Efficient Structural System Reliability Updating with Subspace-Based Damage Detection Information

Michael Döhler, Sebastian Thöns

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Michael DÖHLER[1,3], Sebastian THÖNS[2,3]

[1] Inria/IFSTTAR, I4S
Campus de Beaulieu, 35042 Rennes, France, michael.doehler@inria.fr

[2] Department of Civil Engineering, Technical University of Denmark
Brovej, 2800 Lyngby, Denmark, sebt@byg.dtu.dk

[3] Federal Institute for Materials Research and Testing (BAM), Berlin, Germany

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Abstract

Damage detection systems and algorithms (DDS and DDA) provide information of the structural system integrity in contrast to e.g. local information by inspections or non-destructive testing techniques. However, the potential of utilizing DDS information for the structural integrity assessment and prognosis is hardly exploited nor treated in scientific literature up to now. In order to utilize the information provided by DDS for the structural performance, usually high computational efforts for the pre-determination of DDS reliability are required. In this paper, an approach for the DDS performance modelling is introduced building upon the non-destructive testing reliability which applies to structural systems and DDS containing a strategy to overcome the high computational efforts for the pre-determination of the DDS reliability. This approach takes basis in the subspace-based damage detection method and builds upon mathematical properties of the damage detection algorithm. Computational efficiency is gained by calculating the probability of damage indication directly without necessitating a pre-determination for all damage states. The developed approach is applied to a static, dynamic, deterioration and reliability structural system model, demonstrating the potentials for utilizing DDS for risk reduction.

1 INTRODUCTION

Research focus within structural health monitoring (SHM) has been, amongst others, on the development of damage indicators [1] as well as on the utilization of SHM information in structural reliability analysis (e.g. [12] and [2]) during the last decade. Both foci have been pursued mostly independently and only recently the research community started to interact, e.g. within the COST Action TU1402. For example, it is essential to know if a detected damage actually has a significant impact on the structural integrity. Conversely, given the information that no damage has been detected, the current structural performance is of no lesser interest. The connection of damage detection systems (DDS) and algorithms (DDA) with the evaluation of the structural performance is the aim of this study.

In order to utilize the information provided by DDS and DDA for the structural performance, usually high computational efforts for the pre-determination of DDS reliability are required. In this paper, an approach for the DDS performance modelling is introduced building upon the non-destructive testing reliability [3] which applies to structural systems and DDS containing a strategy to overcome the high computational efforts for the pre-determination of the DDS reliability. This approach takes basis in the subspace-based damage detection approach, see e.g. [4]-[6]. Analysing its mathematical properties regarding the probability of damage indication in connection with a structural model, we show how the reliability of the structural system can be updated using the damage detection information from measurement data. In this way, the probability of damage indication can be calculated directly without the necessity of pre-determination of the DDS reliability.

We introduce a deteriorating structural system and the DDA modelling in Sections 2 and 3. Section 3 contains a detailed description of the DDA properties which can be exploited for the approach to directly calculate the probability of indication. In Section 4, the structural system and the DDA models are utilised together for updating the structural system reliability. Section 5 contains a study on how to apply the developed approach and demonstrates which DDA characteristics lead to a reliability increase of the exemplary structural system. The conclusions (Section 6) highlight the potentials of DDA for risk reduction in general and the efficiency of the developed approach.

2 STRUCTURAL SYSTEM MODELLING

The performance of a structural system both in regard to system failure and system damage can be described by methods such as logical systems, Daniels Systems and Bayesian networks. The probability $P(F_S)$ of a structural system failure can be calculated by integrating the joint probability density over the space of system failure $\mathbf{\Omega}_{F_S}$ in dependency of the vectors of the system performance random variables $\mathbf{X}$ and the system degradation random variables $\mathbf{D}$, i.e.

$$P(F_S) = \int_{\Omega_{F_S}} f_{F_S}(\mathbf{X},\mathbf{D})\, d\mathbf{X} d\mathbf{D}, \tag{1}$$

where the system failure space Ω_{F_S} is defined with limit state functions.

Based on logical system modelling, a deteriorating structural system can be described as a series system with n_k parallel subsystems consisting of n_j components:

$$P(F_S) = P\left(\bigcup_{j=1}^{n_k} \bigcap_{i}^{n_j} \left(M_{R,i,j} R_{i,j}(t_{SL}) - M_S S_{i,j} \right) \leq 0 \right) \text{ with } R_{i,j}(t_{SL}) = R_{i,j,0}\left(1 - D_{i,j}(t_{SL})\right) \tag{2}$$

The vector of the system performance random variables $\mathbf{X}$ comprises then the resistance model uncertainties $M_{R,i,j}$, the time dependent component resistances $R_{i,j}(t_S)$, the loading model uncertainty M_S and the component loading $S_{i,j}$, i.e.

$$\mathbf{X} = \left[M_{R,1,1} \ldots M_{R,n_i,n_j}, R_{1,1}(t_{SL}) \ldots R_{n_i,n_j}(t_{SL}), M_S, S_{1,1} \ldots S_{n_i,n_j} \right]^T. \tag{3}$$

The vector of the system degradation random variables contains the time dependent component damages $\mathbf{D} = \left[D_{1,1}(t_{SL}) \ldots D_{n_i,n_j}(t_{SL}) \right]^T$ including zero damages.

3 SUBSPACE-BASED DAMAGE DETECTION

3.1 Theoretical background

Automatic vibration monitoring aiming for damage detection is one of the most known and developed techniques for long term structural health monitoring (SHM) and has been recognized as an addition or alternative to visual inspections or local non-destructive testing performed manually. The rationale is that damages have an effect on the structural stiffness, and thus on the modal parameters (modal frequencies, damping ratios and mode shapes) which characterize the dynamics of the structure. A network of vibration sensors (usually accelerometers) is attached to the structure, measuring continuously the structural vibration response to ambient excitation like wind, traffic, waves or other sources. Changes in the measured signals with respect to the dynamic characteristics of the structure then indicate damage.

The subspace-based damage detection evaluates such changes with a statistical test value: vibration measurements from the current system are compared to a reference state in a subspace-based residual vector. In a hypothesis test, the uncertainties of the residual are taken into account and the respective χ^2 test statistic is compared to a threshold in order to decide if the structure is damaged or not. Based on these properties, the χ^2 test statistic is considered as the damage indicator value for damage monitoring. It is defined as follows.

The vibration behaviour of the monitored structure is assumed to be described by a linear time-invariant dynamical system

$$\mathbf{M}\ddot{z}(t) + \mathbf{C}\dot{z}(t) + \mathbf{K}z(t) = v_F(t) \tag{4}$$

where t denotes continuous time, $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K} \in \mathbb{R}^{m \times m}$ are the mass, damping and stiffness matrices, vector $z \in \mathbb{R}^m$ collects the displacements of the m degrees of freedom of the structure, and $v_F(t)$ is the external force which is usually unmeasured for long-term monitoring, being modelled as white noise. Observing the structural system (4) with a set of r acceleration sensors yields the measurements

$$y(t) = L\ddot{z}(t) + e(t) \tag{5}$$

where $y \in \mathbb{R}^r$ is the measurement vector, matrix $L \in \mathbb{R}^{r \times m}$ indicates the sensor locations and e is the measurement noise. Measurements are taken at discrete time instants $y_k = y(k\tau)$, where τ is the sampling rate.

From a set of outputs $\{y_k\}_{k=1,\dots,N}$ from the healthy reference state of the structure, the left null space S of a Hankel matrix $\mathcal{H}_{ref}$ containing the output covariance estimates is computed with the property $S^T\mathcal{H}_{ref} \approx 0$. For damage detection, a Hankel matrix $\mathcal{H}$ is computed on measurements $\{y_k\}_{k=1,\dots,N}$ during the monitoring phase. It is confronted to the reference state in residual vector $\zeta = \sqrt{N}\ \mathrm{vec}(S^T\mathcal{H})$, which deviates from zero if the system is damaged. This residual vector is asymptotically Gaussian (for large N) with zero mean in the reference state and non-zero mean in the damaged state, see also details in [4]-[6]. Let θ be a vector of system parameters that shall be monitored, and let θ_0 be its value in the reference state. Then, the respective test statistic writes as

$$s = \zeta^T\Sigma^{-1}\mathcal{J}(\mathcal{J}^T\Sigma^{-1}\mathcal{J})^{-1}\mathcal{J}^T\Sigma^{-1}\zeta \tag{6}$$

where $\mathcal{J}$ is the residual sensitivity with respect to θ and $\Sigma = \mathbf{E}(\zeta\zeta^T)$ is the residual covariance matrix. Both can be computed from measurement data in the reference state. The test statistic (6) is asymptotically χ^2 distributed with a non-centrality parameter in the damaged state.

To decide if the monitored structure is damaged or not, the test statistic is compared to a threshold λ. The threshold is typically chosen so that the probability of false alarms (type I error) is below some chosen level. In the following, the event of *indication* is defined as a detection of damage, i.e. when the test statistic exceeds the threshold. Hence, the test statistic s is our damage indicator value (DIV).

3.2. Performance properties

The probability distribution of the test statistic s is well-known (see [4]-[6]). Its χ^2 distribution has the following parameters: its degrees of freedom are the dimension of θ, and its non-centrality parameter in the damaged state is given by $\delta^T \mathcal{J}^T \Sigma^{-1} \mathcal{J} \delta$, where vector δ is linked to the change in the system parameter due to damage and to the number of data samples by

$$\delta = \sqrt{N}(\theta - \theta_0)\,. \tag{7}$$

In Fig. 1 an example of the respective distributions in the reference and damaged state is given.

With these known parameters, the probability of indication (PoI) can be calculated for a given damage vector δ. It holds

$$PoI(\delta) = P(s > \lambda) = \int_{\lambda}^{\infty} p_{\chi^2}(x; \dim(\theta), \delta^T \mathcal{J}^T \Sigma^{-1} \mathcal{J} \delta) \mathrm{d}x \tag{8}$$

where $p_{\chi^2}(x; \mathrm{dof}, \mathrm{nc})$ is the probability density function of the non-central χ^2 distribution with dof degrees of freedom and non-centrality parameter nc. The integral is linked to the cumulative distribution function of the non-central χ^2 distribution, which is given by the generalized Marcum Q-function [7] and can be easily evaluated.

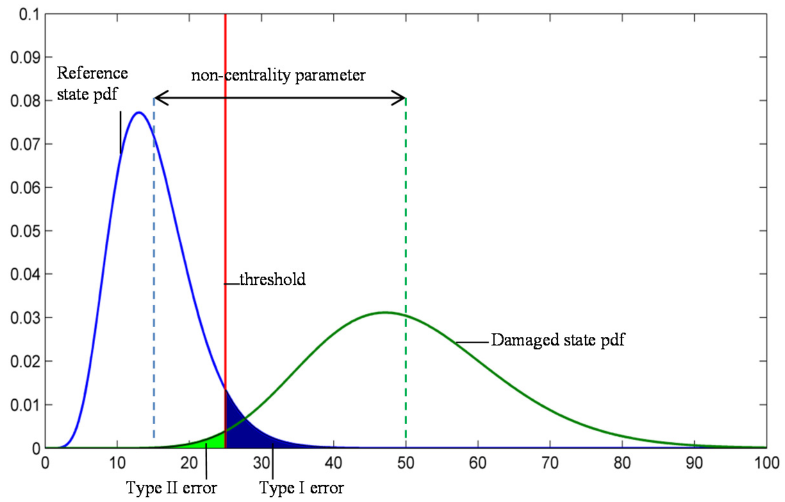

Figure 1: Scheme of probability density functions (pdf) of damage detection test statistic χ^2 in the reference and in a damaged state.

Note that an alternative to the direct computation of the PoI in (8) are Monte Carlo simulations of the DIV s as in [8]. In this way, the empirical distribution of the DIV is obtained, from which the PoI can be evaluated. While this approach is applicable to other DIV's than the one presented in this paper, generalizing the approach, it is computationally much more complex. In particular, it becomes unfeasible for an increasing dimension of the system parameters when the evaluation of PoI's for many damages is necessary.

4 UPDATING THE STRUCTRAL SYSTEM RELIABILITY WITH DAMAGE DETECTION INFORMATION

In order to update the structural system reliability with the damage detection information, the structural damages have to be related to the performance of the damage detection system. The performance of the damage detection is dependent on the static and dynamic properties of the system (Equation (4)) and the damage detection algorithm.

Each structural damage **D** defined in Section 2 affects the structural components. This means that if one or several system components are damaged, then the respective damage **D** corresponds to a particular set of matrices describing the monitored system in (4) with modifications of the matrices **M**, **C** and **K**. Vice versa, the structural reliability of the system is also affected by the damage **D**.

In order to model this relation, the system parameter vector θ in Section 3 is linked to the possible damages **D**, i.e. a damage **D** corresponds directly to a change in θ and is thus related to the damage vector δ in (7). In general this relation may be described as a functional relationship, i.e. $\delta = f(\mathbf{D})$. Such a "compatible" modelling can be assured when the structural components defined in Section 2.1 correspond to the elements of finite element model in (4), and the parameter vector θ is the collection of the selected damage-sensitive parameter for each structural component.

In the remainder of this paper, the damage **D** is modelled as a stiffness loss in the components of a structure. These components are the elements of the finite element model described in (4), and system parameter θ contains the element stiffnesses. Then, it follows from (7) that a damage **D** is linked to the damage vector δ by

$$\delta = \sqrt{N}\mathbf{D} . \tag{9}$$

The probabilities of indication for such damages are required to update the structural system reliability given DDS information as follows. The probability of structural system failure given the DDS information of no-indication, $P\left(F_S\left(t_{SL}\right)|\,\overline{I}_S\right)$, can be determined utilizing Bayesian updating for any point in time during the service life t_{SL} with

$$P\left(F_S\left(t_{SL}\right)|\,\overline{I}_S\right) = \frac{P\left(\overline{I}_S\,|\,F_S\left(t_{SL}\right)\right)P\left(F_S\left(t_{SL}\right)\right)}{P\left(\overline{I}_S\right)} = \frac{P\left(F_S\left(t_{SL}\right)\cap\overline{I}_S\right)}{P\left(\overline{I}_S\right)} . \tag{10}$$

The marginal probability of no-indication $P\left(\overline{I}_S\right)$ can be calculated based on the developed approach for the DDS performance calculation and following [11] with

$$P\left(\overline{I}_S\right) = \int_{\Omega_{I_S}} \left(1 - P\left(I_S\,|\,\mathbf{D}\left(t_{SL}\right)\right)\right) f_{\mathbf{D}\left(t_{SL}\right)}\left(\mathbf{D}\left(t_{SL}\right)\right) d\mathbf{D}\left(t_{SL}\right) \tag{11}$$

$$\text{with } \Omega_{I_S} = \left\{ g_{\bar{I}_S} = P\left(I_S \mid \mathbf{D}\left(t_{SL}\right)\right) - u \right\},$$

where u is a uniformly distributed random variable. It should be noted that the vector of component damages $\mathbf{D}\left(t_{SL}\right)$ includes also zero damages and therefore the probability of indication includes both the undamaged and the damaged system state. The integral in Equ. (11) can then be solved with structural reliability methods such as e.g. Monte Carlo simulation approaches replacing the integration boundaries with infinity and utilizing an indicator function based on limit state function to exclude samples outside the integration space.

The probabilities of indication $P\left(I_S \mid \mathbf{D}\left(t_{SL}\right)\right)$ are calculated for the realizations of the damages **D** during the Monte-Carlo simulation to compute the numerator and denominator in (10). For each realization of **D**, the PoI's are readily computed using Equ. (8), where δ depends on **D** e.g. as described in (9). In contrast, the pre-computation of the PoI's for all possible **D** results in non-polynomial computational demands (see e.g. [8]). The computation "on the fly" during the computation of the updated probability in (10) as introduced here removes the necessity of the PoI computation in advance. In particular, this has the advantage of only computing PoI's for the actually needed realizations of **D**, instead of computing all possible combinations of **D**. The latter becomes computationally almost infeasible when the dimension of **D** grows due to the exponential growth of the possible damage combinations.

5 APPLICATION

To illustrate the developed approach, a simplistic structural system of two components subjected to deterioration with a DDS comprising two sensors and a subspace based DDA (Figure 2) is considered. The structural system is described with its static, dynamic, deterioration, reliability characteristics.

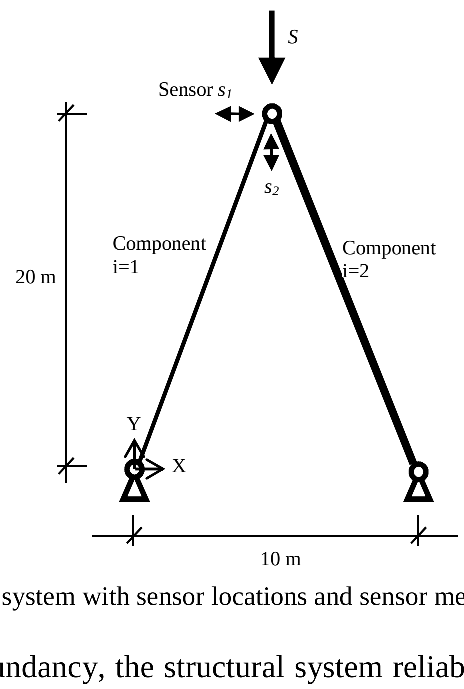

Figure 2: Structural system with sensor locations and sensor measurement directions

Due to the absence of redundancy, the structural system reliability is modelled as a series system, yielding

$$P(F_S) = P\left(\bigcup_{i=1}^{n_c=2}\left(M_{R,i}R_i(t_{SL}) - M_S S_i\right) \le 0\right) \tag{12}$$

The formulation contains the number of components $n_c = 2$ with the random variables component resistance R_i (dependent on the time t_{SL}), system loading S and their associated model uncertainties $M_{R,i}$ and M_S, respectively. The time dependent resistance $R_i(t_{SL})$ of the component *i* is modelled with the initial resistance $R_{i,0}$ and the time dependent damage $D_i(t_{SL})$, i.e.

$$R_i(t_{SL}) = R_{i,0}\left(1 - D_i(t_{SL})\right) \tag{13}$$

For clarity, the temporal dependence of the damage is neglected in the further.

The structural reliability model is summarised in Table 1. The system loading is represented with a Weibull distributed random variable *S* which results by equilibrium in the component loading S_i. The loading and resistance model uncertainties and the resistance model are determined according to [9] as Lognormal distributed with a standard deviation of 10%. The component probability of failure is calibrated to $1 \cdot 10^{-3}$ by adjusting the mean of the component resistance in the undamaged state. The correlation of the resistances and the deterioration are modelled with a coefficient of correlation of 0.5 or varied.

The static and dynamic system properties are modelled with distributed component stiffness and mass subjected to a structural damping of 2% for each mode, see Table 1. The static and dynamic system behaviour is calculated with the Finite Element method.

Table 1 : Static and dynamic and structural reliability model properties

Random variable	Distribution	Mean	Standard deviation
Mass per component	-	0.5	-
Stiffness of component 1: EA_1	-	1000	-
Stiffness of component 2: EA_2	-	2000	-
Damping ratio	-	2 %	-
Loading *S*	WBL	3.50	0.1
Model uncertainty M_S	LN	1.00	0.1
Component resistance in undamaged state $R_{0,i}$	LN	Calibrated	0.1
Model uncertainty $M_{R,i}$	LN	1.00	0.1
Damage D_i	N	0.07	0.03

The DDS is modelled with the acceleration sensors s_1 in X-direction and s_2 in Y-direction recording the responses using the subspace-based DDA. Based on the dynamic structural system model, a reference dataset of length *N* = 10000 at a sampling frequency of 50 Hz is simulated in the undamaged state for both sensors from white noise excitation in order to compute the reference parameters of the damage detection method (null space S, residual sensitivity with respect to both component stiffnesses $\mathcal{J}$ and covariance Σ, threshold λ for type I error of 0.01). For the computation of the probabilities of indication for the damages in (10), a data length of $N = 10000$ is assumed. The distribution of the DIV takes into account the uncertainties related to the measurement data of finite length N, the unknown ambient excitation and the measurement uncertainty. Human errors in the application and operation are accounted by the multiplication of the PoI with a factor of 0.95, see [10].

5.1 DDA performance: probability of indication

From Equ. (8), the probabilities of indication of the DDA (without accounting for human errors) are determined for the damages **D** that are required to evaluate the probabilities in Equ. (10) for updating the structural reliability. In Figure 3, the respective damages are shown (black dots) at which the probability of indication is evaluated during the computation of (10). A correlation of 0.5 between the damages in both components was assumed in this example. Furthermore, Figure 3 shows the theoretical PoI curve for all possible damages. It is observed that the probability of indication is very high for the majority of the damage samples whereas the size of the damage samples is on a low level. This is caused by the high sensitivity of the utilised subspace based DDA.

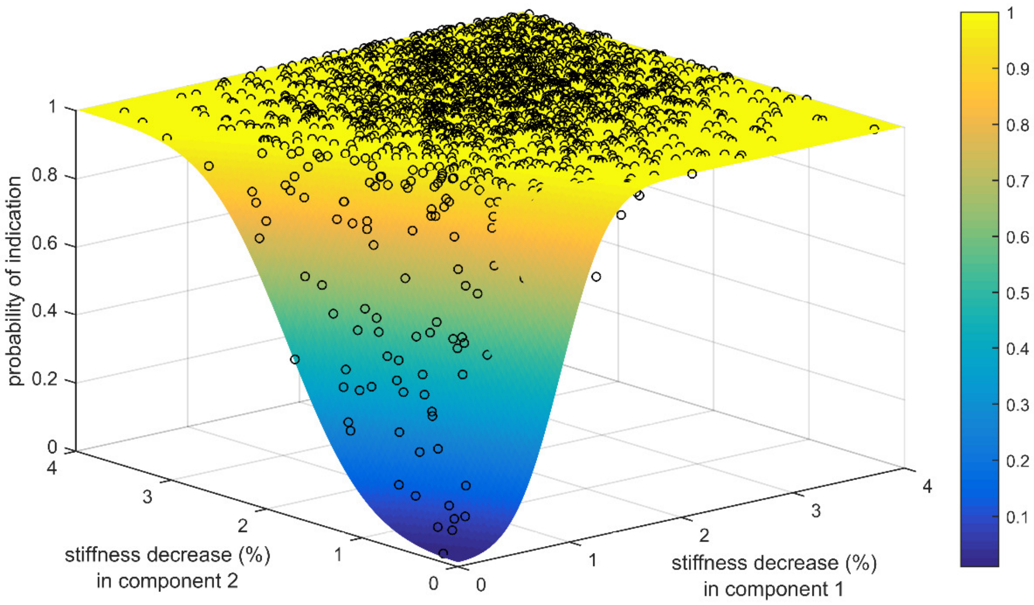

Figure 3: Probability of indication for damages in both structural components.

5.2 Updated structural reliability with DDA information

The DDS information are utilised to update the structural reliability of the deteriorated structural system. The deteriorated series system without updating by DDS information shows a slight decrease of the structural system failure probability for both increasing the initial resistance and the deterioration correlation ρ_{R_0} and ρ_D (Figure 4). The decrease of the system reliability is slightly higher for the resistance correlation considering that the system is not highly deteriorated.

When utilizing the DDS information, the system failure probability drops and decreases with increasing correlation for both sensors s_1 and s_2. With increasing resistance correlation, the decrease rate is comparable to decrease rate without monitoring. The decrease rate for increasing the damage correlation is significantly higher. This can be explained by the higher probability of indication for correlated damages as the dynamic system properties change is more pronounced and can thus be more reliably detected. Sensor s_2 has a better performance as the system the absolute stiffness and thus the stiffness change is in the vertical direction higher and can thus be more reliably detected.

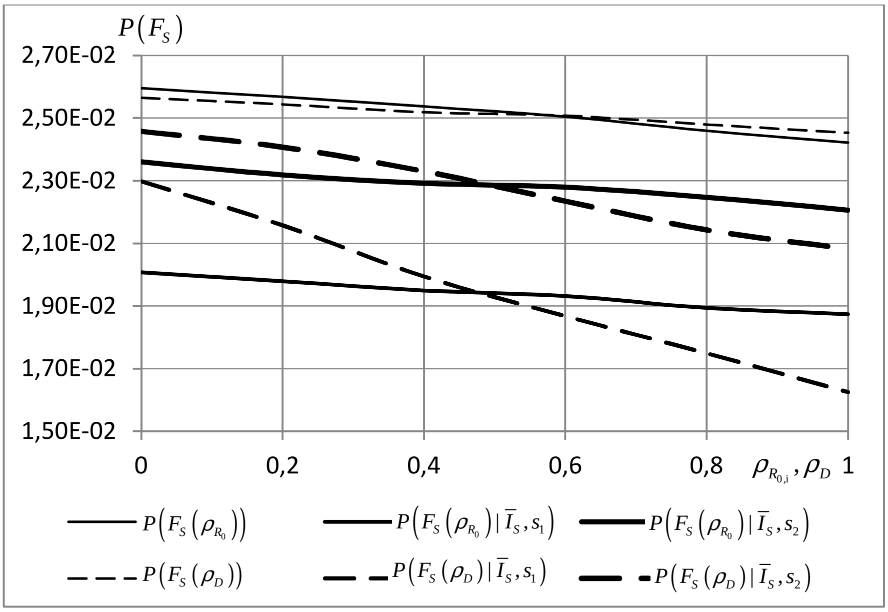

Figure 4: Prior and posterior system probability of failure ($P(F_S)$ and $P(F_S \mid \bar{I})$) in dependency of the resistance correlation, the damage correlation and the probability of component failure for different sensor positions.

6 CONCLUSIONS

An approach for the DDS and DDA performance modelling has been introduced which applies to structural systems and contains a strategy to overcome the high computational efforts for the pre-determination of the DDS reliability.

The introduced approach facilitates (1) that the probability of damage indication can be calculated directly without necessitating a pre-determination and (2) the updating of structural reliability with DDS information on structural system level. The developed approach requires a consistent structural performance modelling in terms of the static, dynamic and deterioration characteristics in order to derive the structural reliability and the DDS performance models.

With an example it is demonstrated that the DDS and DDA performance is evaluated on a system level for both the DDS and the structural system. As such the DDS performance modelling accounts for the dependencies of the structural system damage states and encompasses the measurement system (number of sensors, sensor positions, precision of the system) and the employed damage detection algorithms.

In the perspective of further research, the approach taken here can be used to design efficient DDS and DDA, quantifying the value of DDS and DDA and optimising their performance.

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