

On Integral Sum Numbers of Cycles

Ergen Liu, Qing Zhou, Wei Yu

► **To cite this version:**

Ergen Liu, Qing Zhou, Wei Yu. On Integral Sum Numbers of Cycles. Daoliang Li; Yingyi Chen. 5th Computer and Computing Technologies in Agriculture (CCTA), Oct 2011, Beijing, China. Springer, IFIP Advances in Information and Communication Technology, AICT-370, pp.136-139, 2012, Computer and Computing Technologies in Agriculture V. <10.1007/978-3-642-27275-2_15>. <hal-01361128>

HAL Id: hal-01361128

<https://hal.inria.fr/hal-01361128>

Submitted on 6 Sep 2016

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



On Integral Sum Numbers of Cycles

Ergen Liu, Qing Zhou and Wei Yu

School of Basic Sciences, East China Jiaotong University

Nanchang, jiangxi China, 330013

liueg65@126.com or leg_eg@sina.com

Abstract. In this paper we determine that integral sum number of graph C_n , namely for any integer $n \geq 5$, then $\xi(C_n) = 0$, therefore we prove that the graph $C_n (n \geq 5)$ is an integral sum graph.

Key Words: Integral sum number, Integral sum graph, Graph C_n

1 Introduction

The graph in this paper discussed are undirected, no multiple edges and simple graph, the unorganized state of definitions and terminology and the symbols in this paper referred to reference [1],[2].

F.Harary^[3] introduce the concept of integral sum graphs. The integral sum graph $G^+(S)$ of a finite subset $S \subset Z$ is the graph (V, E) , where $V = S$ and $uv \in E$ if and only if $u + v \in S$. A graph G is an integral sum graph if it is isomorphic to the integral sum graph number of $G^+(S)$ of some $S \subset Z$. The integral sum number of a given graph G , denoted by $\xi(G)$, is defined as the smallest nonnegative integer S such that $G \cup sk_1$ is an integral sum graph. For

convincing, an integral sum graph is written as an integral sum graph in references [3, 4]. Obviously, graph G is an integral sum graph iff $\xi(G) = 0$.

It is very difficult to determine $\xi(G)$ for a given graph G in general. All paths and matchings are verified to be integral sum graph in references [3], and we see from references [4] that $\xi(C_n) \leq 1$ for all $n \neq 4$. And further, an open conjecture was posed in references [4] as follows:

Conjecture^[4]: Is it true that any odd cycle is an integral sum graph?

Definition 1.1. If a graph is isomorphism graph $G^+(S)$, then we call graph G is Integral sum graph, denoted by $G \cong G^+(S)$.

Definition 1.2. For graph G , if it exists nonnegative integer S such that $G \cup sk_1$ is an integral sum graph, then we call number s is integral sum number of G , denoted by $\xi(G) = s$.

2 Main Results and certification

Theorem 2.1. For any integer $n \geq 3$, then

$$\xi(C_n) = \begin{cases} 3, & \text{when } n = 4; \\ 0, & \text{when } n \neq 4. \end{cases}$$

Proof. It is immediate from references [3, 4] that $\xi(C_3) = 0$ and $\xi(C_4) = 3$. And it is clear that $C_5 \cong G^+\{2, 1, -2, 3, -1\}$ and $C_7 \cong G^+\{1, 2, -5, 7, -3, 4, 3\}$.

Next we consider two cases: For all $C_{2j}(j \geq 3)$ and $C_{2j+1}(j \geq 4)$, we will show that the two classes of cycle are integral sum graph.

Let the vertices of $C_{2j}(j \geq 3)$ and $C_{2j+1}(j \geq 4)$ be marked as the methods in Fig.1 and Fig.2.

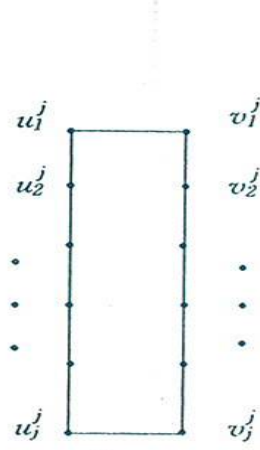


Fig.1. marking of C_{2j}

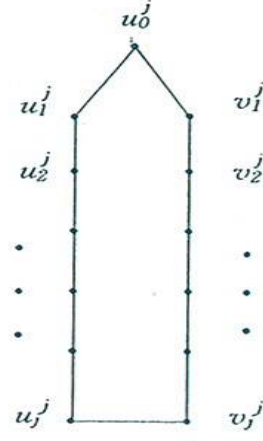


Fig.2. marking of C_{2j+1}

Case 1 When $n = 2j$ ($j \geq 3$)

We first give the labels of C_6 and C_8 as follows:

$$\text{Let } u_1^3 = 4, u_2^3 = -1, u_3^3 = 5; \quad v_1^3 = 1, v_2^3 = 3, v_3^3 = -2.$$

and

$$u_1^4 = 7, u_2^4 = -2, u_3^4 = 9, u_4^4 = -11; \quad v_1^4 = 2, v_2^4 = 5, v_3^4 = -3, v_4^4 = 8.$$

Then $C_6 \cong G^+ \{4, -1, 5, -2, 3, 1\}$ and

$C_8 \cong G^+ \{7, -2, 9, -11, 8, -3, 5, 2\}$, therefore $\xi(C_6) = 0$ and $\xi(C_8) = 0$.

When $j \geq 5$, we give the labels of C_{2j} as follows:

$$\text{Let } u_1^j = u_1^{j-1} + u_1^{j-2} \quad (j \geq 5); \quad u_2^j = u_2^{j-1} + u_2^{j-2} \quad (j \geq 5);$$

$$v_1^j = v_1^{j-1} + v_1^{j-2} \quad (j \geq 5); \quad v_2^j = v_2^{j-1} + v_2^{j-2} \quad (j \geq 5).$$

and

$$u_k^j = u_{k-2}^j - u_{k-1}^j \quad (k = 3, 4, \dots, j);$$

$$v_k^j = v_{k-2}^j - v_{k-1}^j \quad (k = 3, 4, \dots, j).$$

By labeling of above, we know $\xi(C_{2j}) = 0 (j \geq 5)$.

The labeling of above is illustrated in Fig.3.

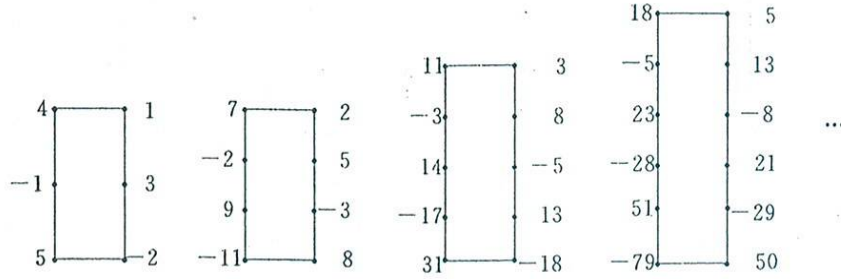


Fig.3. labeling of $C_{2j} (j \geq 3)$

Case 2 When $n = 2j + 1 (j \geq 4)$

We first give the labels of C_9 and C_{11} as follows:

$$\text{Let } u_0^4 = 2, u_1^4 = 5, u_k^4 = u_{k-2}^4 - u_{k-1}^4 \quad (k = 2, 3, 4);$$

$$v_1^4 = -5, v_2^4 = 7, v_k^4 = v_{k-2}^4 - v_{k-1}^4 \quad (k = 3, 4).$$

and

$$u_0^5 = 3, u_1^5 = 8, u_k^5 = u_{k-2}^5 - u_{k-1}^5 \quad (k = 2, 3, 4, 5);$$

$$v_1^5 = -8, v_2^5 = 11, v_k^5 = v_{k-2}^5 - v_{k-1}^5 \quad (k = 3, 4, 5).$$

Then $C_9 \cong G^+ \{2, 5, -3, 8, -11, 19, -12, 7, -5\}$ and

$C_{11} \cong G^+ \{3, 8, -5, 13, -18, 31, -49, 30, -19, 11, -8\}$, therefore $\xi(C_9) = 0$

and $\xi(C_{11}) = 0$.

When $j \geq 6$, we give the labels of C_{2j+1} as follows:

$$\text{Let } u_0^j = u_0^{j-1} + u_0^{j-2} \quad (j \geq 6); \quad u_1^j = u_1^{j-1} + u_1^{j-2} \quad (j \geq 6);$$

$$v_1^j = v_1^{j-1} + v_1^{j-2} \quad (j \geq 6);$$

and

$$u_k^j = u_{k-2}^j - u_{k-1}^j \quad (k = 2, 3, \dots, j);$$

$$v_k^j = v_{k-2}^j - v_{k-1}^j \quad (k = 2, 3, \dots, j).$$

Where $v_0^j = u_0^j$ for $j \geq 6$.

By labeling of above, we know $\xi(C_{2j+1}) = 0$ ($j \geq 6$).

The labeling of above is illustrated in Fig.4.

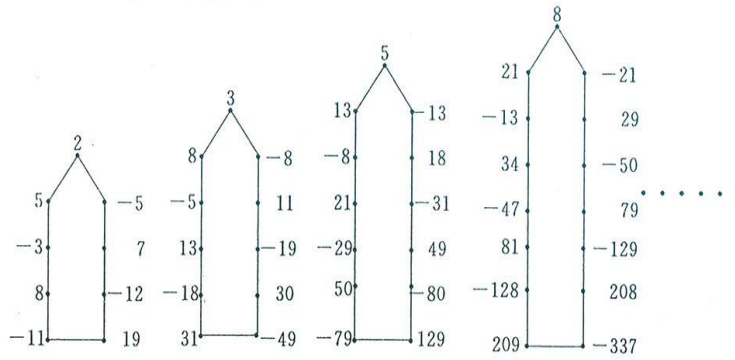


Fig.4. labeling of C_{2j+1} ($j \geq 6$)

Theorem 2.2. For any integer $n \geq 5$, the graph C_n is integral sum graph.

Proof. From theorem 2.1, we know $\xi(C_n) = 0 (n \geq 5)$, therefore for any integer $n \geq 5$, the graph C_n is integral sum graph.

References

1. F. Harary. Graph Theory[M]. Addison Wesley. Reading MA, 1969.
2. Bondy J A, Murty U S R. Graph Theory with Applications[M]. Elsevier North-Holland, 1976.
3. F. Harary. Sum graphs over all the integers[J]. Discrete Math. 1994(124):99~105.
4. Baogen Xu. On integral sum graphs[J]. Discrete Math. 1999(194):285~294.
5. Achaya B D and Hegde S M. Arithmetic graphs[J]. Journal of Graph Theory, 1990, 18(3):275-299.
6. Hui Yu. The Windmill W_n^* is the integral sum Graph and Mod integral sum Graph[J]. Journal of Heze University. 2006, 28(3):18-19.
7. Gao Xiu-lian. Several kinds Integral Sum Tree, Journal of Dezhou University. 2011, 17(2):18-22.