Stochastic subgrid tensor for geophysical flow modeling

Valentin Resseguier, Etienne Mémin, Bertrand Chapron

STOCHASTIC SUBGRID TENSOR FOR GEOPHYSICAL MODELING

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Motivations

- Rigorous identification of sudgrid dynamics effects
- Taking into account likely small-scale dynamics (stochastic backscatter)
- Forecast of likely distinct scenarios
- Quantification of the dynamics errors:
  - Diagnostic for numerical simulations (mesh refinements, …)
  - **Ensemble data assimilation and forecasts**

Transport under location uncertainty

Assumption:

Velocity decomposition: $\boldsymbol{v} = \boldsymbol{w} + \boldsymbol{\sigma}\dot{\boldsymbol{B}}$ with
- $\boldsymbol{w}$: large-scale velocity
- $\boldsymbol{\sigma}\dot{\boldsymbol{B}}$: uncorrelated in time, divergence-free, correlated in space, Gaussian, inhomogeneous and anisotropic

From stochastic calculus theory, we proofed:

$$\frac{D\Theta}{Dt} = \partial_t\Theta + \underbrace{\boldsymbol{w}^\star \cdot \boldsymbol{\nabla}\Theta + \boldsymbol{\sigma}\dot{\boldsymbol{B}} \cdot \boldsymbol{\nabla}\Theta}_{\text{Advection}} \underbrace{- \boldsymbol{\nabla} \cdot \left(\frac{1}{2}\boldsymbol{a}\boldsymbol{\nabla}\Theta\right)}_{\text{Inhomogeneous and anisotropic diffusion}} = 0$$

$$\boldsymbol{w}^* = \boldsymbol{w} - \frac{1}{2}\boldsymbol{\nabla}\cdot\boldsymbol{a}$$

$$\boldsymbol{a} = \boldsymbol{\sigma}\boldsymbol{\sigma}^T$$

- Drift correction
- Multiplicative random forcing
- Balanced energy exchanges

Systematic derivation of random models with the new $\frac{D}{Dt}$

- Conservations (mass, linear momentum, …)
  - Navier-Stokes
  - Boussinesq
    - Uncertainty
      - QG MU
      - SQG under Moderate Uncertainty
      - SQG under Strong Uncertainty

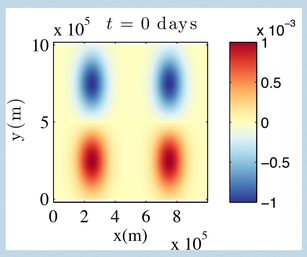
SQG under Moderate Uncertainty

Description

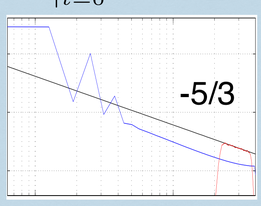
- Usual SQG relationship and transport under uncertainty of buoyancy
- Good illustration of our stochastic transport
- Simulations done with homogeneous small-scale velocity

One realization: better small-scale representation

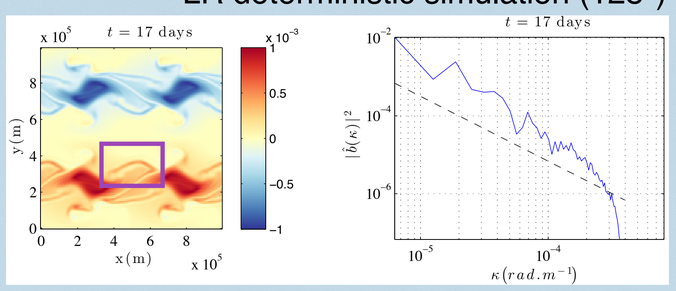
Initial condition



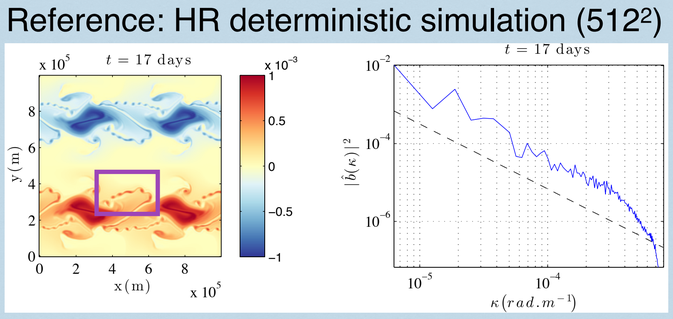
Spectrum of $\boldsymbol{w}|_{t=0}$ and $\boldsymbol{\sigma}\dot{\boldsymbol{B}}$



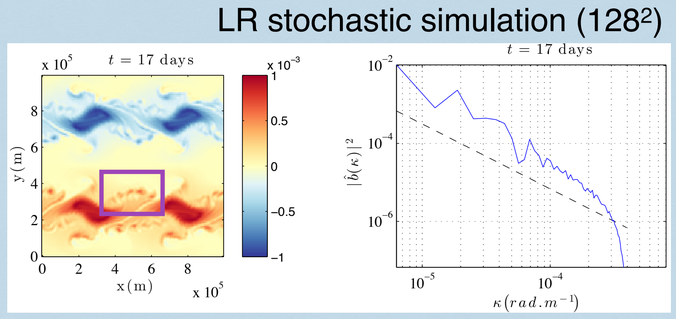
LR deterministic simulation (128^2)



Reference: HR deterministic simulation (512^2)

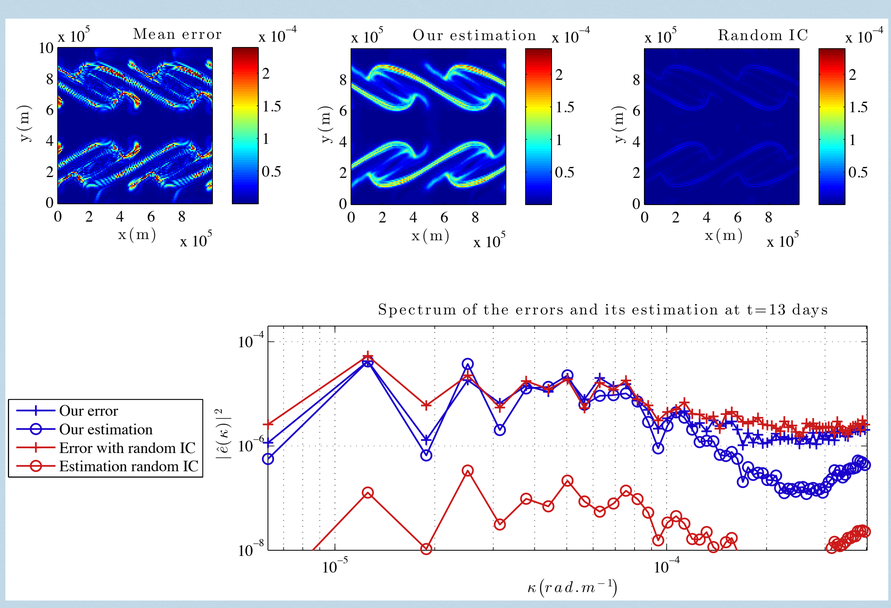


LR stochastic simulation (128^2)

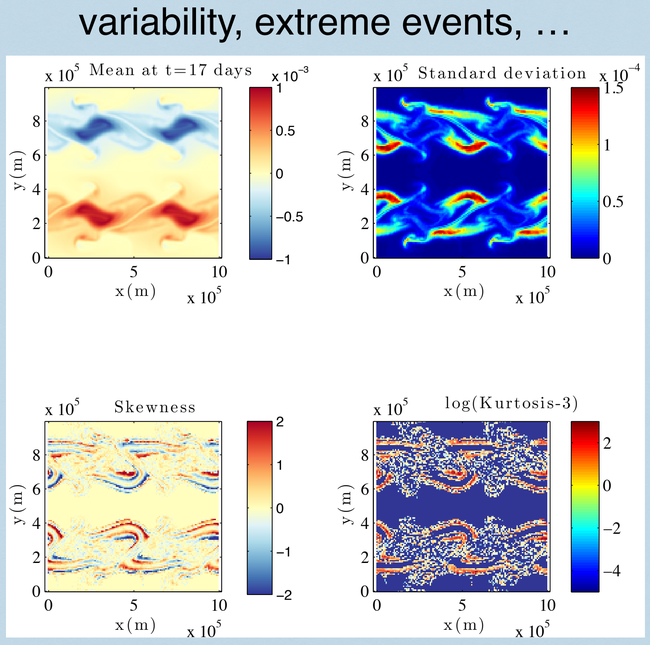


Ensemble of simulations

Errors estimation



Point-wise moments: variability, extreme events, …



Conclusion

- Random transport applicable to any dynamics
- Better representation of small scales
- Extreme events
- **Good errors estimation in location and amplitude**
- Likely scenarios

code available online: link from Fluminance website - V. Resseguier

SQG under Strong Uncertainty

Description

QG assumption under strong uncertainty:
- kills the interior dynamics
- creates horizontal velocity divergence
- provides a diagnostic relation

Modified geostrophic balance

Geostrophic balance $\boldsymbol{f} \times \boldsymbol{u} = -\frac{1}{\rho_b}\boldsymbol{\nabla} p'$ + Horizontal Diffusion $\frac{a}{2}\Delta\boldsymbol{u}$

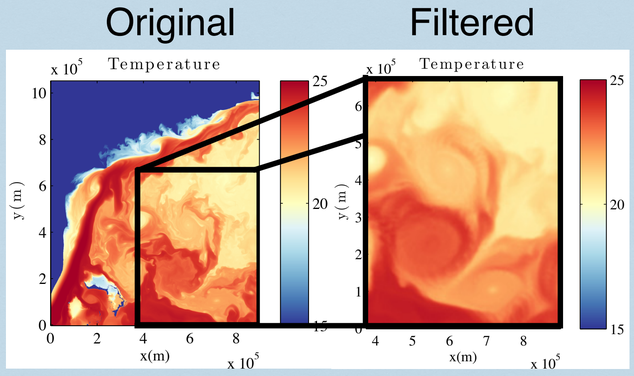
$$\boldsymbol{\nabla}\cdot\boldsymbol{u} \propto \Delta\boldsymbol{\nabla}^\perp \cdot \boldsymbol{u}$$

Testbed data:

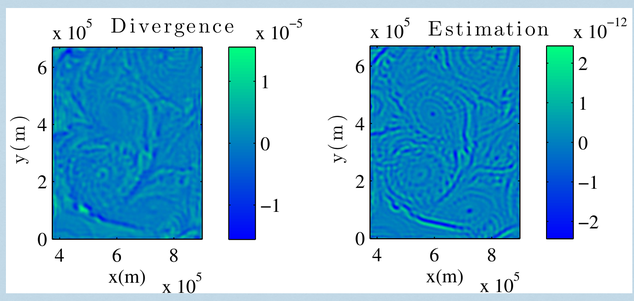
very high-resolution model outputs

Gula, J., Molemaker, J., and McWilliams J.
"Gulf Stream dynamics along the southeastern US seaboard."
Journal of Physical Oceanography 45.3 (2015): 690-715.

Original / Filtered



Results



Conclusion

- Frontolysis / frontogenesis on warm / cold side of fronts
- Estimation of horizontal divergence or diffusive coefficient and subgrid variance
- Strong uncertainty assumption verified *a posteriori* for the Gulf Stream during wintertime at mesoscales