambda_1, \lambda_{2,1} \cdots \lambda_{2,n}$, which are determined by the constraints entailed by equation (3), (4) and (5). These equations are nonlinear and have to be solved numerically. A standard Newton method, for example, in Djafari (2000). Buchen and Kelly (1996) and Avellaneda (1998) show that the objective function is convex, and that the solution is unique.

Then the last unknown parameter default barrier D, also has to be solved numerically with following constraint:

$$\lim_{\Delta \to 0} \frac{L(f(V_T, D+\Delta)) - L(f(V_T, D))}{D+\Delta} = 0 \qquad (8)$$

The whole probability distribution function can be established.

It is of importance that the solved probability distribution function is a “partial function” with “partial domain of definition”. Because constraints of option-iPoD come from real option market, which shows that constraints are meaningful if option market’s price information is effective. Stock prices and option prices are insignificant when corporate defaults. For investors of stock or option, there is no trading of stocks, no trading of options in default state. Equity options are not suited to describe the market value of asset in default state.

While equity options don’t contain information on shape of the probability density function in the default state, they do contain information on the cumulative distribution function, the probability of default. As it can be found in equation (7), $f^0(V_T)$ has same information as $f(V_T)$when $V_T < D$, so it provides sufficient “cumulative information” in $V_T = D$. For example, default probability and other risk exposure indicators can be measured.

Table 4.Comparison in mathematical methods

	CCA	Option-iPoD
Emphasis of mathematical methods	Analytic expression of all elements of risk-adjusted balance sheet	Solution to probability distribution of asset and default barrier
Similarity	Mathematical methods are regarded as tools for solution in different purpose	

3.4 Empirical analysis with data

Ⅰ） CCA

There are 3 important basic input variables: market value of asset, standard variation of assets return rate, default barrier. Unlike trading equity, market value of asset and its variation can't be observed directly. So two equations are needed to solve this problem.

$$S_t = h(V_t, \sigma_V, r, D, T)\ (9)$$

$$\sigma_S = g(\sigma_V)\ (10)$$

S_t is equity's market price, V_t is market value of asset, σ_V is standard variation of asset, r is risk-free interest rate, and D is market value of default barrier.

Equation (9) is BSM option pricing model and equation (10) can be expressed as following:

$$\eta_{S,V} = \frac{\Delta S}{S} / \frac{\Delta V}{V} \longrightarrow \sigma_S = \eta_{S,V}\sigma_V\ (11)$$

$$\eta_{S,V} = \Phi(d_1)\frac{V}{S}\ (12)$$

As for market value of default barrier D, it's always be set manually. Taking KMV for example, default barrier equals 100% short-term debt plus 50% long-term debt.

The economy-wide CCA can be used with scenario, simulation, and stress-testing analysis. The level of analysis depends on practical issues related to data availability, data reliability, and goals of the analysis.

a) Risk exposure indicators from CCA can be analyzed with other macroeconomic factors. Factor model is a suitable model[6]. The time pattern of asset returns of each financial institution can be used as the dependent variable in a factor model.
b) "Sub risk-adjusted balance sheet for household sectors"[7].
c) Stress-testing and assessing capital adequacy using CCA model of financial institutions.
d) Making connections between traditional monetary policies based on interest rate & money supply and CCA risk exposure indicators.

Ⅱ）option-iPoD

One of main empirical analysis of option-iPoD is what Capuano (2008) has done with Citi Bank. In that research, option-iPoD is well used to estimate how default barrier and leverage ratio generate through financial crisis in 2008, which proves that option-iPoD is suitable for systemic risk measurement. Based on results from option-iPoD, some advices, such as "de-leveraging" should be applied for Citi Bank, are suggested. At the same time, Capuano observes how Moody's KMV EDF[8] generates, which shows what the systemic risk Citi Bank faces.

Table 5.Comparison in empirical analysis

	CCA	Option-iPoD
Application fields	Wider than option-iPoD does including macro-economy, financial sector, households sector	Fields with trading equity and options
Data	Suitable balance sheets for different sectors	Price information from option market

[6] Based on data from Chile central bank, Dale et al (2008) build a factor model with 4 main economic factors and asset return rate calculated by CCA. This factor model well simulates how the systemic risk generates

[7] See more details in Dale F. Gray et al, "NEW FRAMEWORK FOR MEASURING AND MANAGING MACROFINANCIAL RISK AND FINANCIAL STABILITY"

[8] Estimated Default Frequency: a measure of probability of default given distance to default

For risk supervisors	All four sectors[9]	Corporate and financial sector
Similarity	Time pattern series are usually used in analysis. It is expected that risk indictors can well predict breakout of crisis	

3.5 Extensions of the methods

Ⅰ）relaxing assumptions of CCA

Assumptions lead to gap between theory methods and reality. Extensions of CCA focus on its assumption's relaxation.

- Some researchers have established a relationship between implied volatility of two equity options, leverage and implied asset volatility. In fact, this is another way of implementing Merton's model to get spreads and risk-neutral default probabilities directly from the implied volatility of equity options
- Shimko, Tejima, and Van Deventer (1993) include a Vasicek interest rate term structure model which allows interest rates and term structure of interest rates to vary

Ⅱ）More constraints about option-iPoD

More constraints from markets make option-iPoD more accurate. Capuano (2008) adds zero-coupon bonds constraints to option-iPoD.

Table 6.Comparison in extension of methods

	CCA	Option-iPoD
Extension	Relaxing more assumptions	Adding more constraint from market
Similarity	Fewer manual assumptions and more constraints from market	

Out of the potential participants (249 members), 118 filled the questionnaire, resulting in a response rate of 47%. Table 1 contains the non-respondent analysis results.

Table 7. Reasons for non-participation in the survey

Reason for non-participation	N:o of persons (total = 131)	Percentage
Too busy or forgot	27	21%
Dislike of such surveys or not interested	10	8%
Technical difficulties with the survey	19	15%
Did not feel competent enough to fill the survey despite being a health journalist	7	5%
Unknown or person could not be contacted	67	51%

[9] Corporate sector, financial sector, households sector, sovereign sector

4 Conclusion

4.1 Data from different fields of the economy solve different problems

Based on Merton's analysis, economy can be divided into 4 parts: corporate sector, financial sector, households sector and sovereign sector. Different sector prefers different risk measurement approaches because of the accessibility to different data.

For sectors with trading equities and options, some corporates, some financial institutions, even some public sectors, option-iPoD provides a more accurate and effective systemic risk measurement approach. Compared with CCA, option-iPoD gets fewer manual assumptions which makes it closer to real market. Option-iPoD is more suitable in this situation for its data used in option-iPoD comes from high-frequency option market. More constraints come from markets makes option-iPoD more accurate for various goals of analysis.

CCA shows its wide range of application because of Merton's risk-adjusted balance sheet. For sectors like sovereign sector, with no trading options, CCA is better at estimating systemic risk. The core function[10] is calibrated to correspond with various sectors thus CCA can be used in all sectors, even in public sector or households sector. CCA works at sectors with trading equity or "analogous equity" and option is not necessarily needed.

All above, CCA has wider range of application than option-iPoD does but option-iPoD has less manual assumptions, which makes it closer to reality.

4.2 CCA needs release of assumptions & option-iPoD needs more constraints

Assumptions simplify the problem in theory but deduce the freedom of variable, which leads to inconsistency with reality. CCA will be better if more assumptions are released. In classical Merton Model, constant interest rate and probability distribution are required. For example, Shimko et al. (1993) include a Vasicek interest rate term structure model which allows interest rates and term structure of interest rates to vary, which is a release to interest rate assumption.

Option-iPoD needs more constraints of reliable, available market. More reliable constraints makes prior probability distribution function closer to real probability distribution function by Lagrangianoptimization process. Christian Capuano (2008) adds zero-coupon bonds constraints to option-iPoD which makes option-iPoD more accurate.

Above all, subjectivity should be reduces and objectivity should be increased both in CCA or option-iPoD.

4.3 The improvement of the quality of data tells more

CCA and option-iPoD are two main credit risk measurement methods. These two methods are better than traditional FSIs[11] at solutions to deal with high-frequency

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[10] Assets = Equity + Risk Debt

[11] FSIs: Financial Stability Indicators system, it is a system of indictors measuring financial risk

market value. CCA applies the thinking of option while option-iPoD applies the price information of options.

The two methods focus on option market. On one perspective, option market is a highly financial market whose information if effective. On another perspective, price information from option market is easy to get. With the development of option market, it can provide more information in financial market which are constraints for credit risk measurement model.

Information from option market and theories of option market can work in more sectors not only in financial and corporate sectors. Like CCA, contingent claim analysis works well in sovereign sector that represents macroeconomic systemic risk exposure. Contingent claim analysis can also be used in key banks to simulate systemic risk in financial sector after weighting process.

Above all, macro financial engineering is one of extensions of credit risk measurement method.

4.4 Future work

This paper makes comparison and analysis between two main systemic risk measurement methods based on option in 5 perspectives. But effectiveness of two methods in empirical analysis is not estimated which requires more related researches to support.

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