Stochastic parameterization of geophysical flows through modelling under location uncertainty

Valentin Resseguier, Etienne Mémin, Bertrand Chapron, Pierre Dérian

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Motivations

- Rigorously identified sudgrid dynamics effects
- Injecting likely small-scale dynamics
- Studying bifurcations and attractors

  → Climate projections

- Quantification of modeling errors

  → Ensemble forecasts and data assimilation

Contents

- Randomized dynamics
- SQG under Moderate Uncertainty
- Lorenz under location uncertainty

Randomized dynamics

Random equations

- Random initial conditions → Underdispersive + need large ensemble
- Arbitrary Gaussian forcing → Adding energy + wrong phase
- Averaging, homogenization → Assumptions and energy issues
- Adding white random velocity → $\boldsymbol{v} = \boldsymbol{w} + \boldsymbol{\sigma}\dot{\boldsymbol{B}}$

Advection of tracer $\boldsymbol{\Theta}$

$$\frac{D\Theta}{Dt} = 0$$

Advection of tracer Θ

Advection of tracer $\boldsymbol{\Theta}$

$$\partial_t \Theta + \boldsymbol{w}^\star \cdot \boldsymbol{\nabla} \Theta + \boldsymbol{\sigma} \dot{\boldsymbol{B}} \cdot \boldsymbol{\nabla} \Theta = \boldsymbol{\nabla} \cdot \left(\frac{1}{2} \boldsymbol{a} \boldsymbol{\nabla} \Theta \right)$$

Advection of tracer $\boldsymbol{\Theta}$

$$\partial_t \Theta + \underbrace{\boldsymbol{w}^\star \cdot \boldsymbol{\nabla}\Theta + \boldsymbol{\sigma}\dot{\boldsymbol{B}} \cdot \boldsymbol{\nabla}\Theta}_{\text{Advection}} = \boldsymbol{\nabla} \cdot \left(\frac{1}{2} \boldsymbol{a} \boldsymbol{\nabla} \Theta \right)$$

Advection of tracer $\boldsymbol{\Theta}$

$$\partial_t \Theta + \underbrace{\boldsymbol{w}^\star \cdot \boldsymbol{\nabla}\Theta + \boldsymbol{\sigma}\dot{\boldsymbol{B}} \cdot \boldsymbol{\nabla}\Theta}_{\text{Advection}} = \underbrace{\boldsymbol{\nabla} \cdot \left(\frac{1}{2}\boldsymbol{a}\boldsymbol{\nabla}\Theta\right)}_{\text{Diffusion}}$$

Advection of tracer $\boldsymbol{\Theta}$

Advection

Diffusion

$$\partial_t \Theta + \boldsymbol{w}^{\star} \cdot \boldsymbol{\nabla}\Theta + \boldsymbol{\sigma}\dot{\boldsymbol{B}} \cdot \boldsymbol{\nabla}\Theta = \boldsymbol{\nabla} \cdot \left(\frac{1}{2}\boldsymbol{a}\boldsymbol{\nabla}\Theta\right)$$

Drift correction

Advection of tracer $\boldsymbol{\Theta}$

$$\partial_t \Theta + \overbrace{\boldsymbol{w}^\star \cdot \boldsymbol{\nabla}\Theta + \boldsymbol{\sigma}\dot{\boldsymbol{B}} \cdot \boldsymbol{\nabla}\Theta}^{\text{Advection}} = \overbrace{\boldsymbol{\nabla}\cdot\left(\frac{1}{2}\boldsymbol{a}\boldsymbol{\nabla}\Theta\right)}^{\text{Diffusion}}$$

Drift correction

Multiplicative random forcing

Advection of tracer $\boldsymbol{\Theta}$

$$\partial_t \Theta + \overbrace{\boldsymbol{w}^{\star} \cdot \boldsymbol{\nabla}\Theta + \boldsymbol{\sigma}\dot{\boldsymbol{B}} \cdot \boldsymbol{\nabla}\Theta}^{\text{Advection}} = \overbrace{\boldsymbol{\nabla} \cdot \left(\frac{1}{2}\boldsymbol{a}\boldsymbol{\nabla}\Theta\right)}^{\text{Diffusion}}$$

Drift correction

Multiplicative random forcing

Advection of tracer $\boldsymbol{\Theta}$

$$\partial_t \Theta + \underbrace{\boldsymbol{w}^{\star} \cdot \boldsymbol{\nabla} \Theta + \boldsymbol{\sigma} \dot{\boldsymbol{B}} \cdot \boldsymbol{\nabla} \Theta}_{\text{Advection}} = \underbrace{\boldsymbol{\nabla} \cdot \left(\frac{1}{2} \boldsymbol{a} \boldsymbol{\nabla} \Theta \right)}_{\text{Diffusion}}$$

Drift correction

Multiplicative random forcing

Advection of tracer $\boldsymbol{\Theta}$

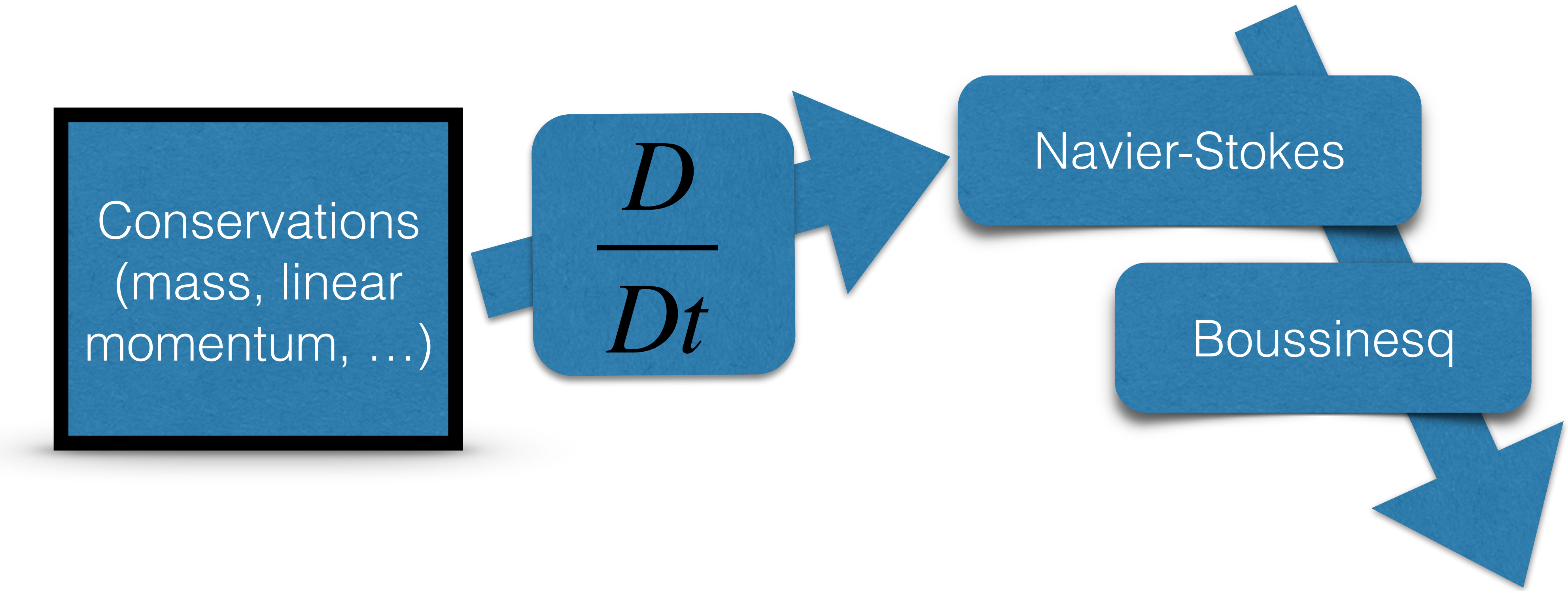
$$\partial_t \Theta + \overbrace{\boldsymbol{w}^{\star} \cdot \boldsymbol{\nabla}\Theta + \boldsymbol{\sigma}\dot{B} \cdot \boldsymbol{\nabla}\Theta}^{\text{Advection}} = \overbrace{\boldsymbol{\nabla} \cdot \left(\frac{1}{2}\boldsymbol{a}\boldsymbol{\nabla}\Theta\right)}^{\text{Diffusion}}$$

Drift correction

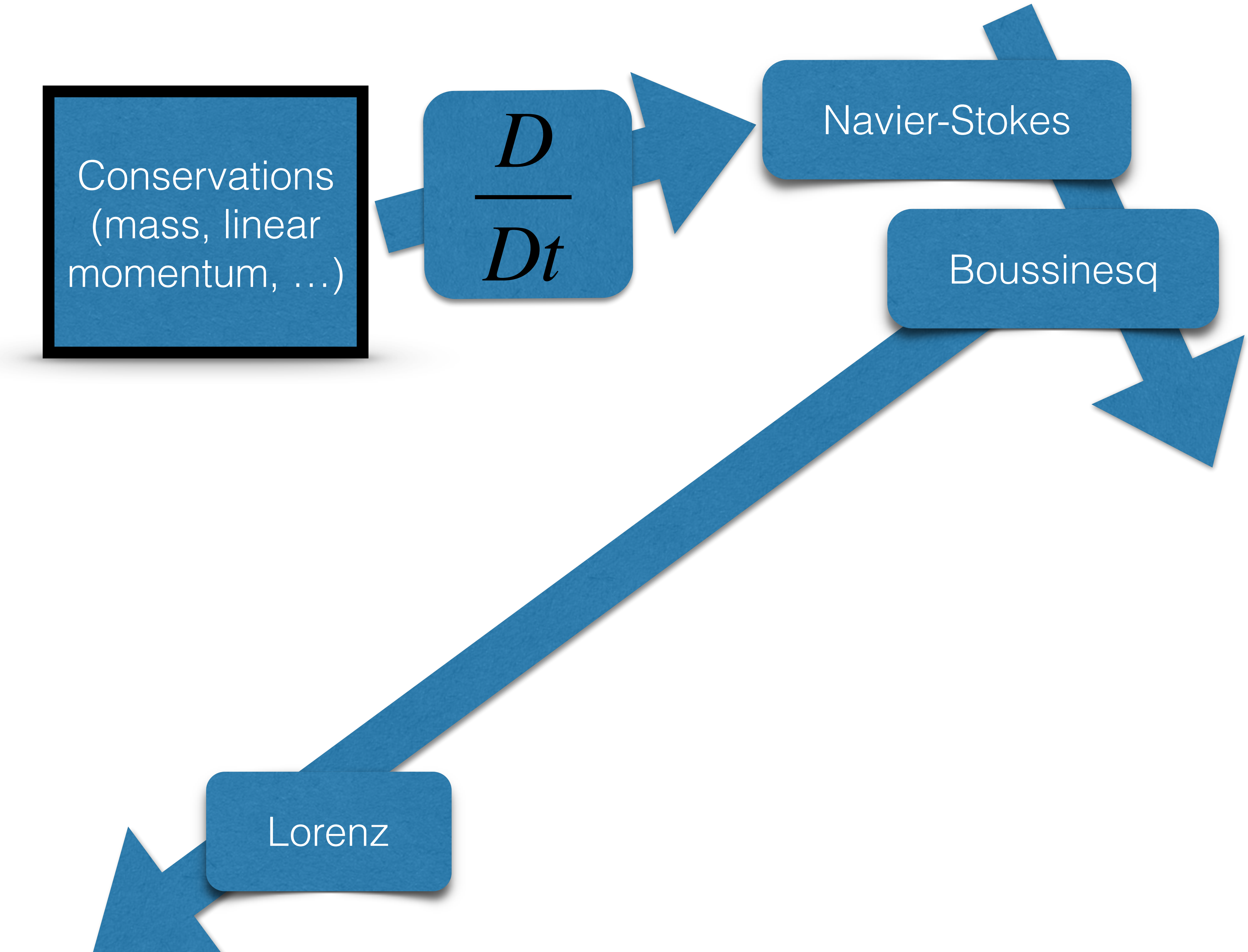
Multiplicative random forcing

Balanced energy exchanges

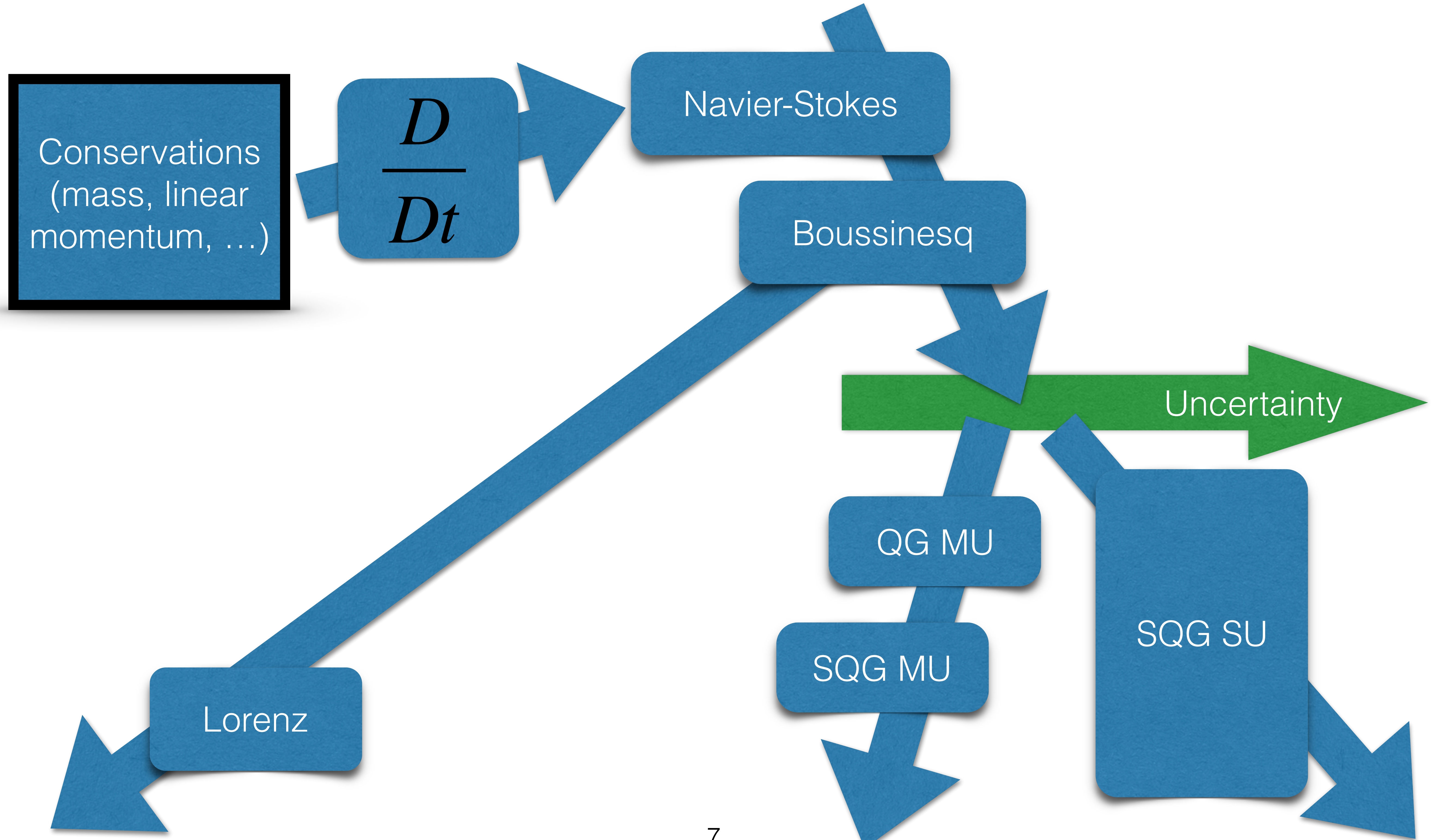
Derived random models



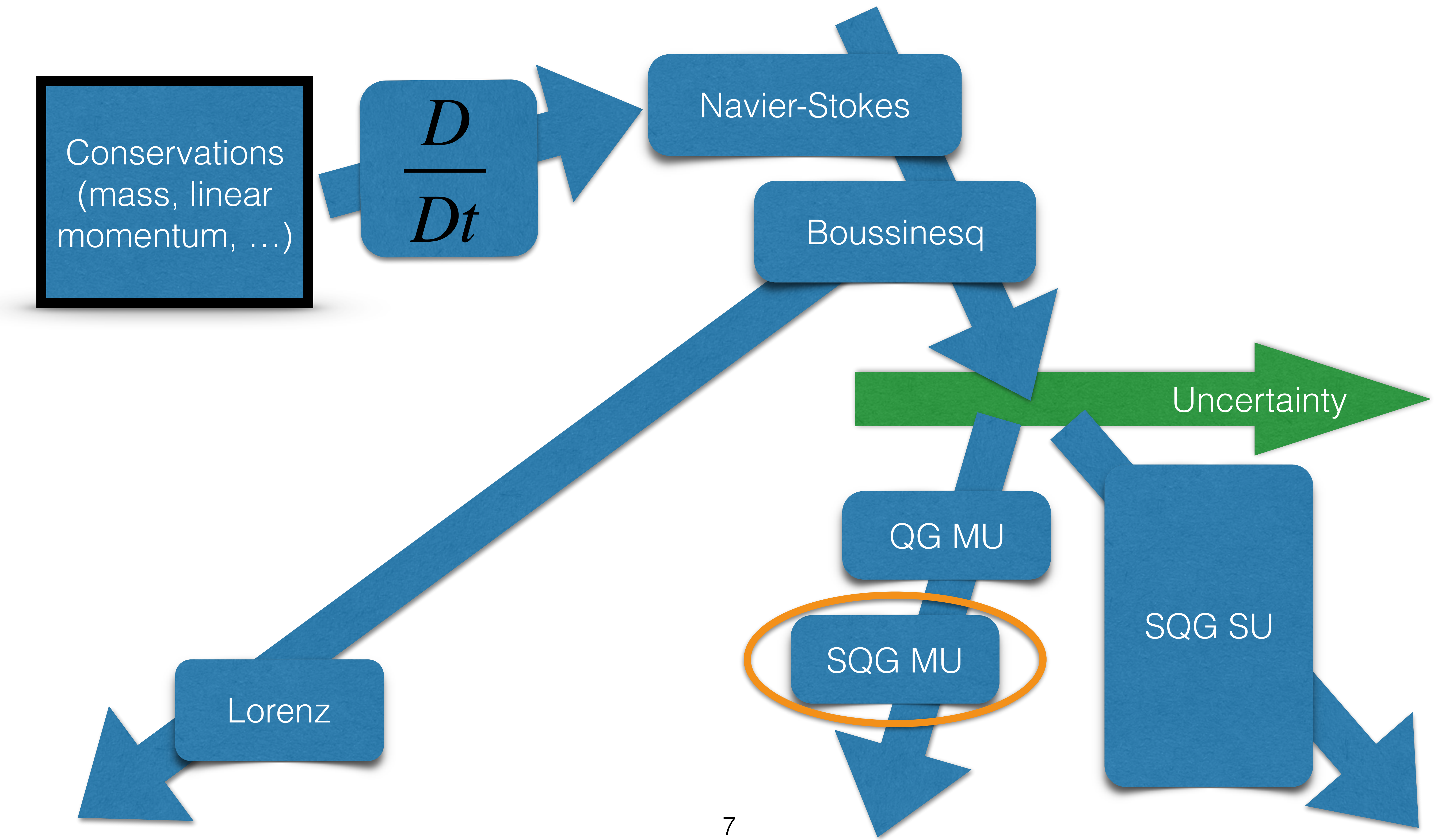
Derived random models



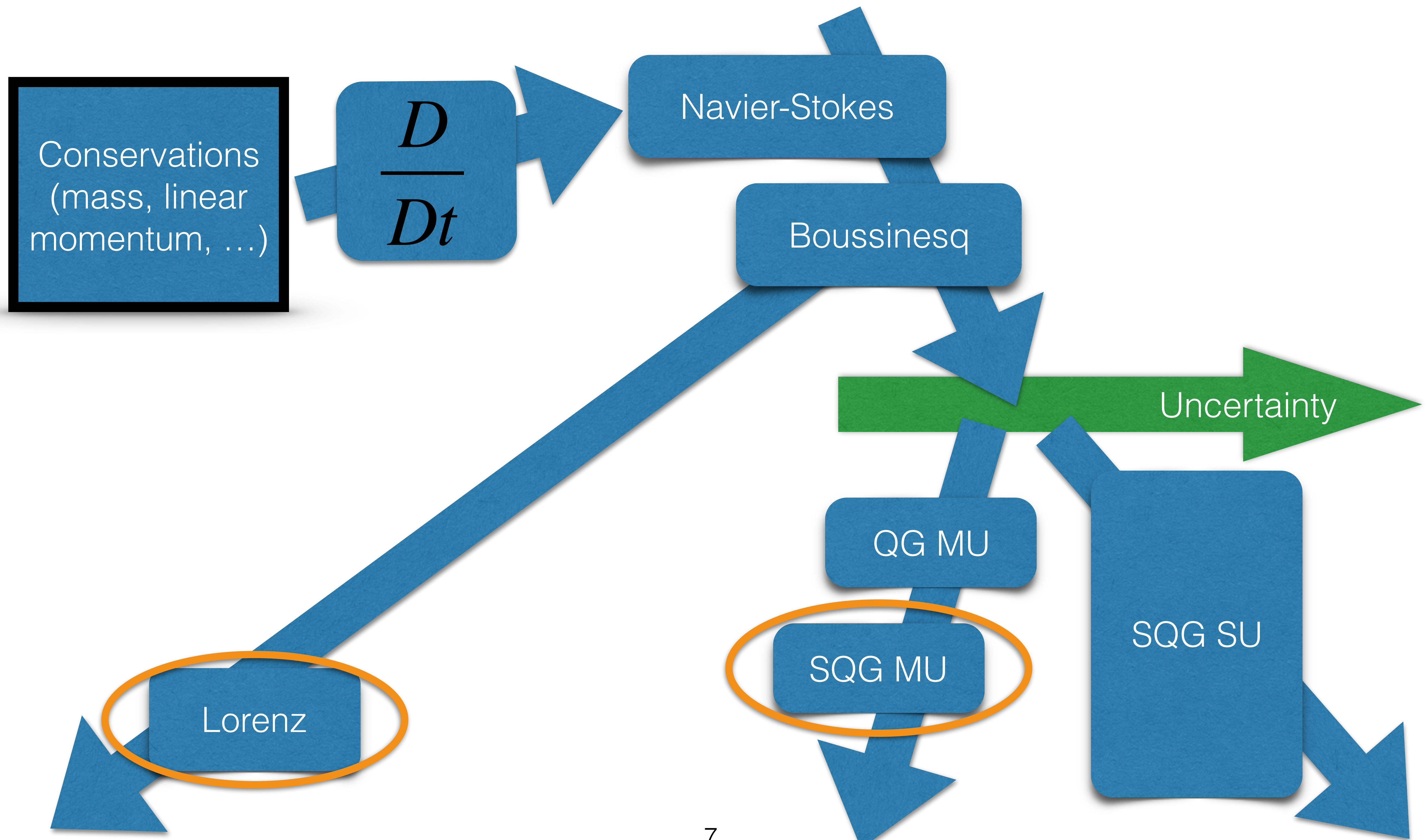
Derived random models



Derived random models



Derived random models



SQG under Moderate Uncertainty

SQG MU

Code available online

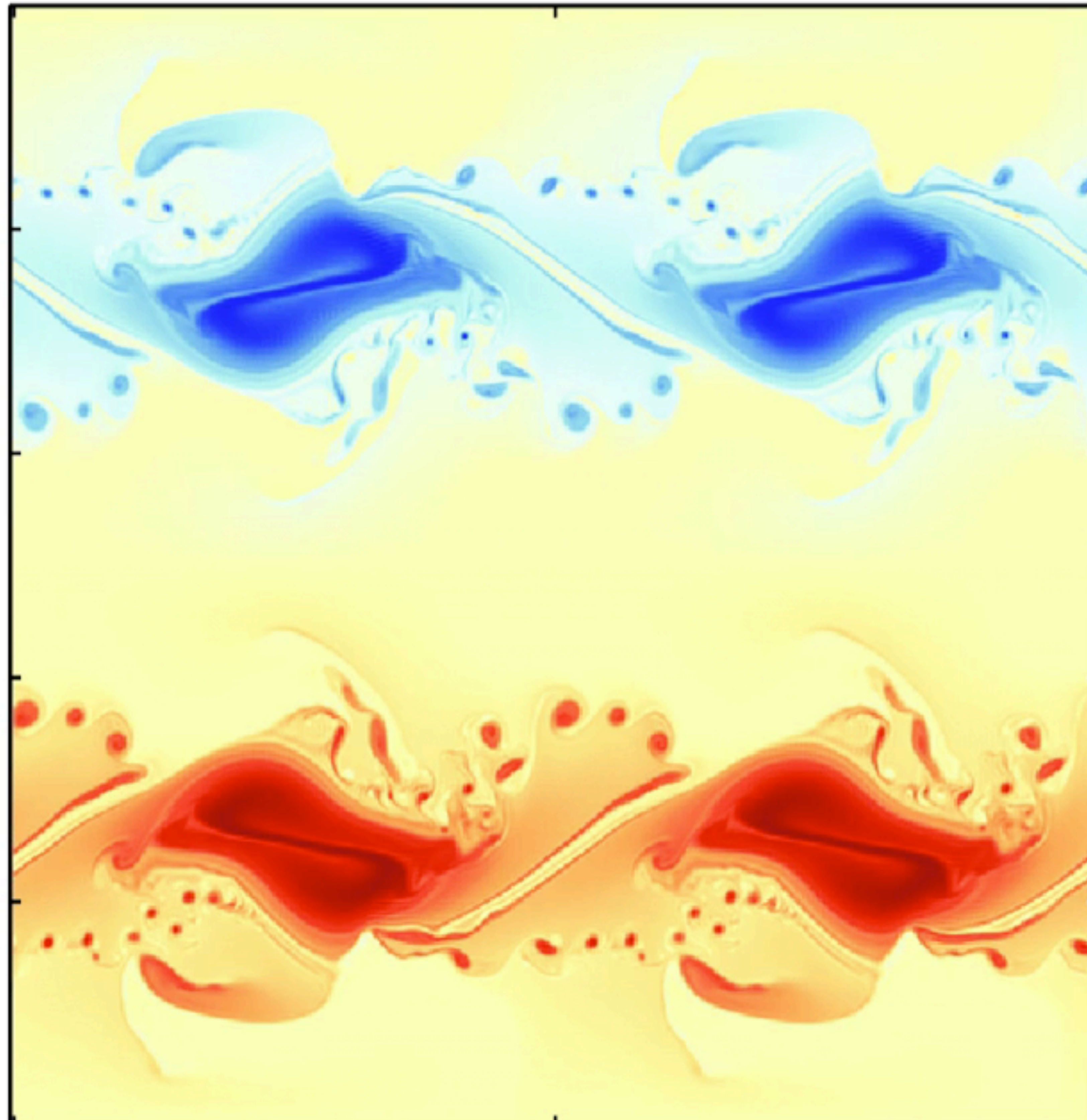
$t = 17$ days

Reference flow:

deterministic

SQG

512 x 512



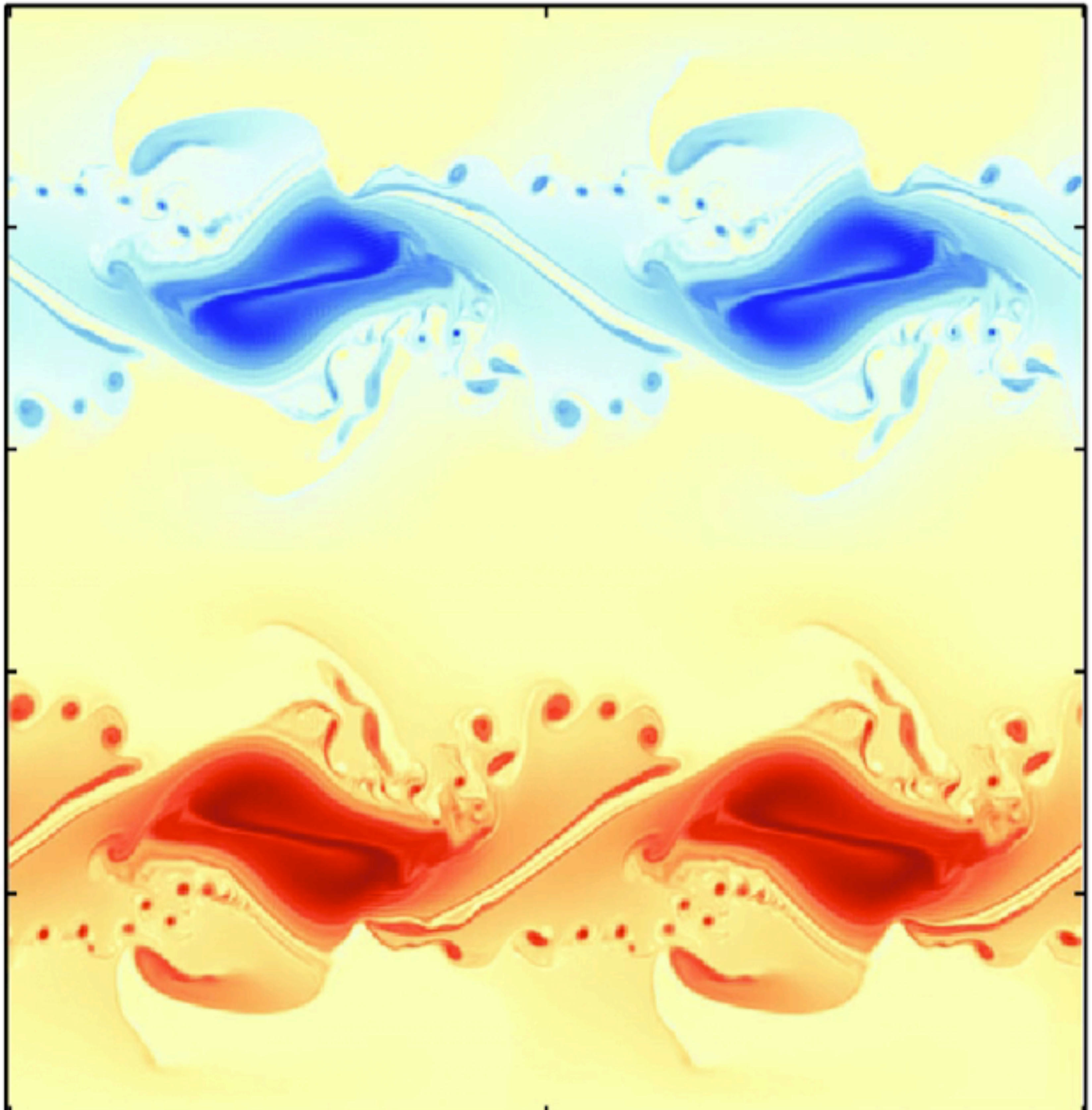
Reference flow:

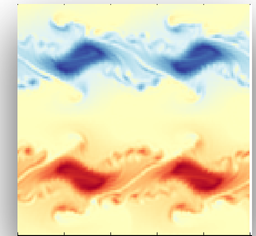
deterministic

SQG

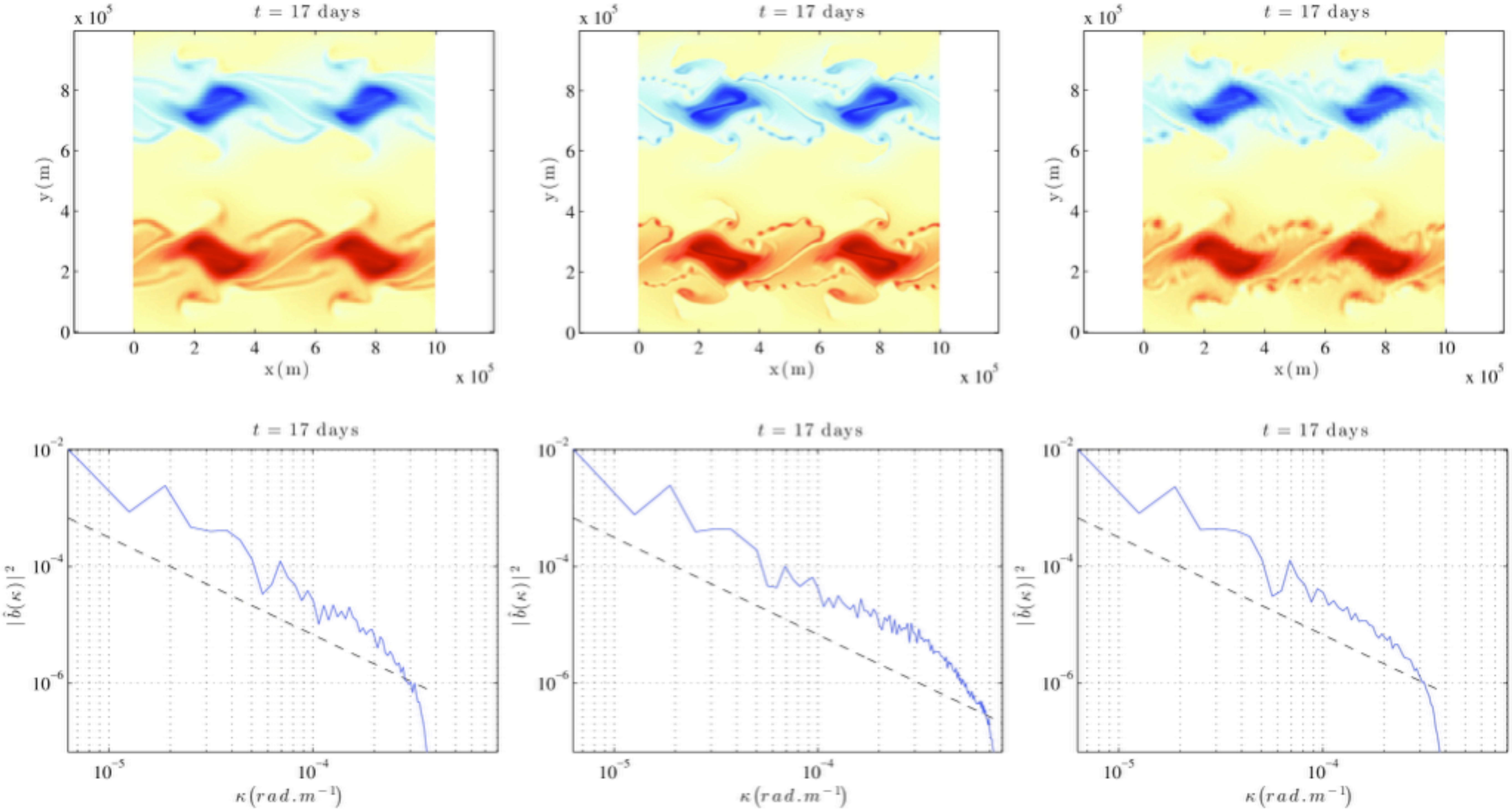
512 x 512

$t = 17$ days

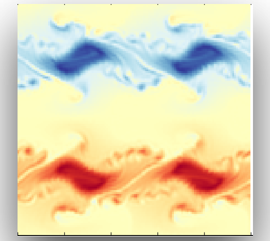




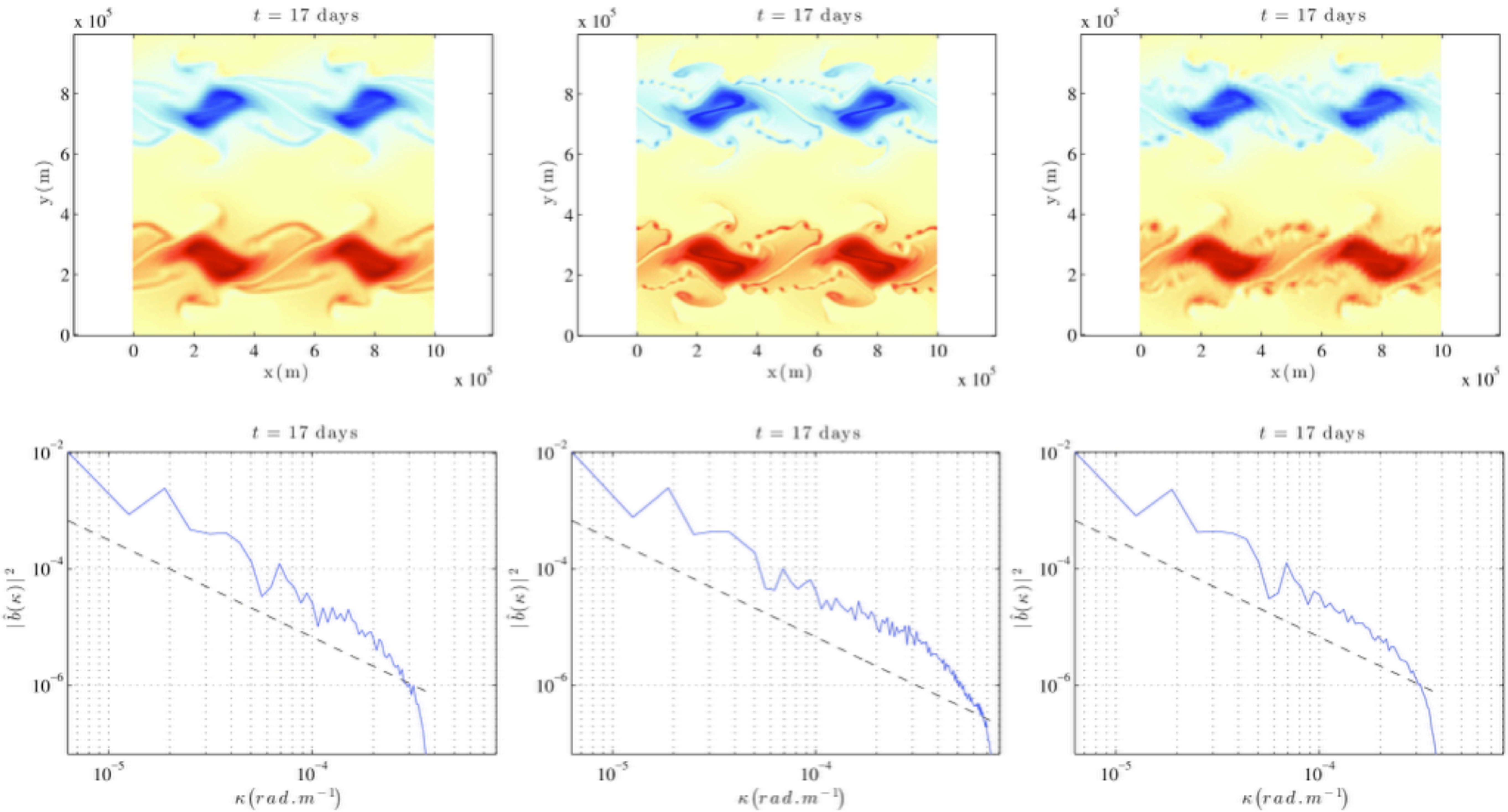
One realization



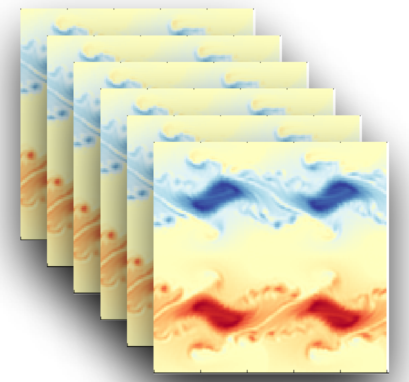
Deterministic 128x128 **Deterministic 512x512** **Stochastic 128x128**



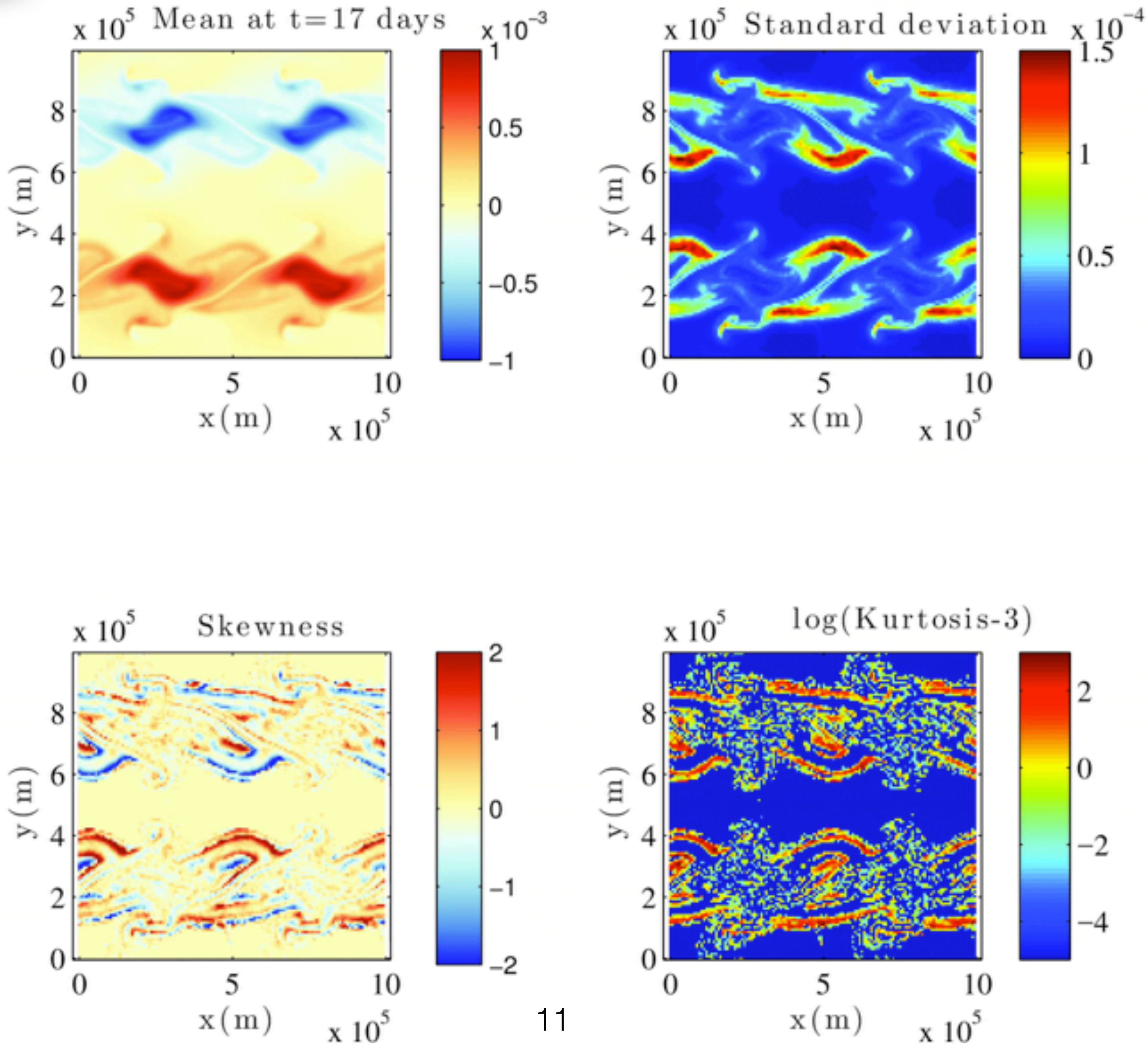
One realization



Deterministic 128x128 **Deterministic 512x512** **Stochastic 128x128**

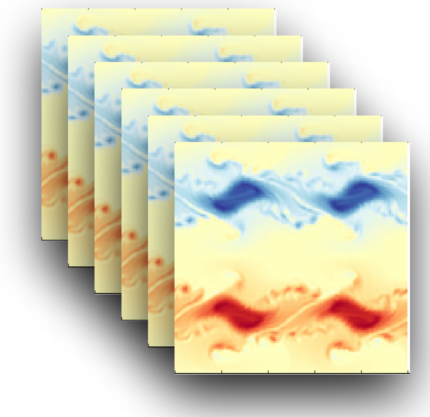


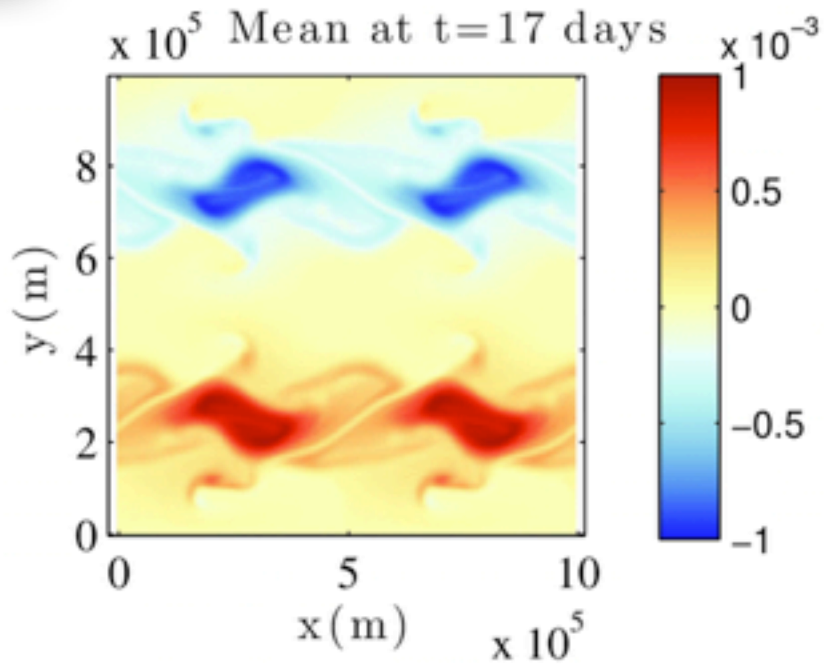
Ensemble



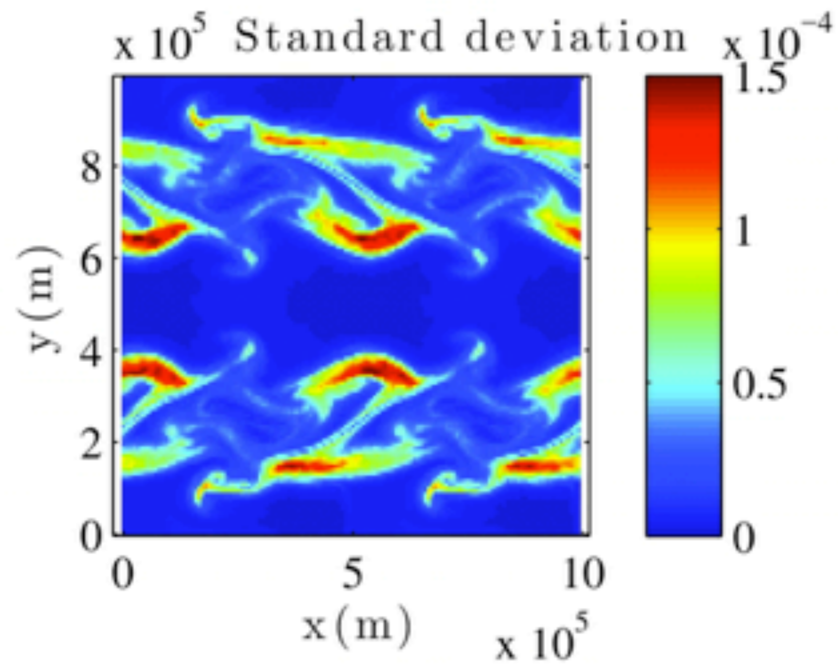
Mean at t=17 days
x 10^5
x 10^-3
y (m)
x (m)
Standard deviation
x 10^-4
Skewness
log(Kurtosis-3)

Ensemble

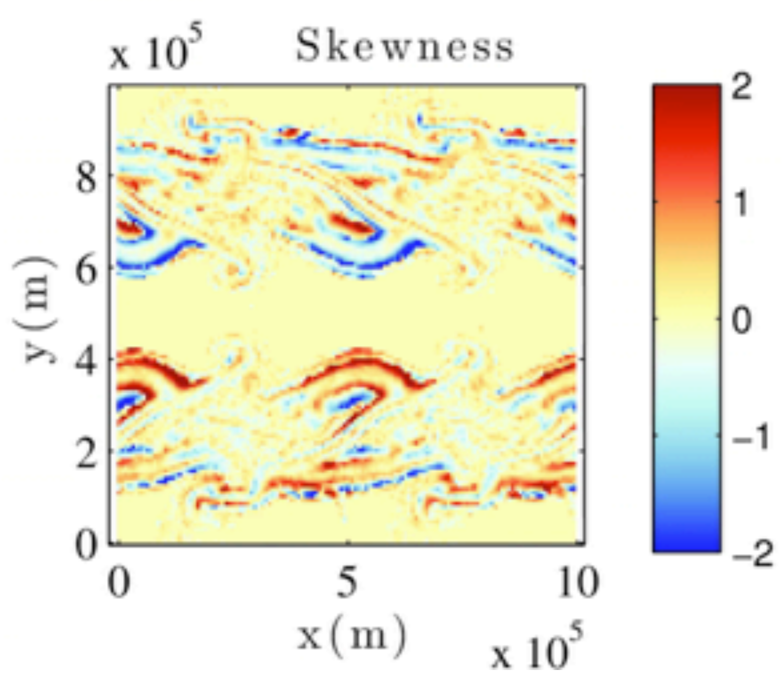




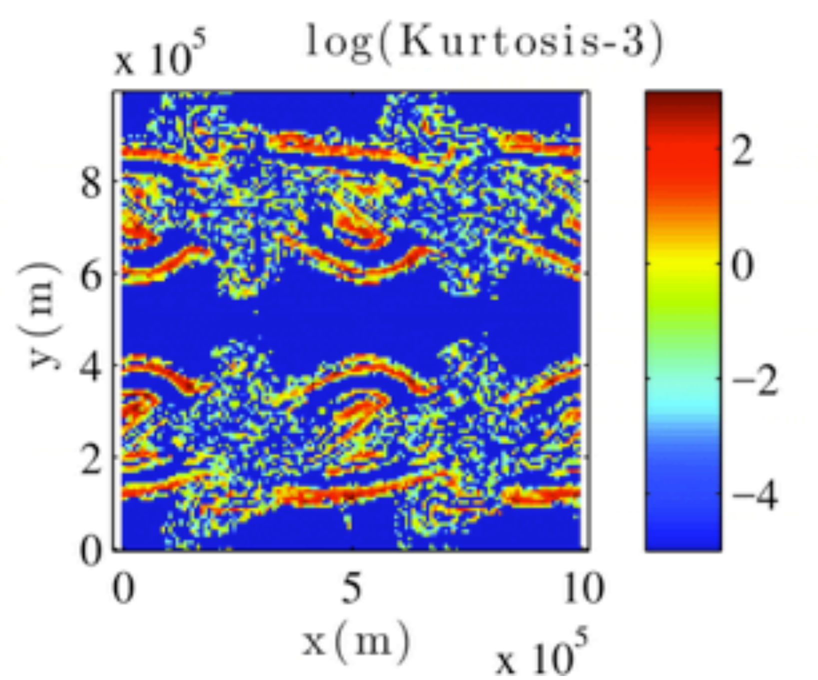
Mean at t=17 days
x 10^5
x 10^-3
y (m)
x (m)
x 10^5



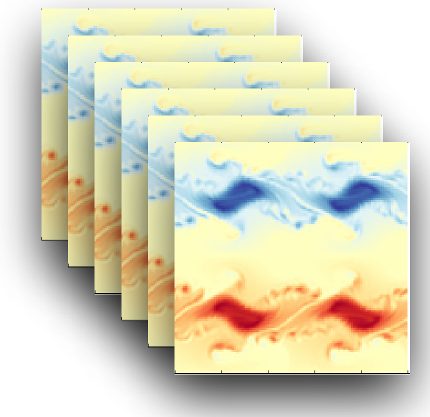
Standard deviation
x 10^5
x 10^-4
y (m)
x (m)
x 10^5



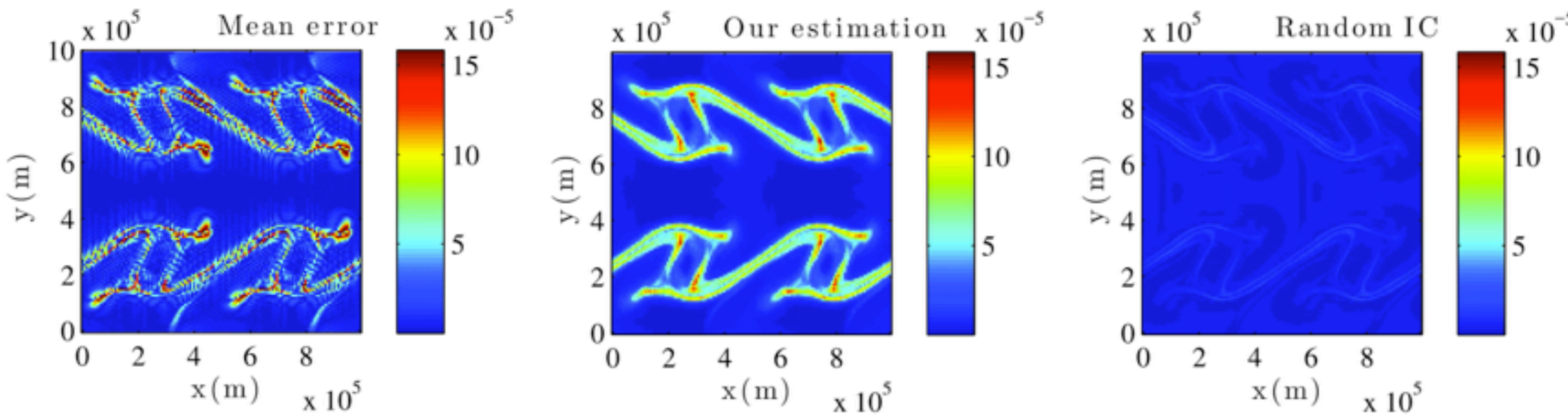
Skewness
x 10^5
y (m)
x (m)
x 10^5



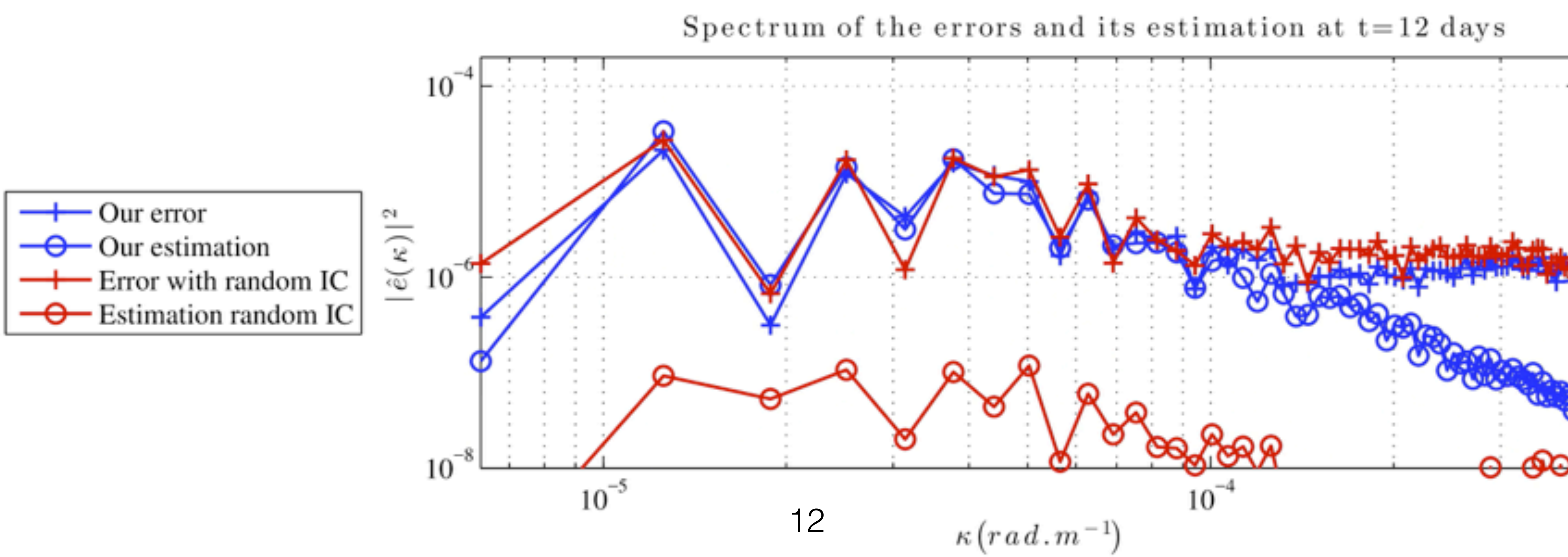
log(Kurtosis-3)
x 10^5
y (m)
x (m)
x 10^5



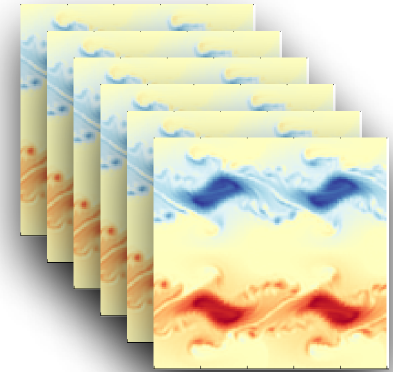
Ensemble



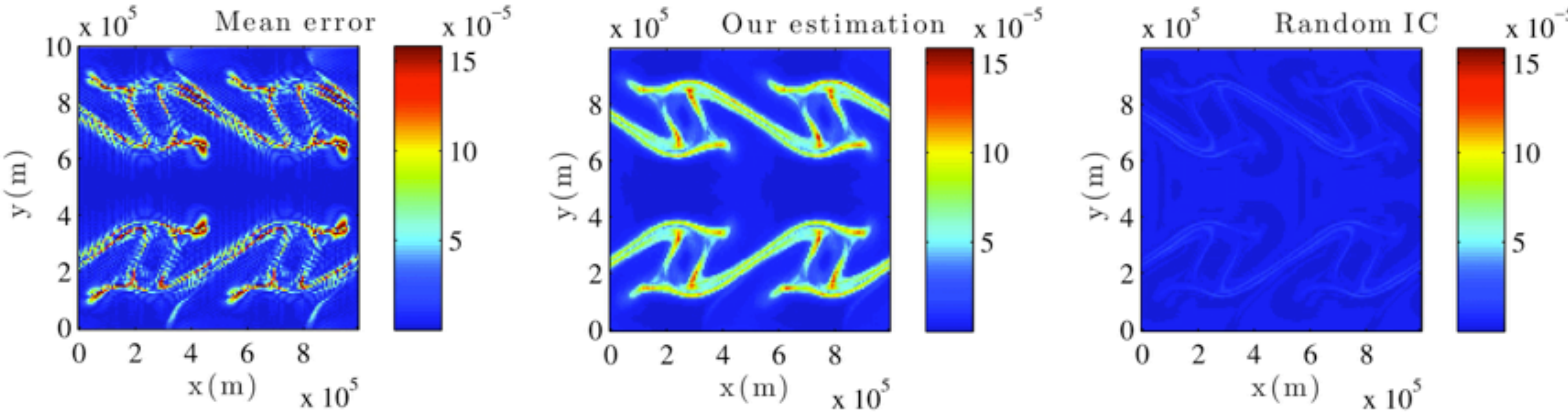
Mean error
Our estimation
Random IC
x 10^5
x 10^{-5}
y (m)
x (m)

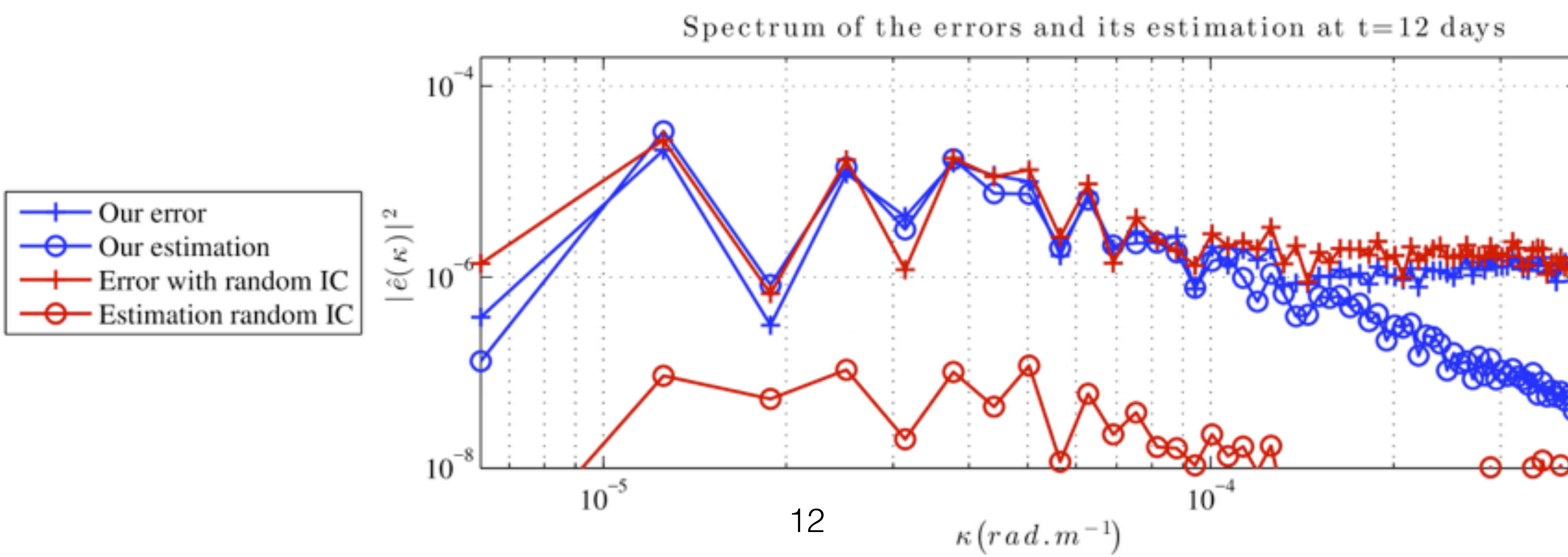


Spectrum of the errors and its estimation at t=12 days
Our error
Our estimation
Error with random IC
Estimation random IC
$|\hat{e}(\kappa)|^2$
$\kappa(rad.m^{-1})$

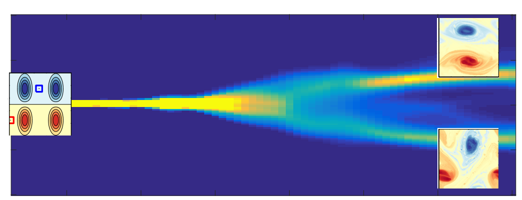


Ensemble


Mean error
Our estimation
Random IC
x(m)
y(m)
x 10^5
x 10^-5


Spectrum of the errors and its estimation at t=12 days
Our error
Our estimation
Error with random IC
Estimation random IC
$|\hat{e}(\kappa)|^2$
$\kappa\,(rad.m^{-1})$

Summary of QG models

- Better small scales
- Estimate position and amplitude of errors
- Extreme events
- Bifurcations 
- under Strong Uncertainty:
  Simple 2D description of frontolysis/frontogenesis

Code SQG MU:
link from Fluminance website - V. Resseguier

Lorenz model under location uncertainty

Do large-scale (diffusive) models lead to over-representing "stable"-states in ensemble simulations?

Lorenz model(s)

$$\frac{\mathrm{d}X}{\mathrm{d}t} = \Pr\left(Y - X\right) - \frac{4}{2\Upsilon}X$$

$$\mathrm{d}Y = \left[X(\rho - Z) - Y - \frac{4}{2\Upsilon}Y\right]\mathrm{d}t + \frac{\rho - Z}{\Upsilon^{1/2}}\mathrm{d}B_t$$

$$\mathrm{d}Z = \left[XY - bZ - \frac{8}{2\Upsilon}bZ\right]\mathrm{d}t + \frac{Y}{\Upsilon^{1/2}}\mathrm{d}B_t$$

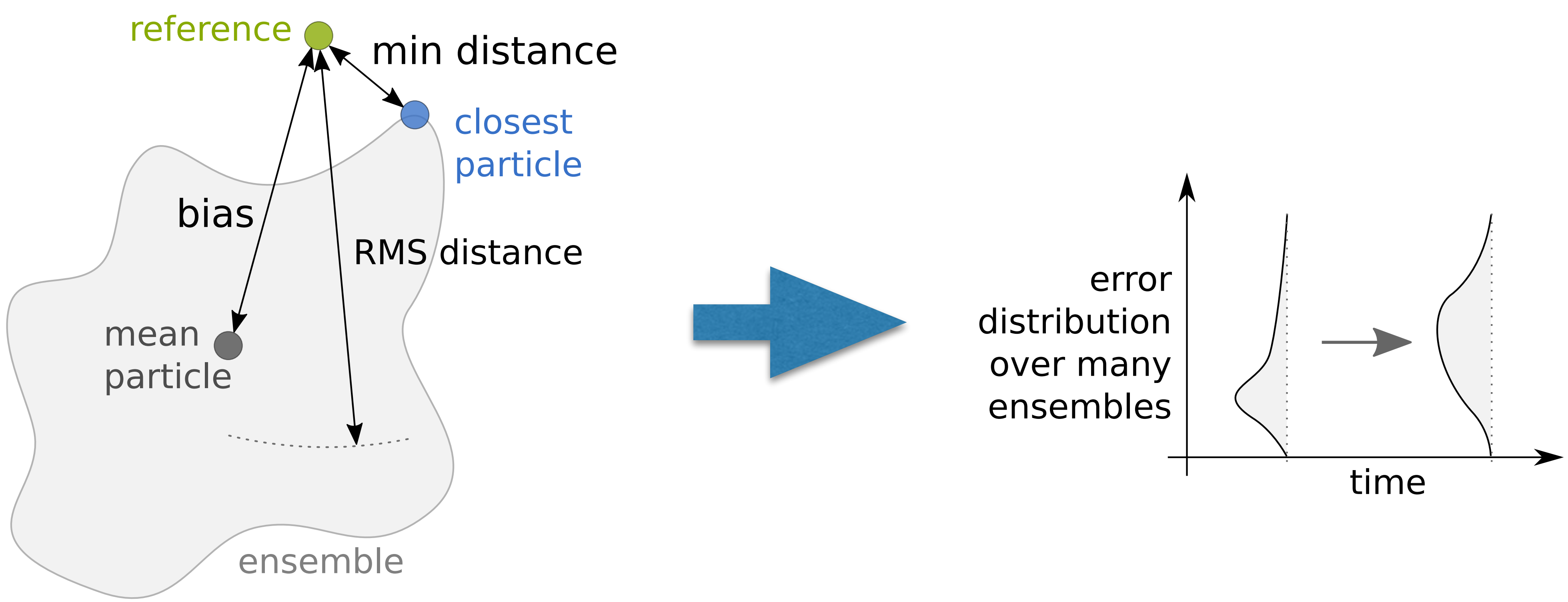
- (usual) **deterministic** model ~ DNS, accurate but impossible to compute
- (deterministic) **diffusive** model ~ LES
- **stochastic** model under location uncertainty

➡ behaviors of ensembles?

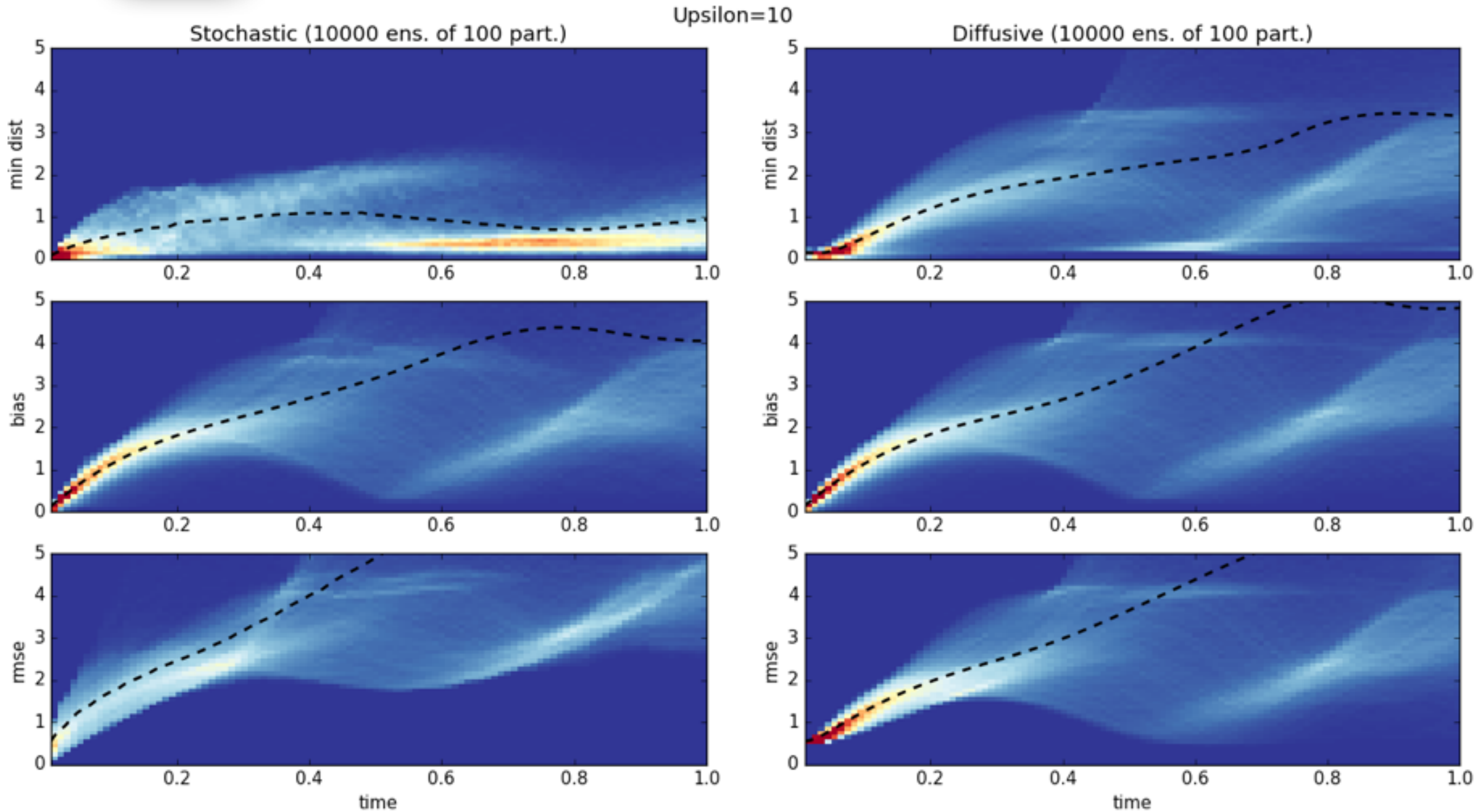
Short time behavior

Comparison **ensemble $\leftrightarrow$ reference**

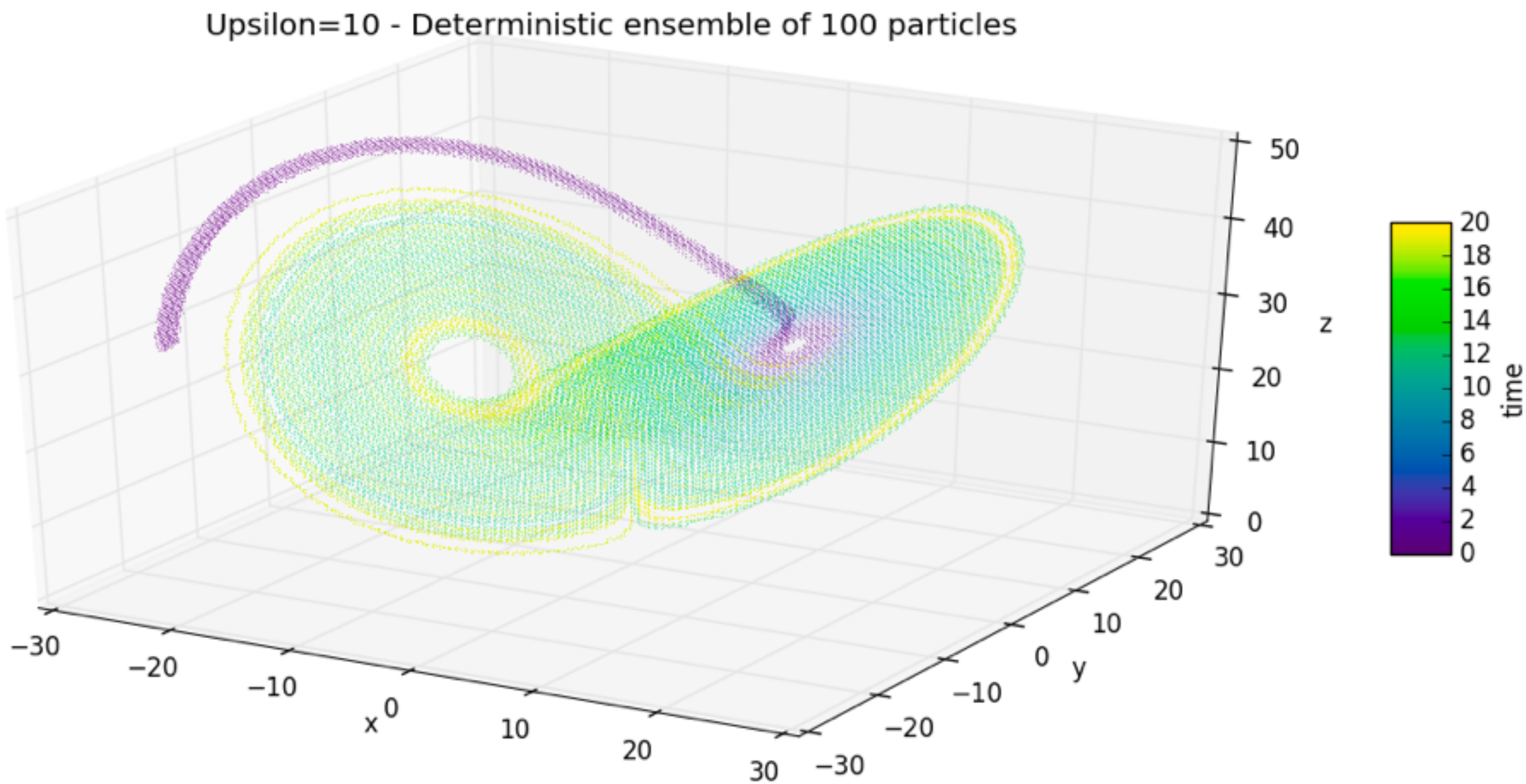
3 metrics: **minimum distance, bias, RMS distance**



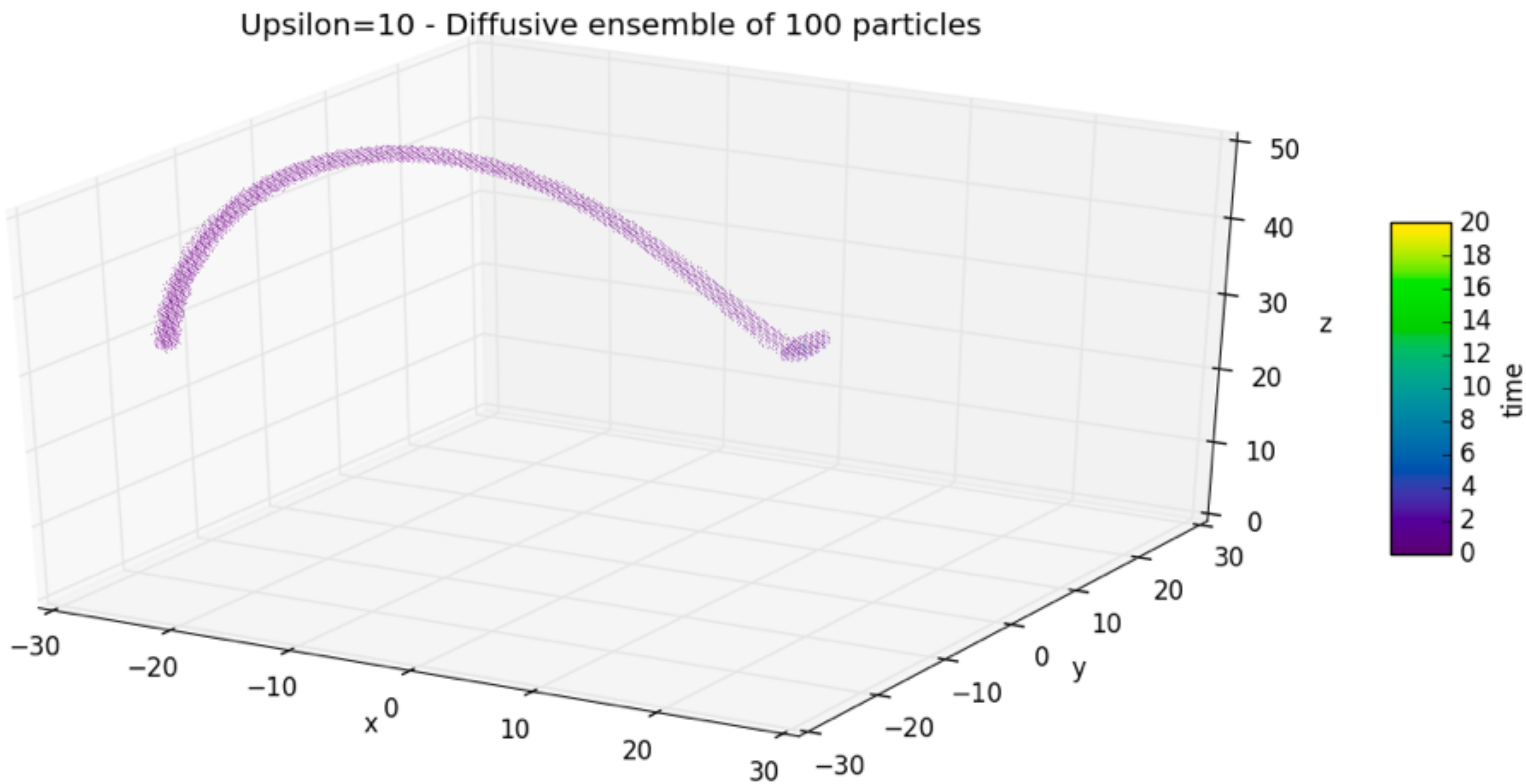
Short time behavior



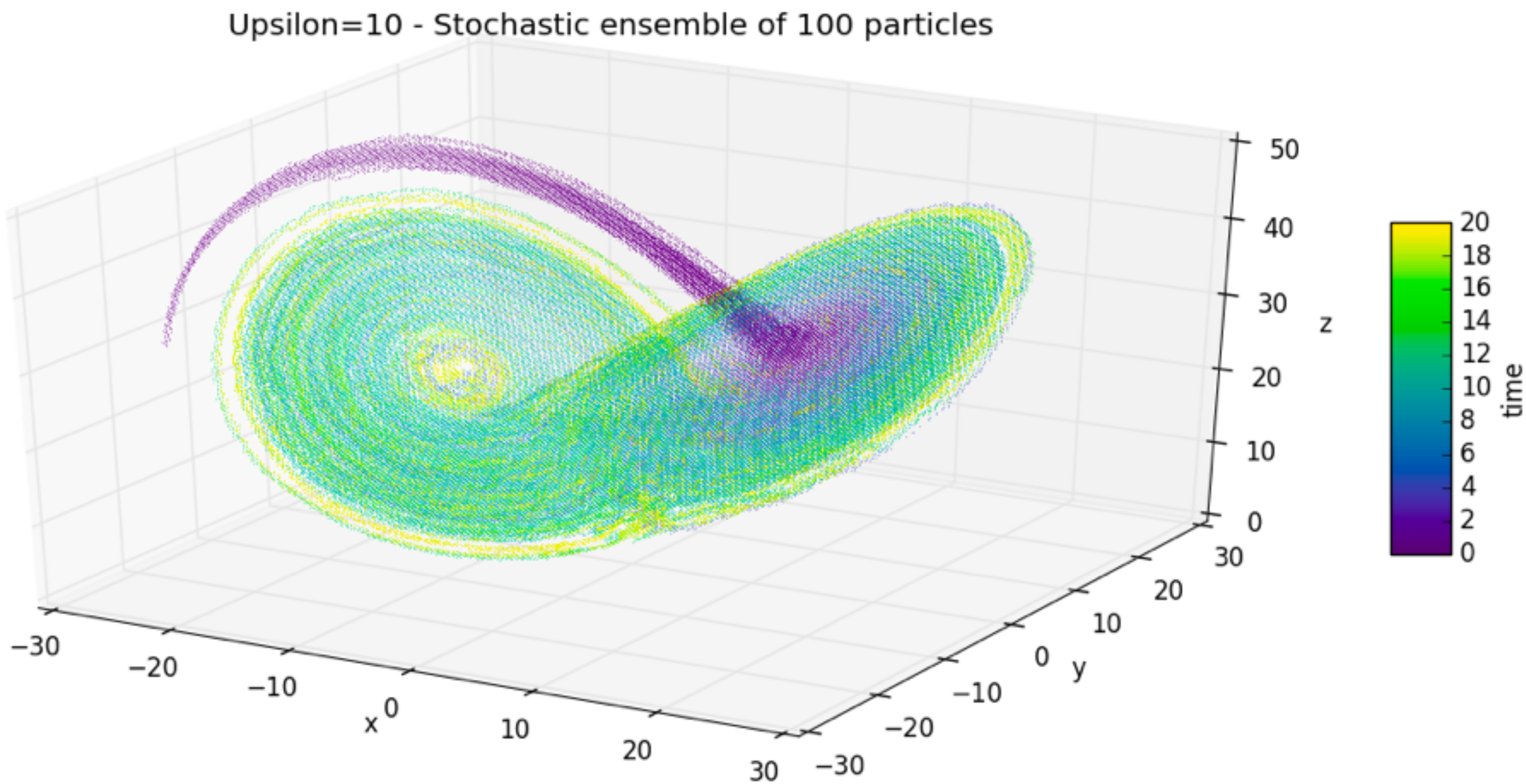
Long time


Upsilon=10 - Deterministic ensemble of 100 particles
x
y
z
time
−30
−20
−10
0
10
20
30
50
40
30
20
10
0
20
18
16
14
12
10
8
6
4
2
0

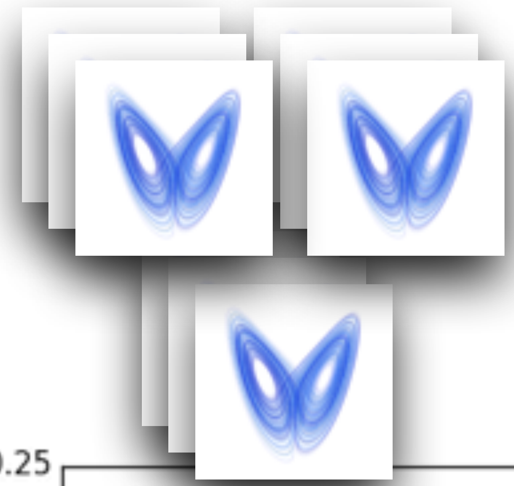
Long time

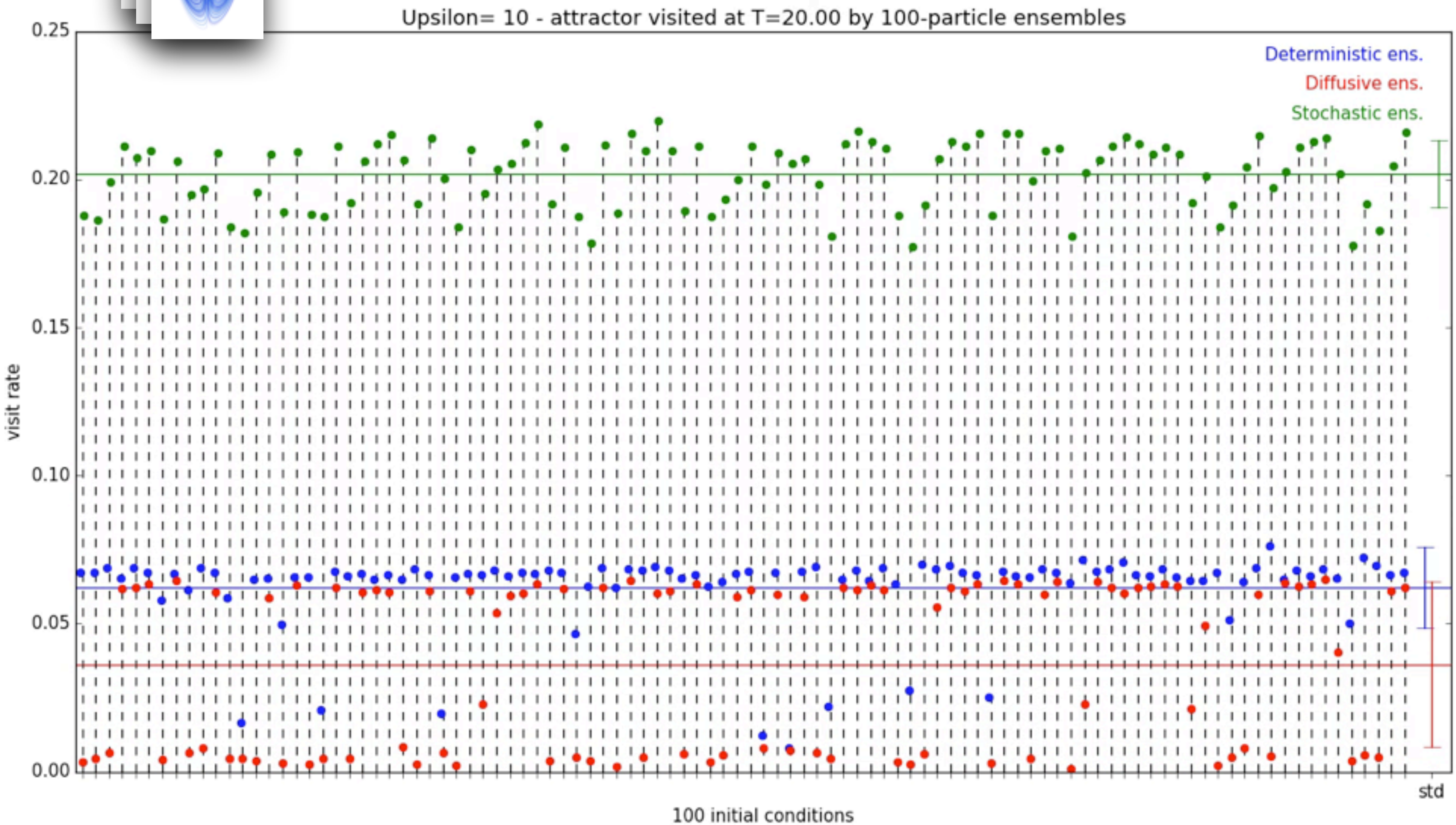

Upsilon=10 - Diffusive ensemble of 100 particles
x
−30
−20
−10
0
10
20
30
y
−30
−20
−10
0
10
20
30
z
0
10
20
30
40
50
time
0
2
4
6
8
10
12
14
16
18
20

Long time

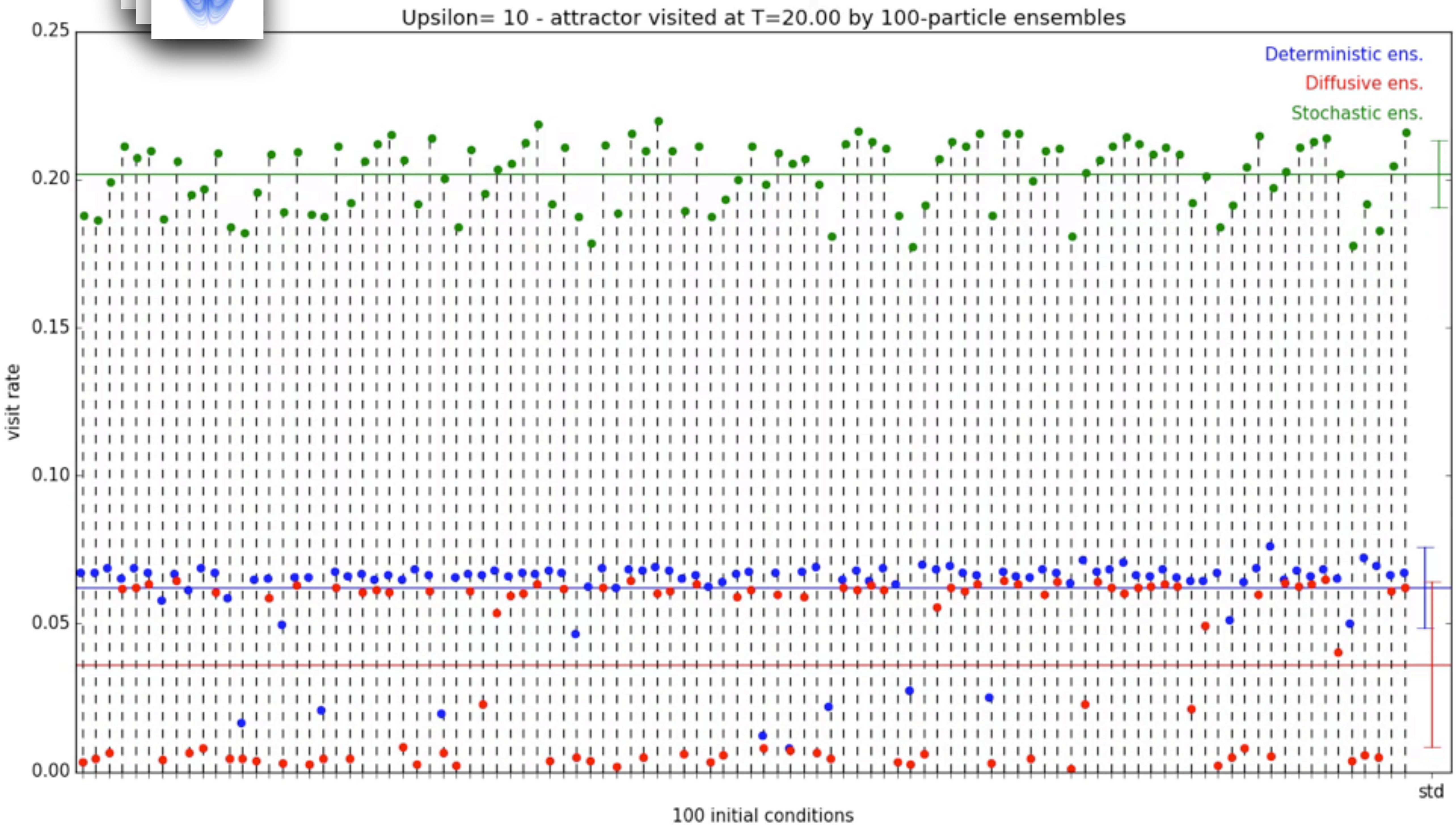

Upsilon=10 - Stochastic ensemble of 100 particles
x
y
z
time
−30
−20
−10
0
10
20
30
40
50
2
4
6
8
12
14
16
18

Attractor visit rates





Attractor visit rates



Conclusion

Conclusion

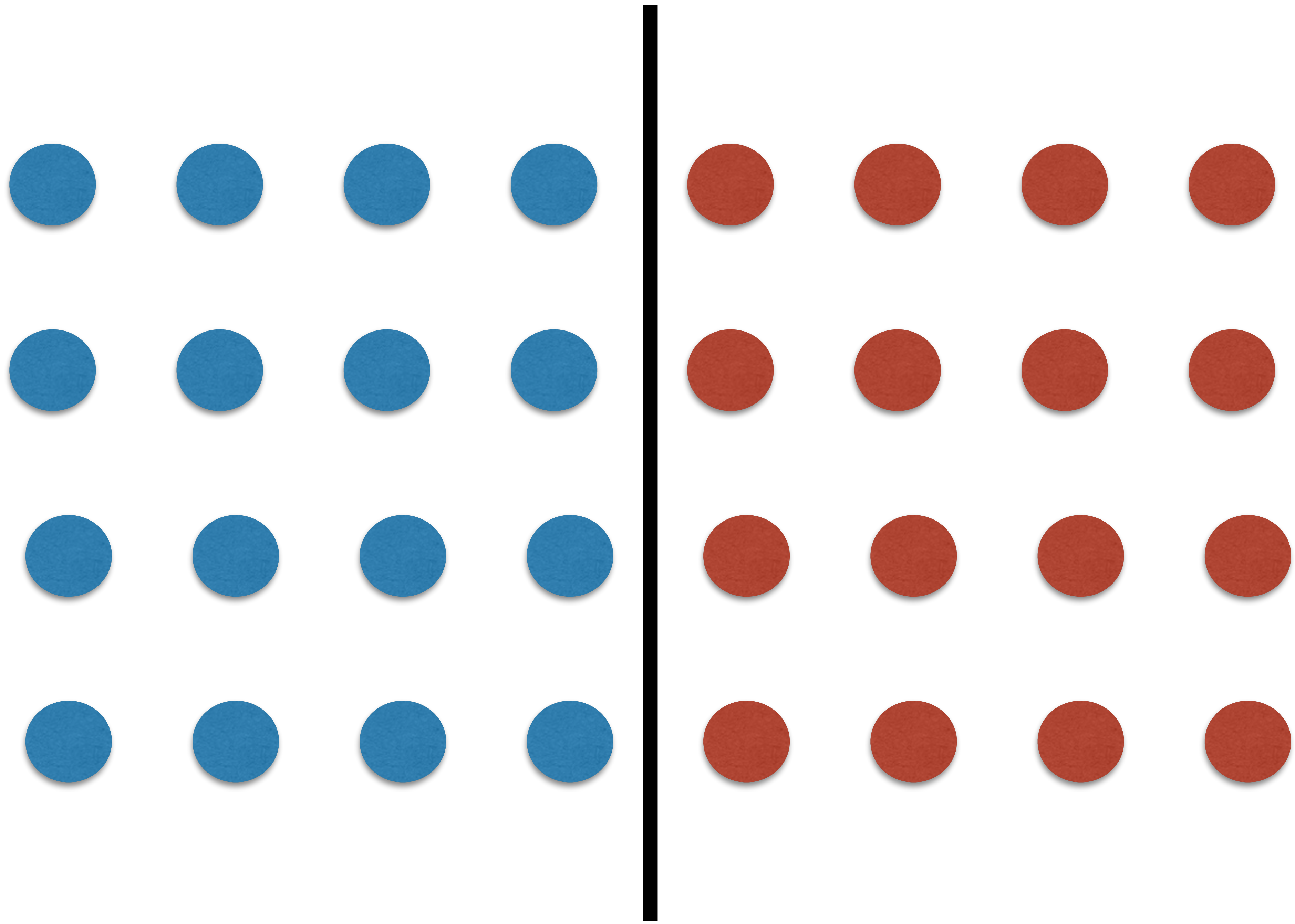
- Random transport applicable to any dynamics
- Better small scales
- Efficient spreading of the ensemble
- Likely scenarios
- Exploration of the attractor

Thank you for your attention

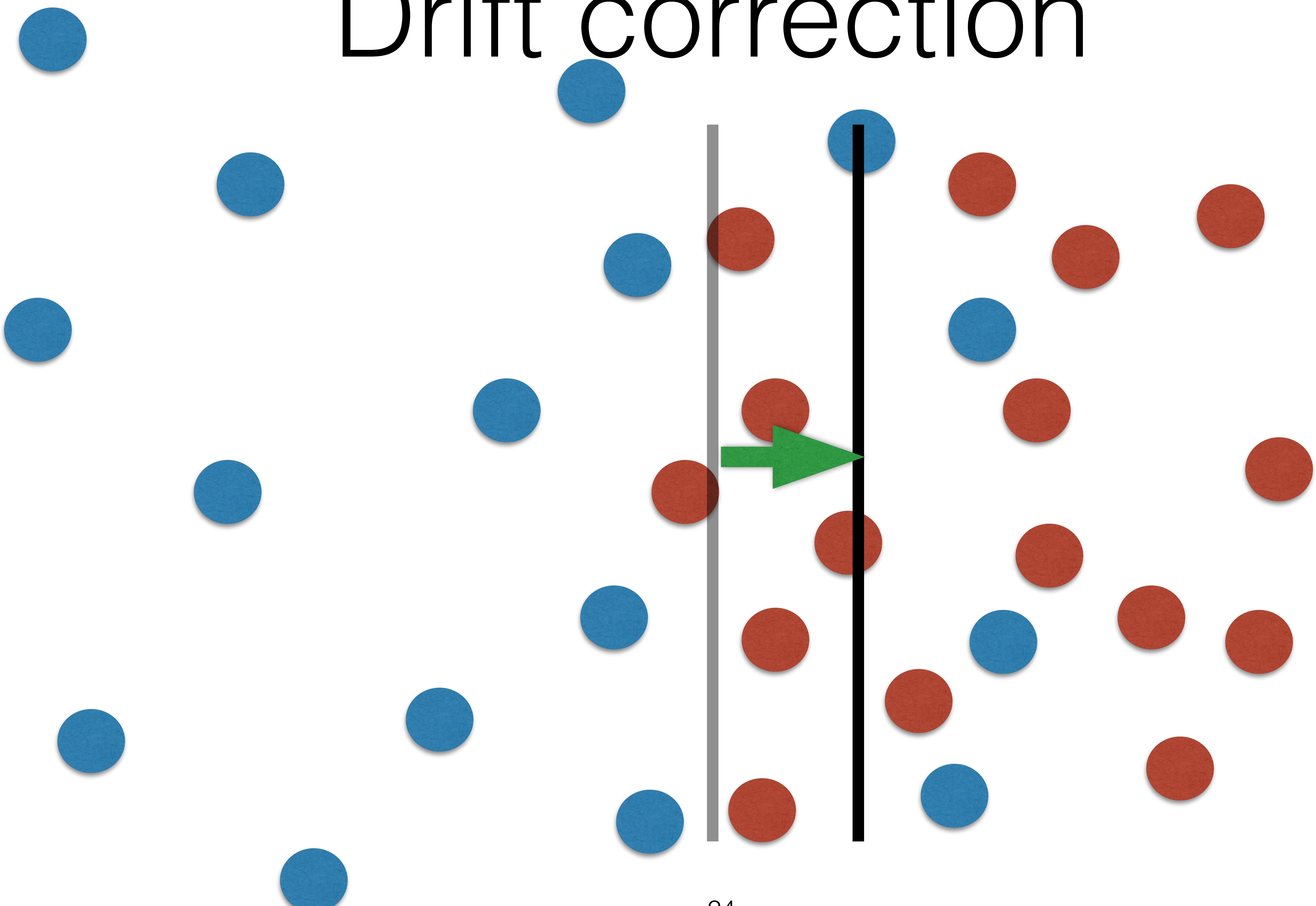
Code SQG MU:
link from Fluminance website - V. Resseguier

Drift correction

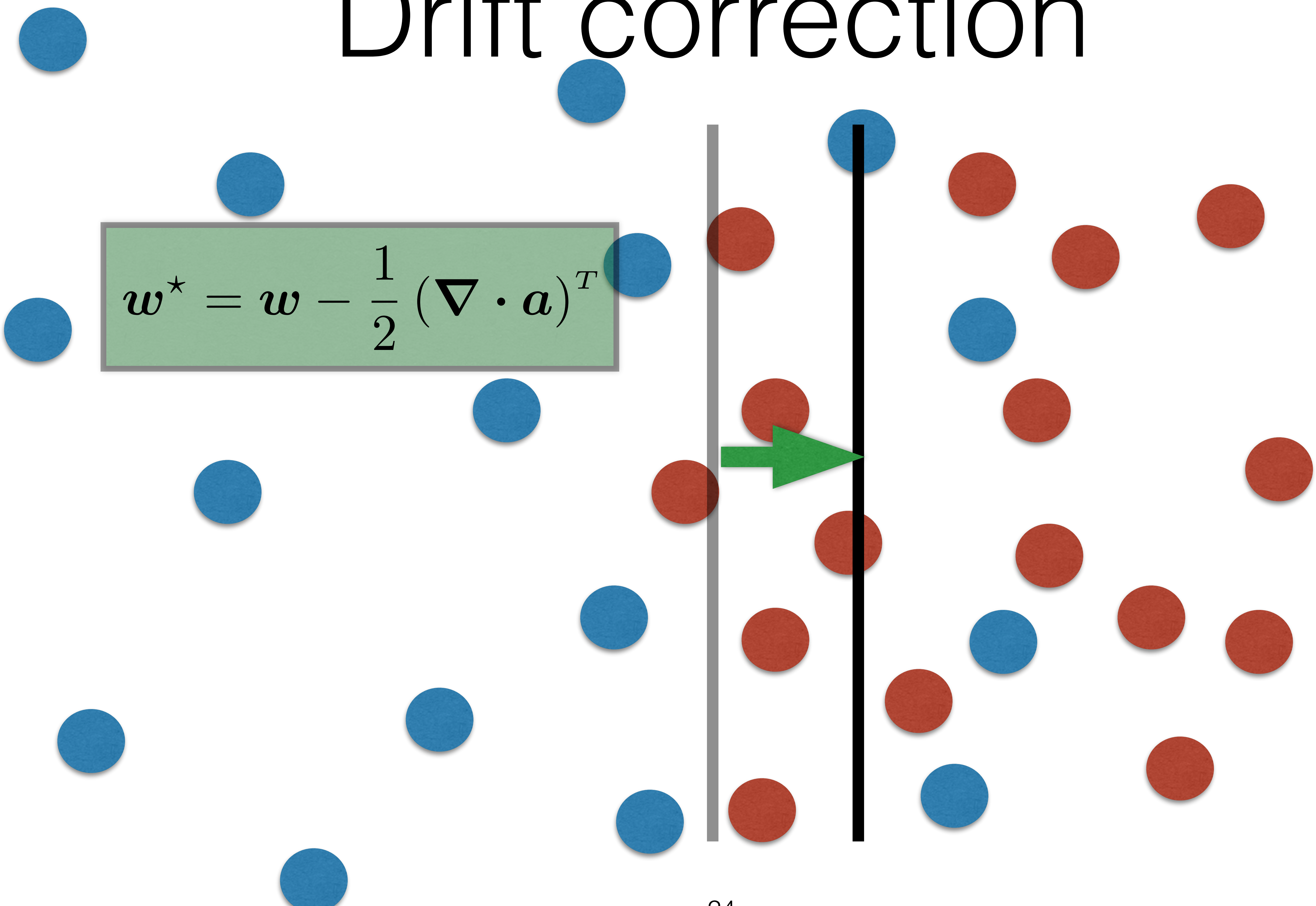
Drift correction



Drift correction



Drift correction



$$\boldsymbol{w}^{\star} = \boldsymbol{w} - \frac{1}{2}\left(\boldsymbol{\nabla} \cdot \boldsymbol{a}\right)^{T}$$

Bifurcations in SQG

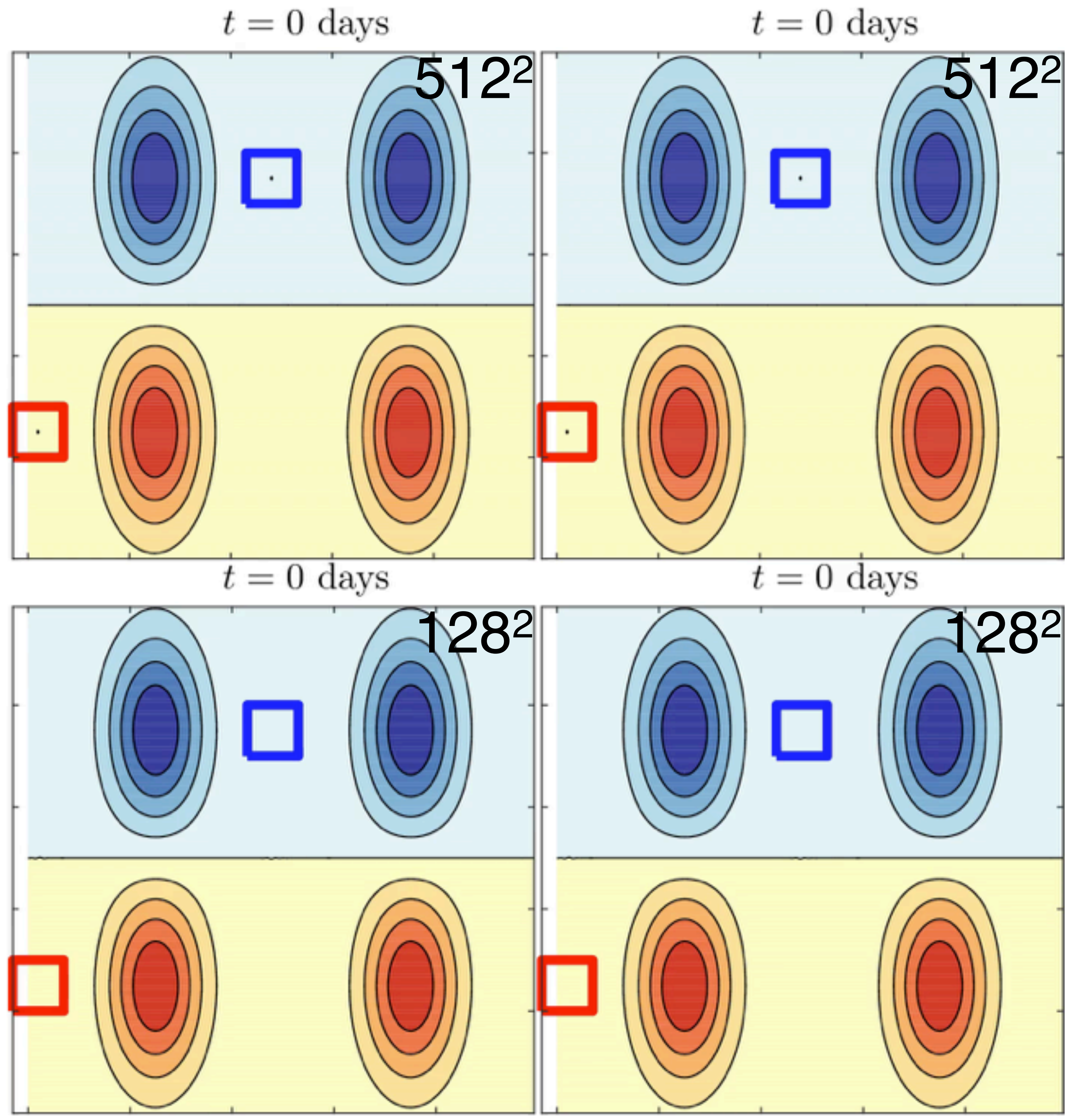
tracked by SQG MU

Reference flow: deterministic SQG

512^2 versus 128^2

Initial condition 1

?

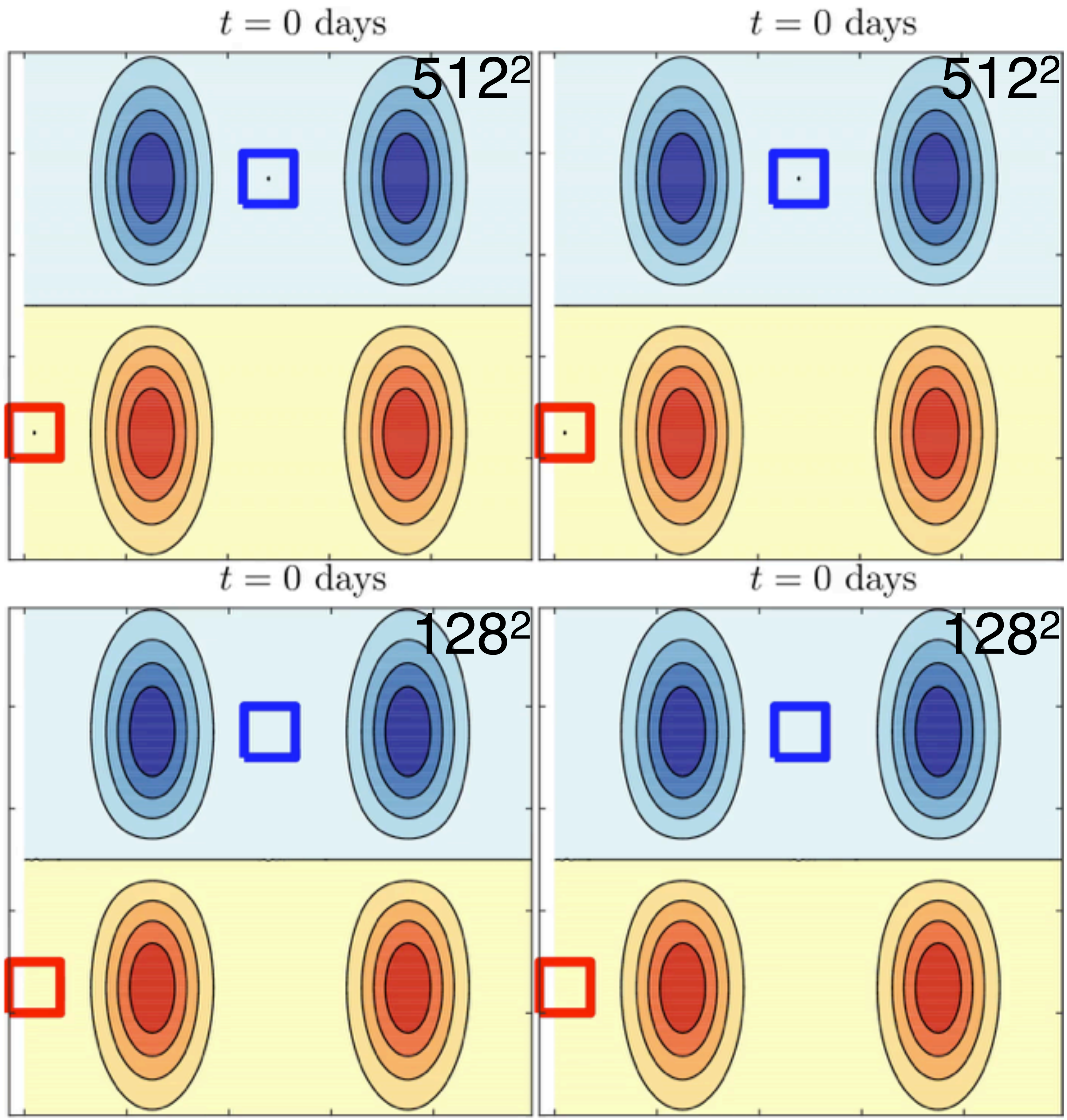


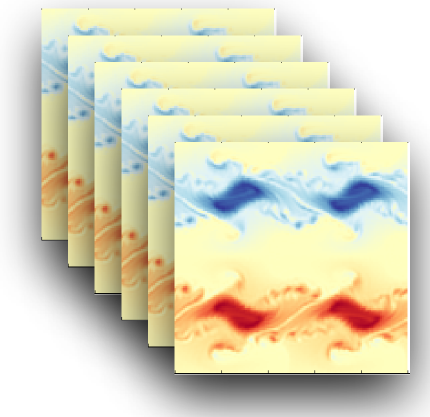
Reference flow: deterministic SQG

512^2 versus 128^2

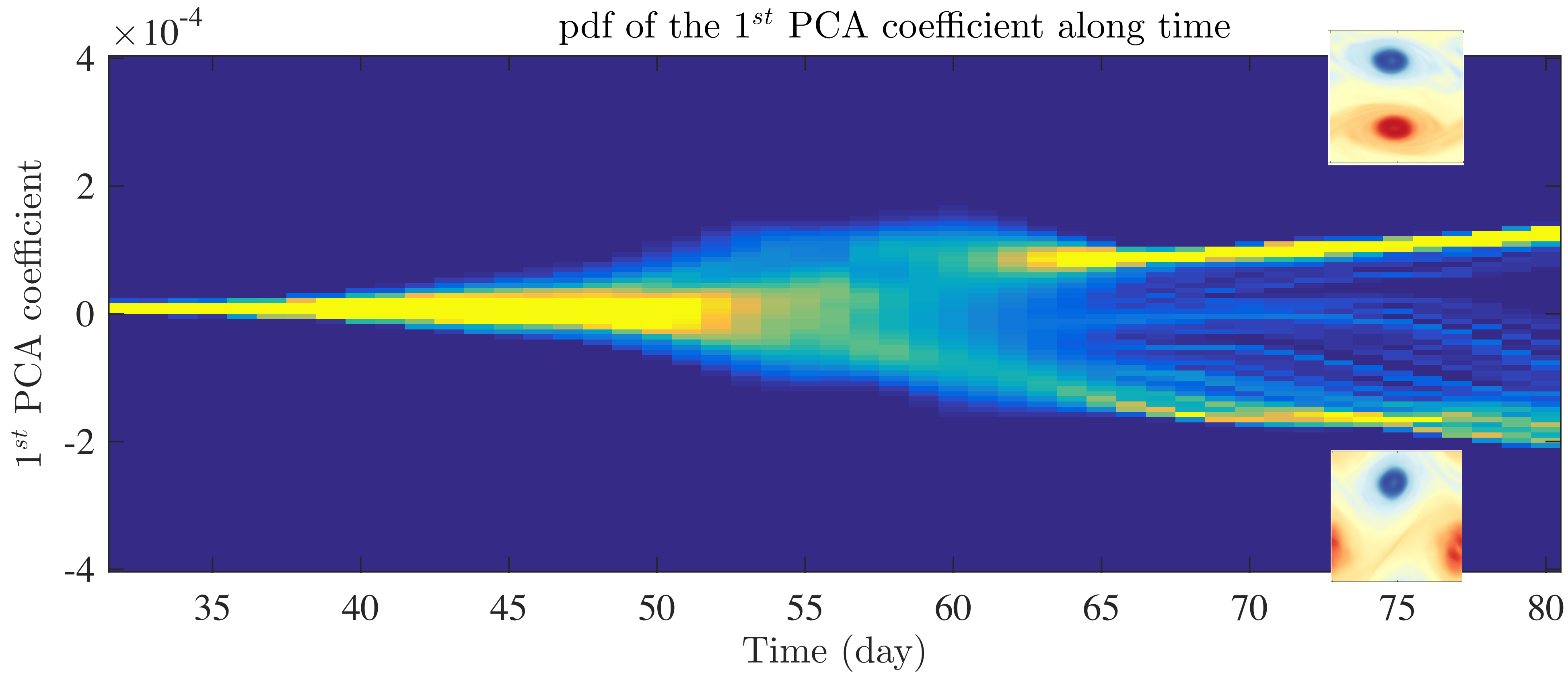
Initial condition 1

?

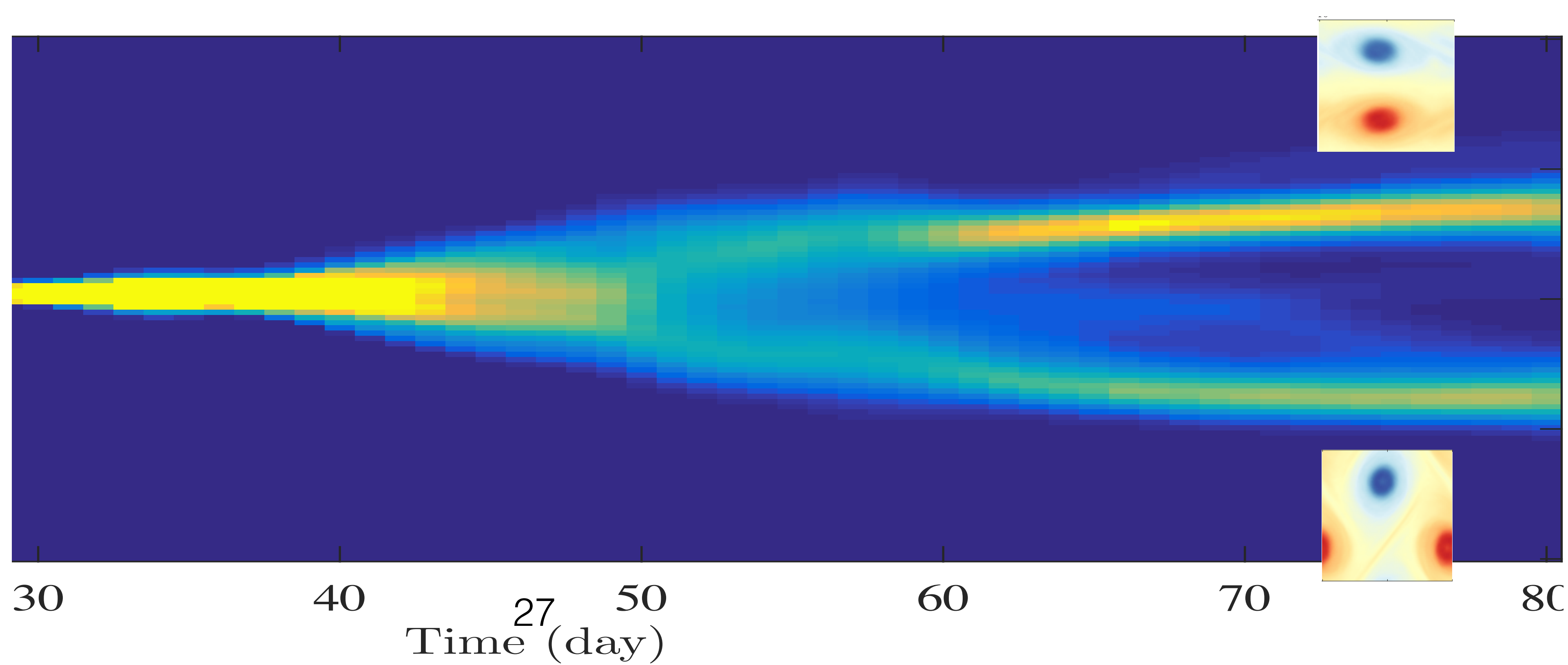




Random initial conditions



Under location uncertainty



SQG under Strong Uncertainty

SQG SU

Mesoscale divergence

Geostrophic balance

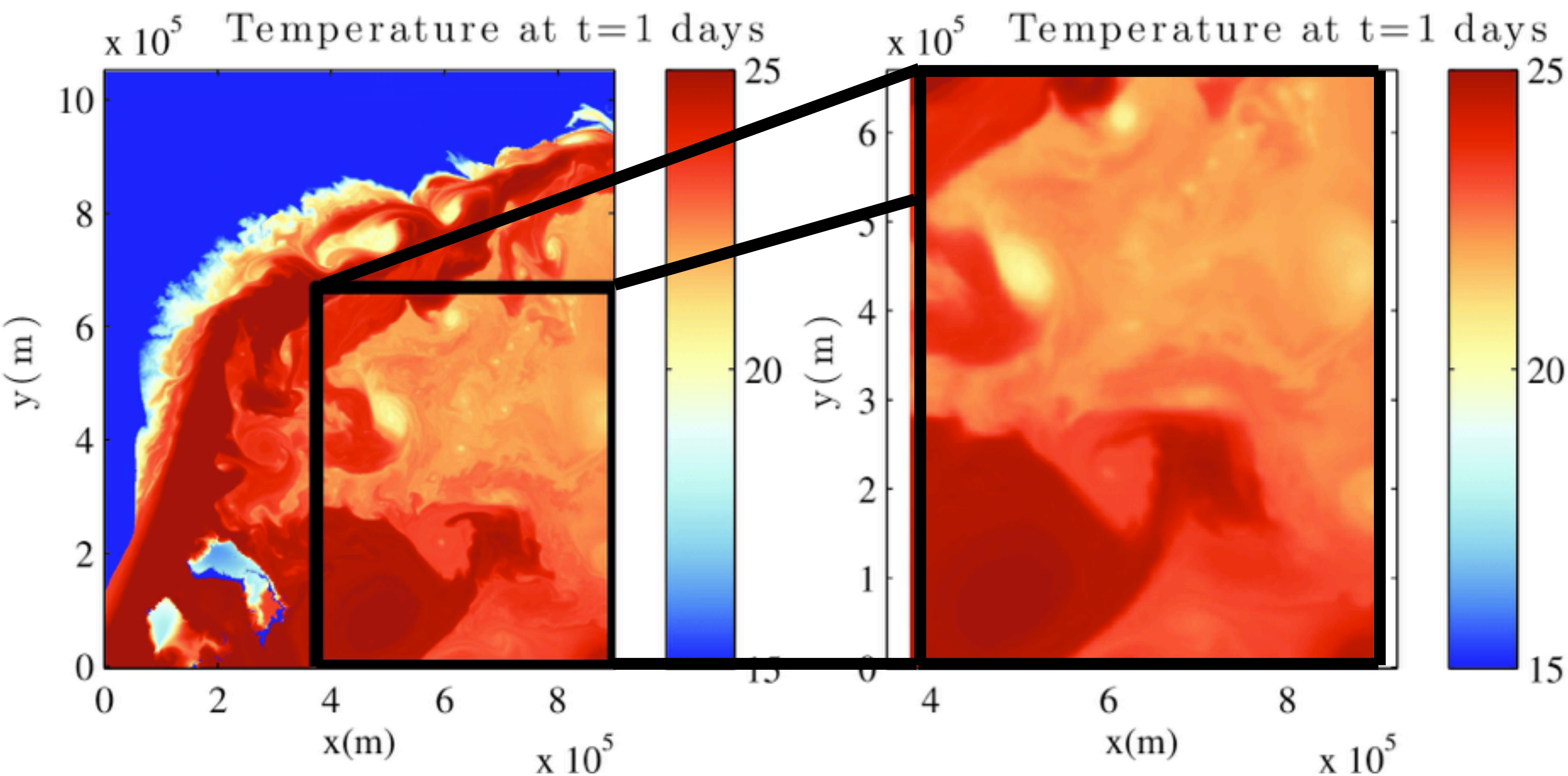
Horizontal Diffusion

$$\boldsymbol{f} \times \boldsymbol{u} = -\frac{1}{\rho_b}\boldsymbol{\nabla} p' + \frac{a}{2}\Delta \boldsymbol{u}$$

$$\Rightarrow \quad \boxed{\boldsymbol{\nabla} \cdot \boldsymbol{u} \propto \Delta \boldsymbol{\nabla}^{\perp} \cdot \boldsymbol{u}}$$

Filtering of model outputs:

Gula, Jonathan, M. Jeroen Molemaker, and James C. McWilliams
"Gulf Stream dynamics along the southeastern US seaboard."
Journal of Physical Oceanography 45.3 (2015): 690-715.



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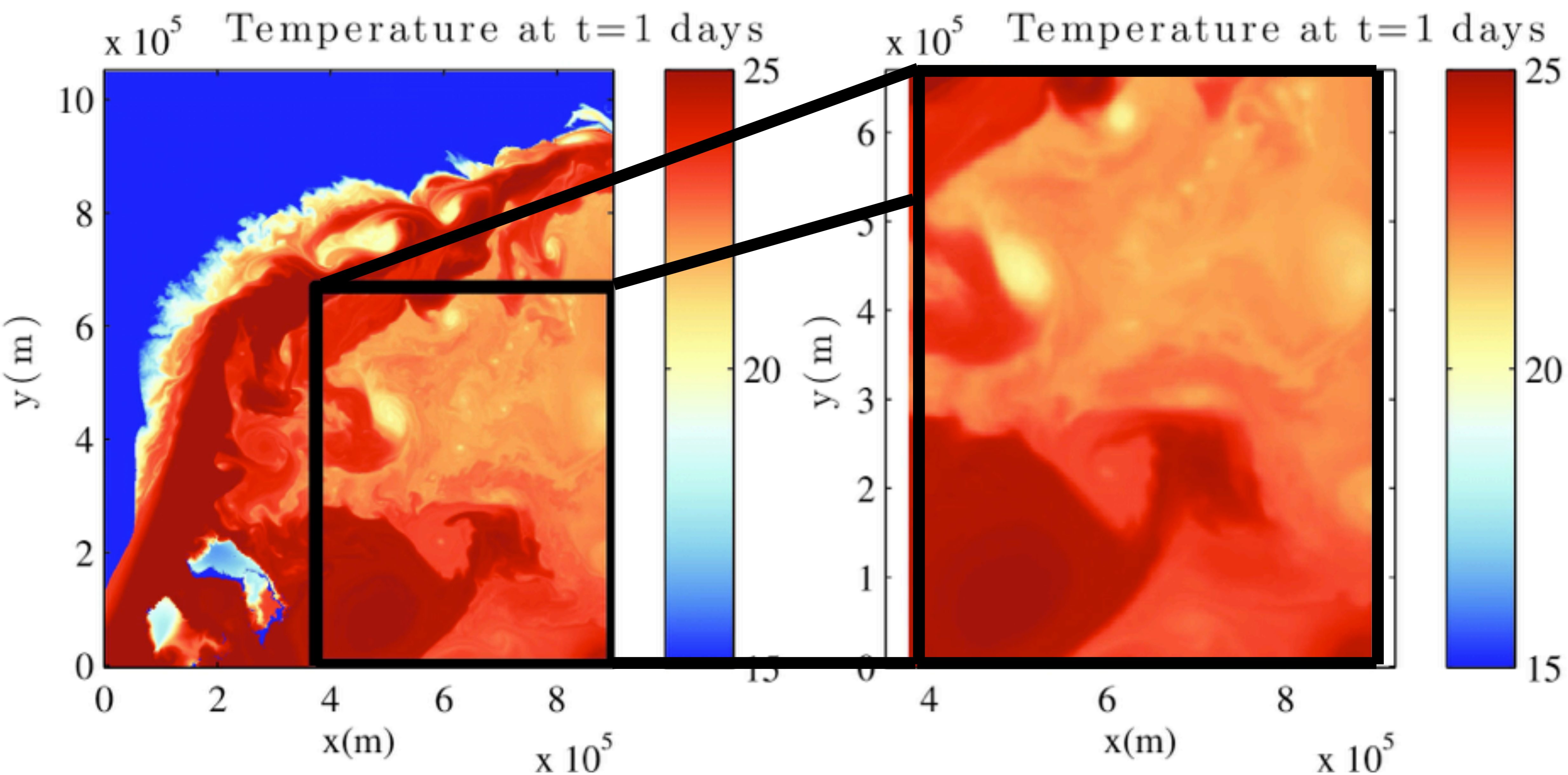
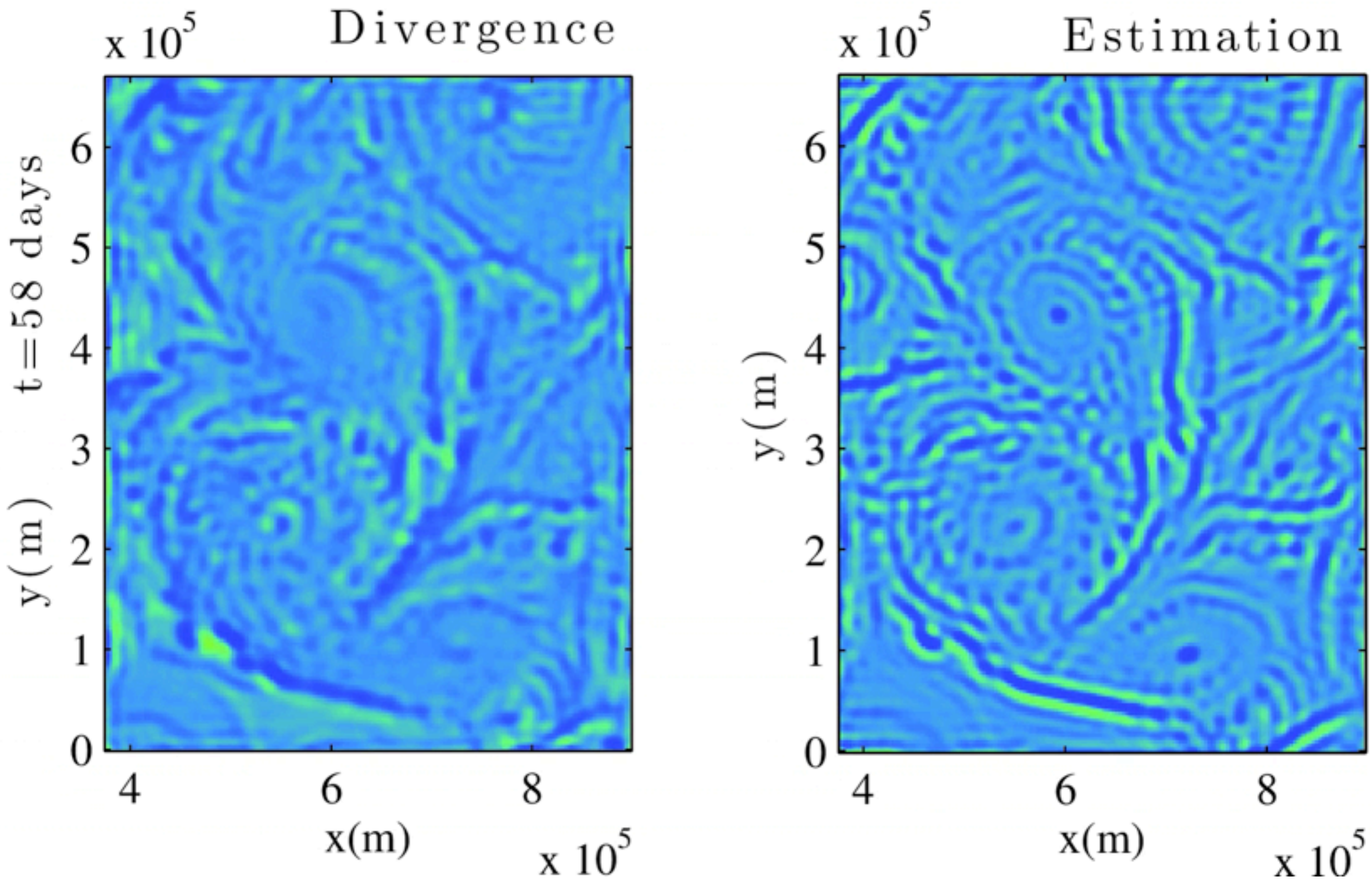
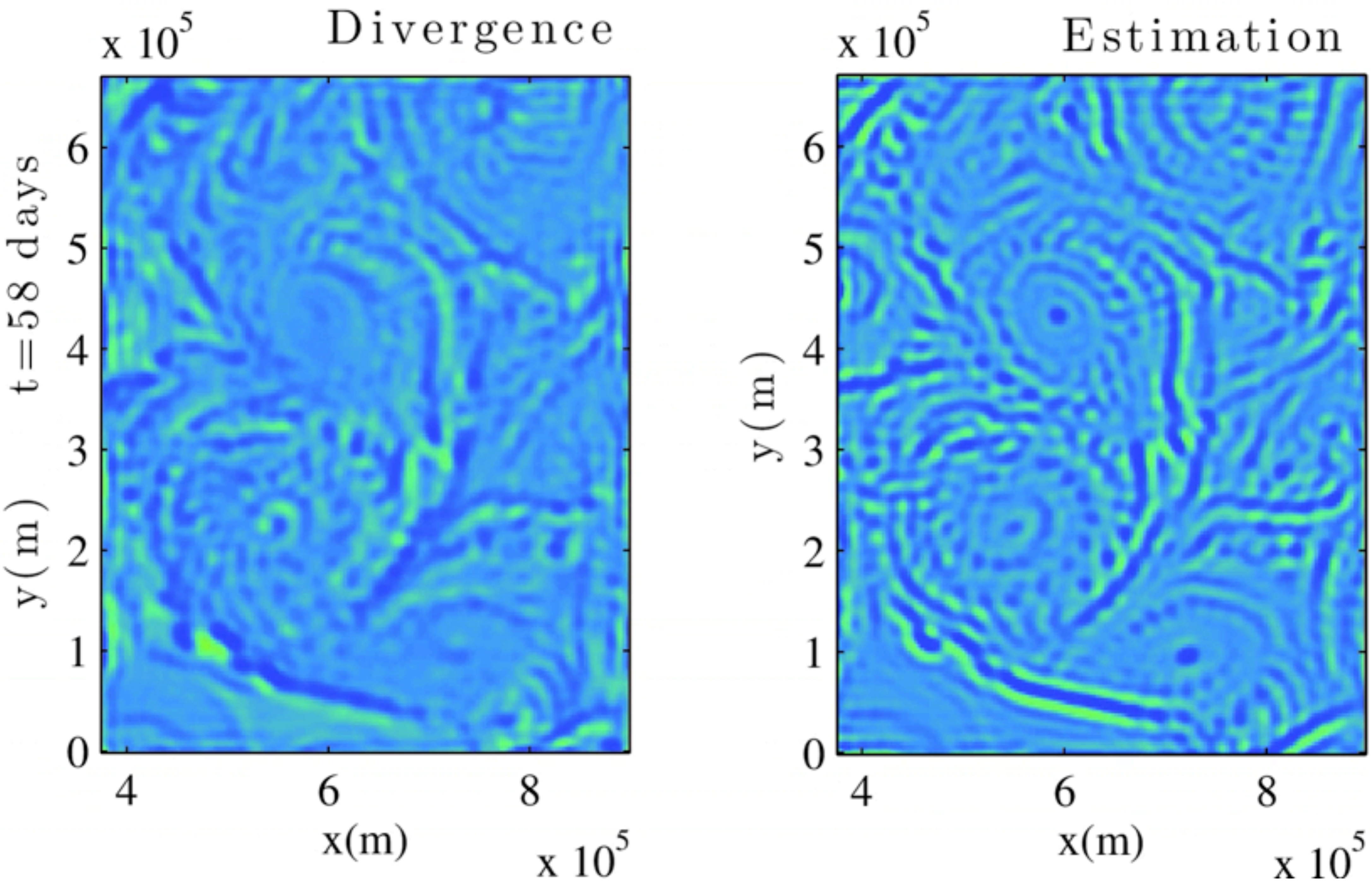


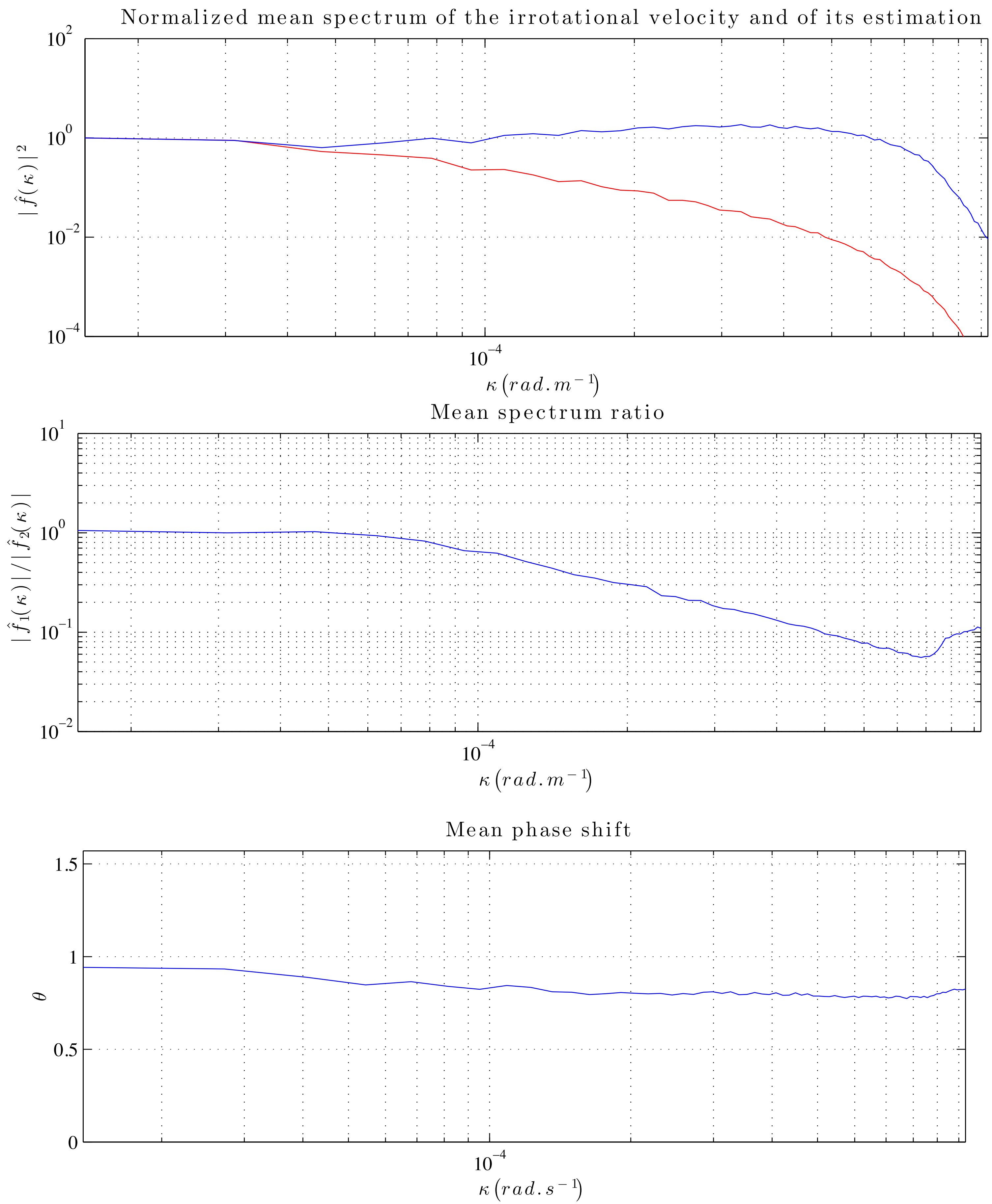
Spatial test



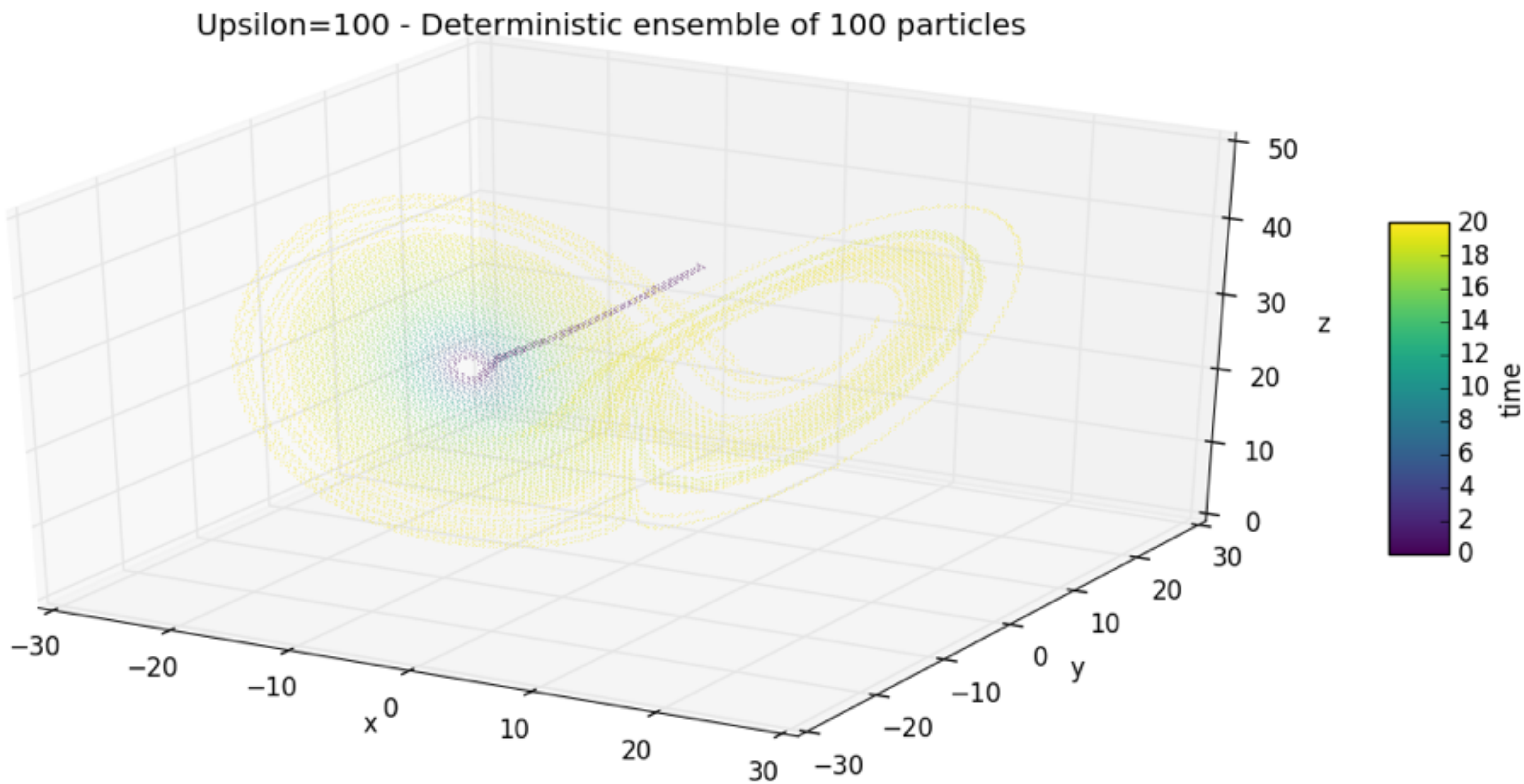
Spatial test



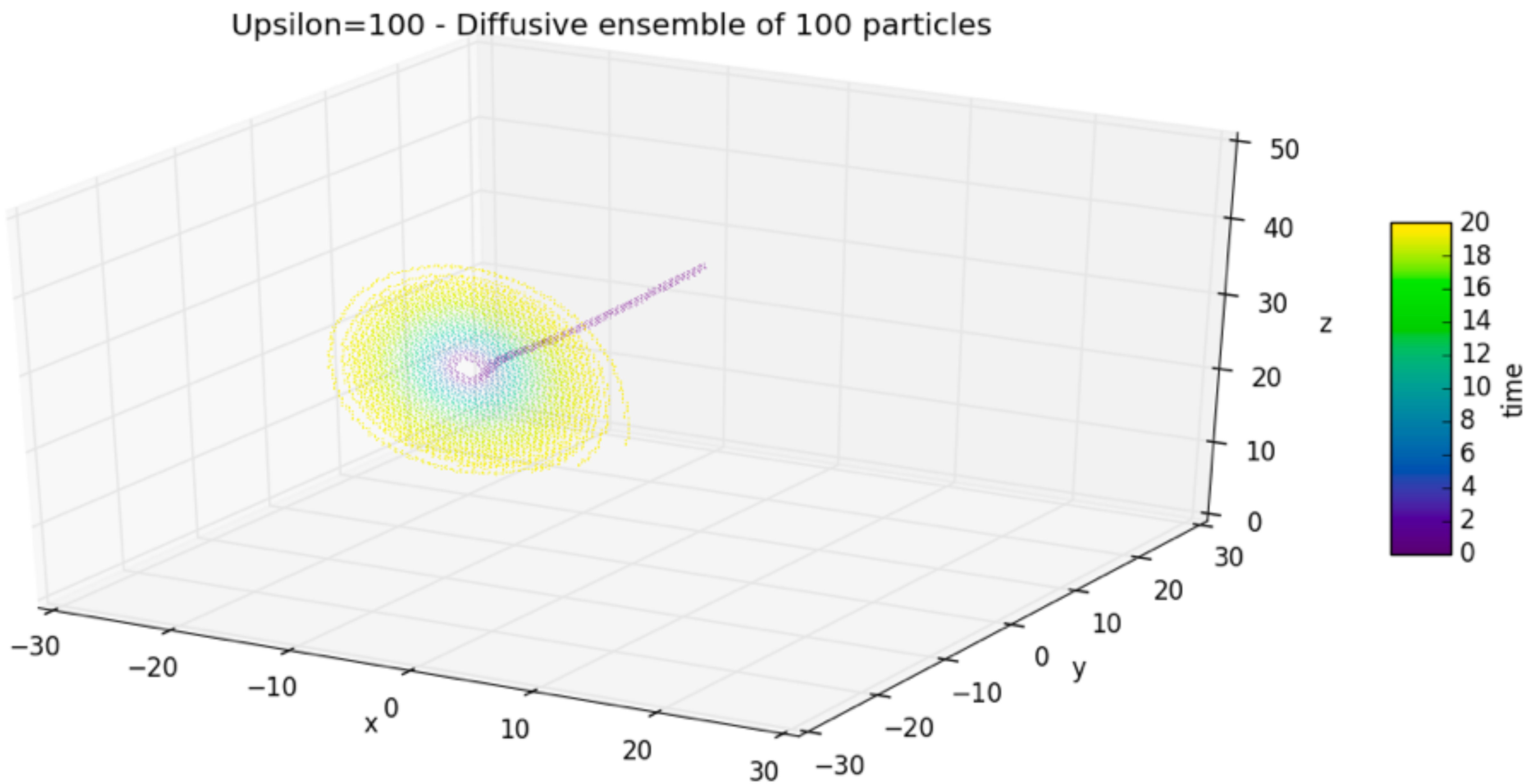
Spectral test



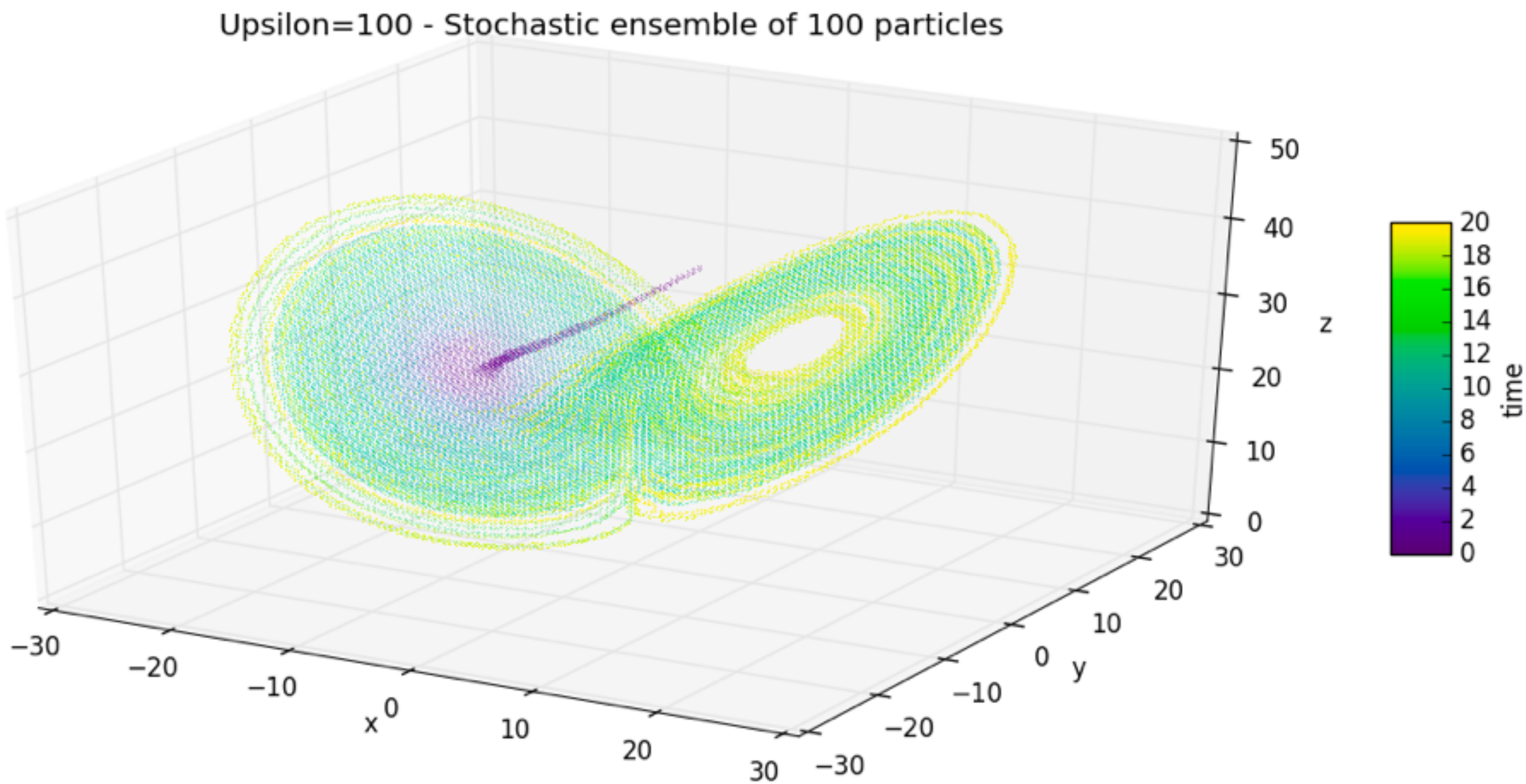
Long time: weak noise



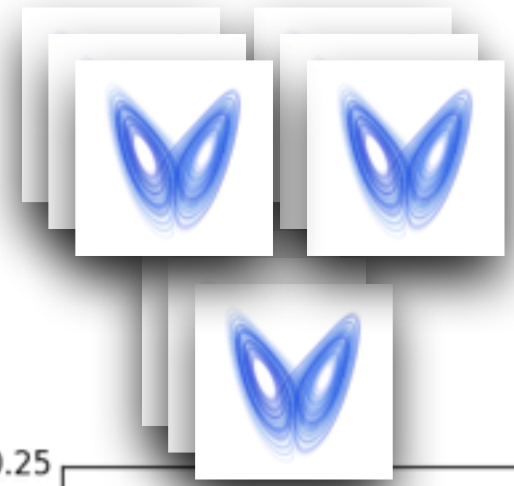
Long time: weak noise

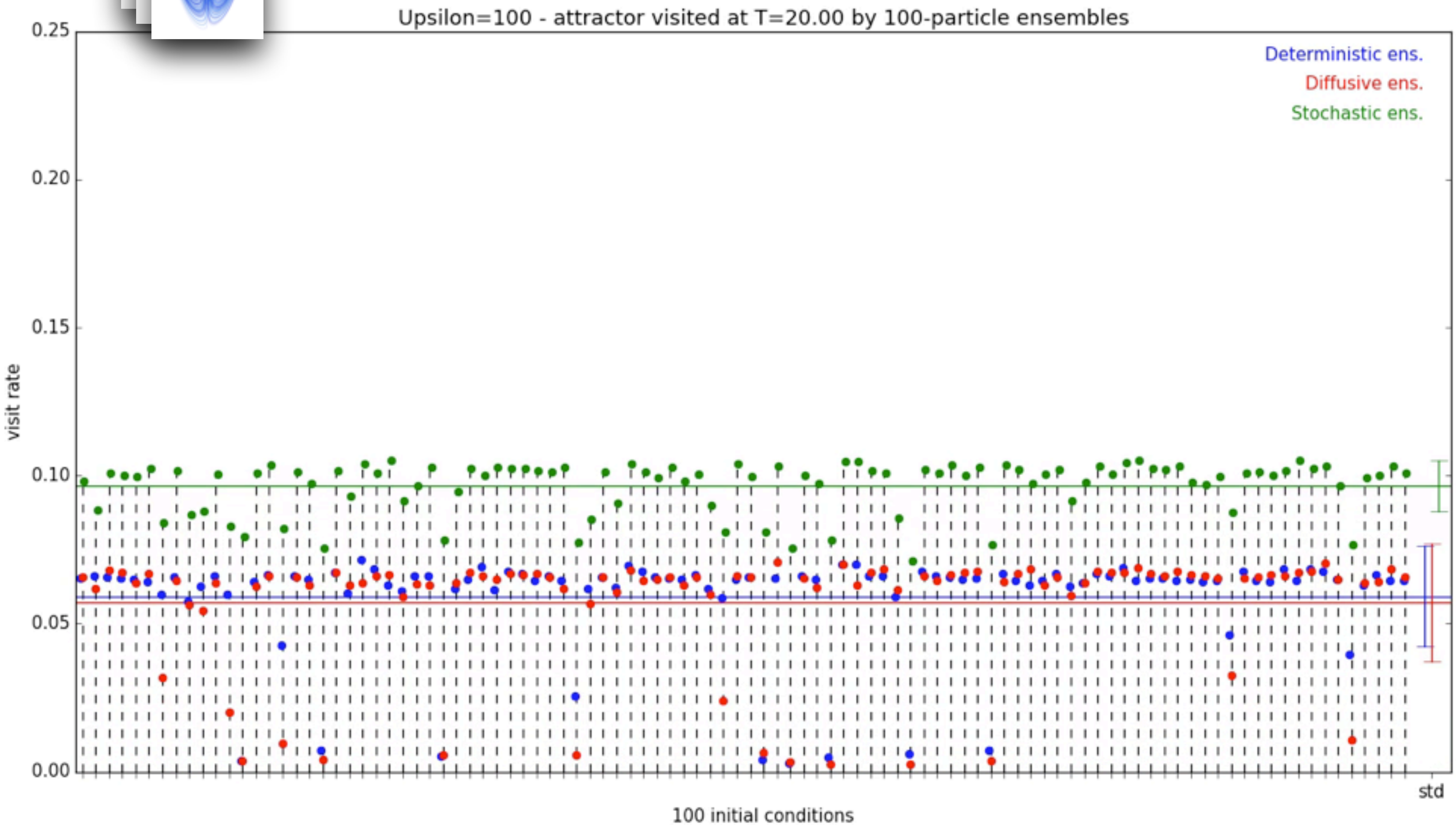


Long time: weak noise

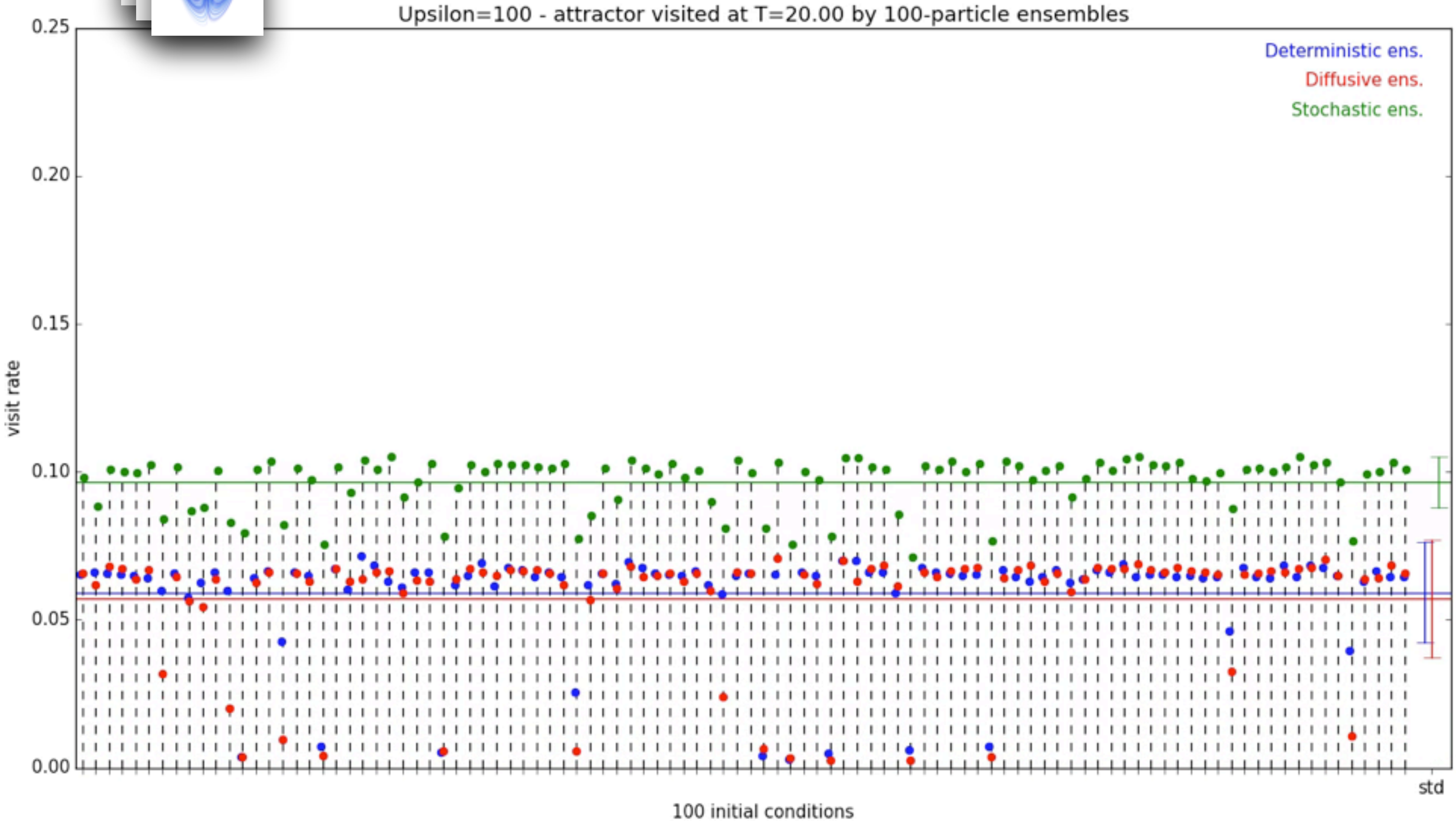


Attractor visit rates





Attractor visit rates



Computing the visit rate

- **Discrete covering** of the Lorenz attractor (GAIO) → **611,550 cubic boxes** of radius=0.15625

- For each ensemble, the visit rate:

$$\tau(T) = \frac{\#\{\text{ unique boxes visited by ensemble over } [0;T]\}}{\text{total } \# \text{ of boxes}}$$