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New Multi-Objective Decision Support Methodology to Solve Problems of Reconfiguration in the Electric Distribution Systems

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Abstract. The distribution systems (DS) reconfiguration problem is formulated in this paper as a multi-objective mixed-integer linear programming (MILP) multi-period problem, enforcing that the obtained topology is radial in order to exploit several advantages those configurations offer. The effects of distributed generation (DG) and energy storage systems (ESS) are also investigated. To address the multi-objective problem, an improved implementation of the ε -constraint method (AUGMECON-2) is used, providing an adequate representation of the Pareto set. The objective functions considered stand for the minimization of active power losses and the minimization of switching operations. The proposed methodology is tested using a real system based on the S. Miguel Island, Azores, Portugal. The potential uses of cloud-based engineering systems, both in terms of exploiting the enhanced decentralized computational opportunities they offer and of utilizing them in order to achieve communication and coordination between several entities that are engaged in DS, are thoroughly discussed.

Keywords: Distribution system reconfiguration; ε -constraint method; switching cost; multi-objective optimization.

1 Introduction

The reconfiguration of power distribution systems (DS) is the process of opening and closing switches, changing the network topology, in order to achieve several operating advantages. The problem is to find a configuration that presents the least amount of active power losses, subject to the following restrictions: voltage levels, power transfer capability of branches, the rated power of transformers and often the radial configuration of the obtained system. A classification of the DS problems together with a literature survey can be found in [1].

Systems with meshed structures are not recommended for power distribution networks, because their protection schemes are more complex in comparison with radial distribution systems. The reconfiguration problem is typically a nonlinear, multi-objective, combinatorial problem, subject to operational constraints of loads. Some of these configurations are not allowed, or because they lead to a disconnected system (with several islands) or non-radial systems. Others are not feasible, because they violate operational and load restrictions. Various objective functions have been used, but the most common one is the minimization of the active power losses [2].

Multi-objective approaches try to optimize a combination of the previous objective functions [3], [4]. In the emerging Smart Grid scheme, Distributed Generation (DG) and Energy Storage Systems (ESS) play a key role and pose new challenges in the operation of the DS [5], [6].

The majority of the related studies in the literature treat the problems using solution techniques based on meta-heuristics, mainly because they are easily applicable to the problem and offer computational advantages, especially in the case of the multi-objective formulations. There are also studies that try to solve the DS reconfiguration problem through the linearization of the AC power flow constraints [7], formulating it as a Mixed-Integer Linear Programming (MILP) problem.

However, treating the DS reconfiguration with a Multi-Objective Mathematical Programming (MOMP) approach poses difficulties, mainly due to the computational effort that is required and the need to guarantee that the obtained solution is the best possible [8]. Although solving a Single-Objective Mathematical Programming problem is a procedure that will return the maximum or the minimum solution among the feasible ones, the solution of a MOMP problem is not a trivial task, since there is not, in general, a single solution that optimizes every objective function simultaneously. In MOMP, the required result is the set of the relatively optimal solutions, called the Pareto set.

The aim of this work is to develop tools that contribute to the quality of service to the end-users at a minimal cost to the companies responsible for power distribution. Therefore, the novel contributions of this work are twofold. The first contribution is the formulation of the radial DS reconfiguration as a multi-period MOMP problem, considering the effects of DG and ESS with two objective functions: minimization of the active power losses and minimization of the total switching cost. The second contribution comes from the solution of the problem using an improved version of the ε -constraint method, namely the Augmented ε -constraint Method (AUGMECON-2), a state-of-the-art methodology introduced by Mavrotas in [8] and further improved in [9], in order to generate an adequate representation of the Pareto optimal solutions of the problem.

2 Contribution to Cloud-based Engineering Systems

The DS reconfiguration (DSR) problem has been a topic of intensive research that has led to several applications during the past decades. Nevertheless, recent changes in the characteristics of the DS, such as the introduction of DG, ESS and flexible demand, as well as the introduction of various stakeholders that utilize the DS, require the reevaluation of the DSR problem using innovative solutions.

In this respect, cloud-based engineering solutions may prove invaluable in several ways. Illustratively, several applications that can immediately benefit the distribution system operator with respect to the DSR problem are:

1. Optimal DSR is formulated as a complex optimization problem. Given emerging elements that have to be considered in optimization procedures, the complexity of the DSR problem dramatically increases. The high computational burden, especially for a large-scale DS, may force distribution system operators to upgrade the informatics infrastructure, incurring high investment costs. A cloud-based engineering system could be used in order to solve the problem in a de-centralized fashion using cloud computing.

2. The introduction of several entities (e.g. load aggregators and DG unit's owners) that utilize the DS to materialize their purposes, imposes the need of transparency regarding the DSR decisions. The configuration of the DS affects several DS operating parameters, such as losses and quality of service, which in turn may affect the allocation of operating costs. A cloud-based engineering system (e.g. an on-line portal) may be used in order to allow stakeholders to access relevant information, promoting the fair allocation of costs.
3. Internet is a utility that is available virtually everywhere since there are many wired and wireless options to access the internet. A cloud-based engineering system may be utilized in order to monitor data, even at remote nodes of the DS at an affordable cost. In this way, better analytics may be obtained and exploited in the development of more accurate optimization problems and real-time operation (e.g. fault detection). Also, commands such as switching actions may be given remotely, eliminating constraints such as crew availability. Finally, advanced control schemes may be performed automatically (e.g. automated reconfiguration after the detection of a fault).

3 Methodology

3.1 Overview of Augmented ϵ -constraint Method

The ϵ -constraint method is a “generation method” that is used to construct an adequate representation of the Pareto-optimal set of solutions, and does not consider the Decision Maker's preferences a priori. Among the different objective functions, one is used as the objective function of the problem, while the others are treated as inequality constraints. Their bounding values are varied parametrically in order to produce the Pareto-optimal set of solutions. The application of the ϵ -constraint method requires the knowledge of the range of objective functions that are used as constraints. The range is usually calculated using the pay-off table that contains the values of the objective functions, resulting from the individual optimization of each single objective function [10]. The pay-off table alone does not guarantee that the range always provides the efficient set. AUGMECON addresses this issue through the utilization of lexicographic optimization. Lexicographic optimization is performed by optimizing the objective functions according to their priority. The highest priority objective function is optimized first and its optimal value is incorporated as an equality constraint in the optimization of all the less priority objective functions. This procedure is repeated for all the objective functions. After lexicographic optimization, the ranges of the objective functions are divided in n equal intervals and $n + 1$ grid points are used to parametrically vary the bound of the objective functions used as inequality constraints in the classical ϵ -constraint method.

This method is guaranteed to provide only efficient solutions [8]. The number of grid points is adjustable and is directly linked to the density of the Pareto-optimal set representation. It also allows determining the desired trade-off between quality of the representation and computational burden. To reduce the computational burden, power flow equations are not incorporated as constraints of the optimization problem. Through a linear approximation of the active power flow losses (using Special Ordered

Sets of Type 2- SOS2) a least-losses equivalent topology may be obtained. If required, branch currents, node voltages and the exact amount of losses may be calculated afterwards for the obtained topology using any available power-flow algorithms. The linearization through SOS2 variables offers computational advantages when using Branch-and-Bound based solution algorithms (e.g. CPLEX).

3.2 Mathematical Formulation

In this section, the proposed mathematical formulation is presented. To simplify the application of the aforementioned multi-objective optimization method and in order to reduce the computational burden, constraints that typically appear in DS reconfiguration problems in the literature, such as node voltage limits, line currents, reactive power, etc. are not considered, assuming that sufficient reactive power compensation is available locally (at the nodes) and that an advanced metering infrastructure (AMI) provides enough information regarding the state of voltage magnitude and angle. The nomenclature used in the formulation is presented in the Nomenclature at the end of the paper.

1) Objective Function

a) Minimization of the Total Operating Costs

Objective function (1) stands for the minimization of the total power losses over the total time period.

$$TL = \sum_t \sum_b P_{b,t}^{loss} \quad (1)$$

b) Minimization of the Total Switching Costs

Objective function (2) stands for the total cost emerging from the switching operations required throughout the horizon in order to change the DS configuration.

$$TSC = \sum_t \sum_b C_{b,t}^{sw} \quad (2)$$

The two objective functions are conflicting. As the load at the different nodes of the DS varies with time, the flows through the branches change, so do the losses. Thus, by reconfiguring the DS in order to minimize the losses at a specific time period, switching operations are contributing in the total switching-cost.

2) Constraints

a) Radiality Constraints

The radial configuration of the system is ensured by (3)-(5). Constraints (4) and (5) consider transfer nodes, i.e. nodes without production or consumption.

$$\sum_{b \in B} x_{b,t} = N - N^f - \sum_{i \in I: i \in (\Omega_i^t \cup \Omega_b^j)} (1 - y_{i,t}) \quad (3)$$

$$x_{b,t} \leq y_{i,t} \quad \forall b \in B, i \in (\Omega_i^t \cup \Omega_b^i \cup \Omega_b^j), t \in T \quad (4)$$

$$\sum_{b \in B: i \in (\Omega_i^t \cup \Omega_b^i \cup \Omega_b^j)} x_{b,t} \geq 2 \cdot y_{i,t} \quad \forall i \in \Omega_i^t, t \in T \quad (5)$$

A DS topology is radial if and only if it constitutes a tree-graph (no loops) and all the nodes are connected.

b) Constraints ensuring the Connection of Nodes with Distributed Elements

The following constraints prevent islanding. Thus, they do not allow a part of the network to be fed from distributed elements (i.e. storage systems, DG). For intentional islanding (e.g. for back-up reasons) these constraints could be handled in order to allow some nodes to play the role of a substation.

$$\sum_{b \in B: i \in \Omega_b^j} k_{b,t} - \sum_{b \in B: i \in \Omega_b^i} k_{b,t} + k_{i,t}^g = k_i^d \quad \forall i \in I, t \in T \quad (6)$$

$$-N^{de} \cdot x_{b,t} \leq k_{b,t} \leq N^{de} \cdot x_{b,t} \quad \forall b \in B, t \in T \quad (7)$$

$$k_{i,t}^g \geq 0 \quad \forall i \in \Omega_i^f, t \in T \quad (8)$$

$$k_{i,t}^g = 0 \quad \forall i \notin \Omega_i^f, t \in T \quad (9)$$

$$k_i^d = 1 \quad \forall i \in \Omega_i^{de} \quad (10)$$

$$k_i^d = 0 \quad \forall i \notin \Omega_i^{de} \quad (11)$$

c) Node Power Balance, Branch Flow Limits, Substation Limits and Distributed Generation Limits

Constraints (12)-(15) stand for the power equilibrium at each node of the distribution system and set the appropriate values for the respective decision variables.

$$\sum_{b \in B: i \in \Omega_b^j} f_{b,t} - \sum_{b \in B: i \in \Omega_b^i} f_{b,t} + P_{i,t}^f + P_{i,t}^{dg} + P_{i,t}^{dis} = D_{i,t} + P_{i,t}^{ch} \quad \forall i \in I, t \in T \quad (12)$$

$$-f_b^{max} \cdot x_{b,t} \leq f_{b,t} \leq f_b^{max} \cdot x_{b,t} \quad \forall b \in B, t \in T \quad (13)$$

$$0 \leq P_{i,t}^f \leq P_i^{f,max} \quad \forall i \in I \in \Omega_i^f, t \in T \quad (14)$$

$$0 \leq P_{i,t}^{dg} \leq P_i^{dg,max} \quad \forall i \in \Omega_i^{dg}, t \in T \quad (15)$$

d) Linear Approximation of the losses

The power losses on a branch are approximated using a quadratic function of the power that flows through the branch. The units of the coefficients b and c are [.] and [MW⁻¹], respectively.

$$P_{b,t}^{loss} = b \cdot |f_{b,t}| + c \cdot f_{b,t}^2 \quad \forall b \in B, \forall t \in T \quad (16)$$

The expression of the losses can be linearized using the concept of Special Order Sets of Type 2 (SOS2). This is described by (17)-(19). It should be noted that variables z are positive and continuous.

$$\sum_{p \in P} z_{b,t,p} = 1 \quad \forall b \in B, \forall t \in T \quad (17)$$

$$f_{b,t} = \sum_{p \in P} X_p \cdot z_{b,t,p} \quad \forall b \in B, \forall t \in T \quad (18)$$

$$F_{b,t} = \sum_{p \in P} Y_p \cdot z_{b,t,p} \quad \forall b \in B, \forall t \in T \quad (19)$$

By the definition of SOS2, it is also stipulated that no more than two adjacent values of z can be greater than zero. The accuracy of the approximation depends on the sampling of the non-linear function, i.e. the number of samples and the intervals that are used.

It is also reported that the linearization of a function using this method has computational advantages when using the Branch-And-Bound algorithm that is implemented in many commercial solvers.

e) *Switching Cost*

Equations (20)-(22) define the switching cost by following the change of status of every line. Through the parameter SC_b a different switching cost can be attached to every branch representing the cost emerging from this operation, considering several factors such as equipment degradation costs and crew constraints.

$$C_{b,t}^{sw} = (x_{b,t} - x_{b,t-1}) \cdot SC_b \quad \forall b \in B, t \in T | t > 1 \quad \text{if } x_{b,t} = 1 \quad (20)$$

$$C_{b,t}^{sw} = (x_{b,t-1} - x_{b,t}) \cdot SC_b \quad \forall b \in B, t \in T | t > 1 \quad \text{if } x_{b,t} = 0 \quad (21)$$

$$C_{b,t}^{sw} = 0 \quad \forall b \in B \quad \text{if } t = 1 \quad (22)$$

f) *Energy Storage System Constraints*

Energy Storage Systems (ESS) are an important asset of the smart-grid. Promising technologies such as Sodium-Sulfur batteries (NAS) have been already used in practice and can efficiently store large amounts of energy. Their primary goal is to support demand response activities, balancing volatile renewable energy sources production and offer back-up and other ancillary services. In this study, a general formulation of an ESS is considered through (23) to (28), in order to investigate their ability to facilitate the operating goals of the DS.

$$SOE_{i,t} = SOE_{i,t-1} + P_{i,t}^{ch} \cdot CE_i - \frac{P_{i,t}^{dis}}{DE_i} \quad \forall i \in \Omega_i^S, t \in T \quad (23)$$

$$P_{i,t}^{ch} \leq CR_i \cdot y_{i,t}^S \quad \forall i \in \Omega_i^S, t \in T \quad (24)$$

$$P_{i,t}^{dis} \leq DR_i \cdot z_{i,t}^S \quad \forall i \in \Omega_i^S, t \in T \quad (25)$$

$$SOE_{i,t} \leq SOE_i^{max} \quad \forall i \in \Omega_i^S, t \in T \quad (26)$$

$$SOE_{i,t} \geq SOE_i^{min} \quad \forall i \in \Omega_i^S, t \in T \quad (27)$$

$$y_{i,t}^S + z_{i,t}^S \leq 1 \quad \forall i \in \Omega_i^S, t \in T \quad (28)$$

4 Tests and Results

The proposed methodology has been coded using General Algebraic Modeling System (GAMS) and the solver CPLEX. The system used to illustrate the presented methodology is adapted from [11] and is presented in Fig. 1. All the branches are considered switchable except for branches L01, L06, L08, L12, L20, L21, L22, L23, L30, L31 and L32. Active power limits for all branches are considered as 500kW and the switching cost for a single open or close operation is 10€. To approximate the losses, 51 pairs of (X_i, Y_i) are used, one for every 5kW in the range [-500kW, 500kW], and constants a and b are 0.001 and 0.0003 MW^{-1} , respectively. The operation of the system is studied over a 7-hour horizon. An ESS is considered at node 27. Its capacity is 16 kWh, having a charging/discharging rate equal to 4 kW and charging/discharging efficiencies of 90%. Its minimum state-of-energy in order to avoid deep discharge is 8 kWh. Its initial state-of-energy is 9 kWh. Two DG units with maximum capacity as 15 kW and 10 kW are considered at nodes 21 and 45, respectively. Substations at Lagoa 1, Lagoa 2 and Lagoa 3 can provide 1136 kW, 480 kW and 2657 kW,

respectively, and 11811 kW at the São Roque power node. The Pareto efficient set of solutions obtained by the AUGMECON method is shown in Fig. 2.

Five grid points are used and the CPU time needed is 54.8 seconds on a laptop with an 8-core processor and 4GB of RAM, running a windows 64-bit distribution. Using more grid points, no more efficient solutions are discovered, thus Fig. 2 provides the complete Pareto efficient solution set. After acquiring an adequate representation of the Pareto efficient solution set, a decision making procedure should be applied, such as the Analytic Hierarchy Process (AHP) or the Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS), in order to make the final decision.

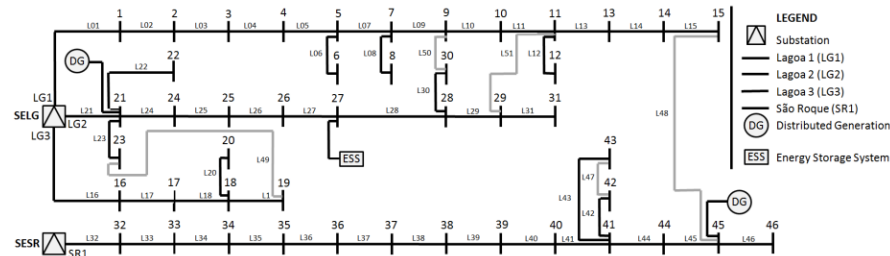


Fig. 1. Single-line diagram of Test System Network MT 10kV Interconnection São Roque - Lagoa adapted [11].

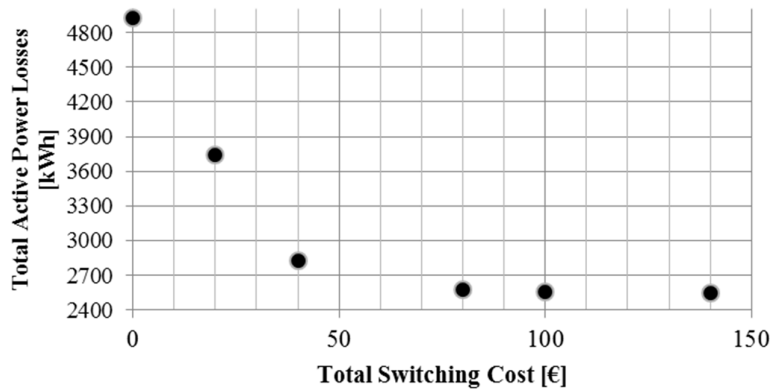


Fig. 2. The Pareto efficient solution set.

However, at this point, an assumption is made about the decision maker’s final selection. It can be noticed that, as the switching cost decreases, the active power losses increase. Furthermore, it can be noticed that the first four solutions slightly increase the active power losses, while the switching cost is significantly decreased. Thus, the final selection considered is the 4-th efficient solution.

The configurations for this solution during the horizon are shown in Figs. 3 and 4. The first reconfiguration occurs at the beginning of period T4-T3 and T7 where the branches L49, L50, L51 and L45 open. At the beginning of period T4 the branch L50 and L45 closes and the branch L10 and L27 opens. The DG units provide power at their maximum capacity during all the periods. In Fig. 5 the ESS state-of-energy is

presented. During period T4 the ESS is charging. This stored energy is used during periods T5 and T6 in order to serve the increased demand needs.

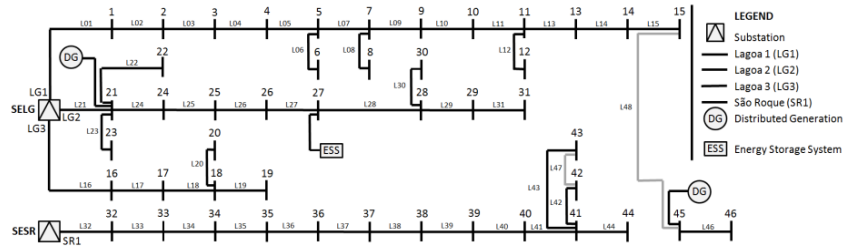


Fig. 3. Configuration of the distribution system during periods T1-T3 and T7.

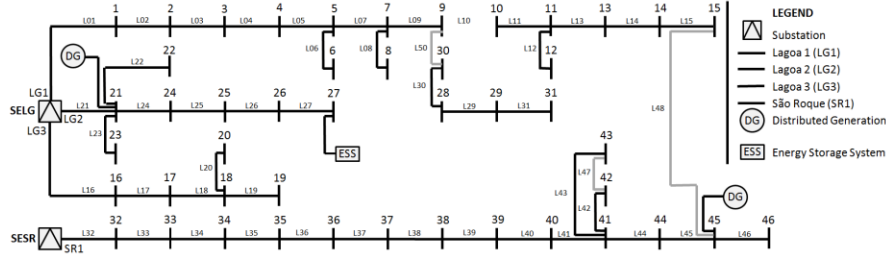


Fig. 4. Configuration of the distribution system during period T4-T6.

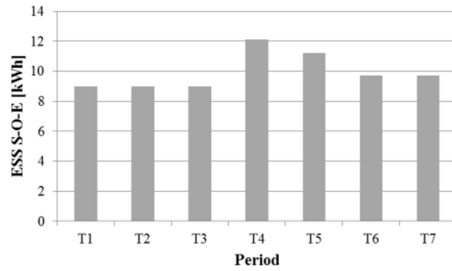


Fig. 5. ESS State-of-energy during the time horizon.

5 Conclusions

In this work the application of a new method to handle the DS reconfiguration problem was presented. The problem was formulated as a multi-period multi-objective mixed-integer linear problem, solved using the augmented ϵ -constraint method in order to produce an adequate representation of Pareto efficient solutions set. The objective of the proposed formulation was to find the DS configuration that has as a result the minimum losses and switching operations. DG and ESS have also been considered in the proposed mathematical formulation. Applying the ϵ -constraint method in order to solve multi-objective problems related to the DS, considering also power flow

equations, such as optimal placement and sizing of DG units and ESS, reliability and other quality of distribution issues, will be the subject of future works by the authors.

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Nomenclature: i (I) - index (set) of nodes; b (B) - index (set) of branches; t (T) - index (set) of time intervals; p (P) - index (set) of points that are used to approximate the non-linear function of losses; Ω_i^f - subset of nodes that are substations; Ω_i^{dg} - subset of nodes that have DG; Ω_i^s - subset of nodes that have ESS; Ω_i^{de} - subset of nodes with DG or ESS ($\Omega_i^{dg} \cup \Omega_i^s$); Ω_i^t - subset of transfer nodes; Ω_b^i, Ω_b^j - mapping of nodes and branches defined as (i, j) . PARAMETERS: N - number of nodes; N^f - number of substation nodes; N^{de} - number of nodes that have DG or ESS; k_i^d - fictitious demand of node i ; $D_{i,t}$ - demand of node i during period t [kW]; f_b^{max} -

flow limit of branch b [kW]; $P_i^{f,max}$ - max. power that substation of node i can provide [kW]; X_p - X- coordinate of point p that is used for approximation; DE_i - discharging efficiency of the ESS of node i ; CR_i - charging rate of node i [kW]; DR_i - discharging rate of node i [kW]; SOE_i^{max} - max. state-of-energy of ESS of node i [kWh]; SOC_i^{min} - minimum state-of-energy ESS of node i [kWh]; DE_i - discharging efficiency of the ESS of node i . VARIABLES: $P_{b,t}^{loss}$ - power losses of branch b during period t [kW]; $C_{b,t}^{sw}$ - switching cost of branch b during period t [€]; $x_{b,t}$ - binary variable- 1 if branch b is closed during period t , else 0; $y_{i,t}$ - auxiliary binary variable that is used to properly handle transfer nodes; $k_{b,t}$ - fictitious flow through branch b during period t ; $k_{i,t}^g$ - fictitious generation of node i during period t ; $f_{b,t}$ - flow through branch b during period t [kW]; $P_{i,t}^f$ - power provided by substation of node i during period t [kW]; $P_{i,t}^{dg}$ - power provided by DG of node i during period t [kW]; $P_{i,t}^{dis}$ - power provided by DG of node i during period t [kW]; $P_{i,t}^{ch}$ - charging power of the ESS of node i during period t [kW]; $z_{b,t,p}$ - SOS2 variables that are used to approximate the power loss.