# Do we need to enforce the homogeneous Neumann condition on the torso for solving the inverse electrocardiographic problem?

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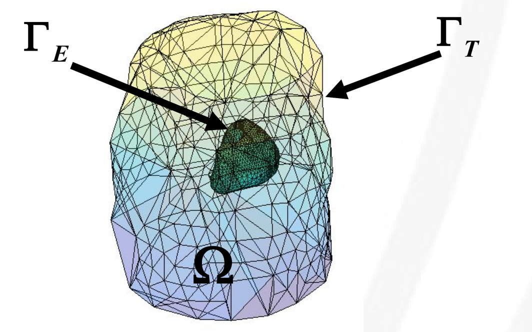
## Introduction

- Robust calculations of the inverse electrocardiographic problem may require accurate specification of boundary conditions at the torso and cardiac surfaces [1].



*Geometrical meshes used: The body is cut at the top and bottom of the torso and the arms.*

$-div(\phi)$, in $\Omega$. Under:

Dirichlet conditions: $\phi=\phi_T$ on $\Gamma_T$ (1)

Homogeneous Neumann conditions: $\partial_n\phi=0$ on $\Gamma_T$

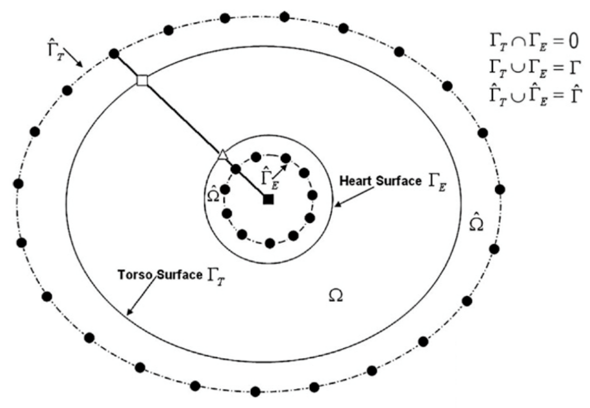
The aim of the inverse problem is to find $\phi=\phi_E$ on $\Gamma_E$

- The classical formulation of the ECGI inverse problem with the method of fundamental solution (MFS) involves a linear system[2]. That can be split into two submatrices:

$$\begin{array}{l} \text{Dirichlet Conditions } B_0 \\ \\ \text{Neumann Conditions } B_1 \end{array}
\begin{pmatrix}
1 & f(\|x_1-y_1\|) & \cdots & f(\|x_1-y_M\|) \\
\vdots & & \ddots & \vdots \\
1 & f(\|x_N-y_1\|) & \cdots & f(\|x_N-y_M\|) \\
0 & \frac{\partial f(\|x_1-y_1\|)}{\partial n} & \cdots & \frac{\partial f(\|x_1-y_M\|)}{\partial n} \\
\vdots & & \ddots & \vdots \\
0 & \frac{\partial f(\|x_N-y_1\|)}{\partial n} & \cdots & \frac{\partial f(\|x_N-y_M\|)}{\partial n}
\end{pmatrix}
*
\begin{pmatrix} a_0 \\ a_1 \\ \vdots \\ a_M \end{pmatrix}
=
\begin{pmatrix} \phi_T(x_1) \\ \vdots \\ \phi_T(x_N) \\ 0 \\ \vdots \\ 0 \end{pmatrix} \quad (2)$$

$$x_i\in\Gamma_T,\ y_j\in\hat{\Gamma}_T\cup\hat{\Gamma}_E \qquad f(\|x_i-y_j\|)=\frac{1}{4\pi\|x_i-y_j\|}$$

$$\phi_E(x)=a_0+\sum_{j=1}^{M}a_j f(\|x-y_j\|),\ x\in\Gamma_E,\ y_j\in\hat{\Gamma}_T\cup\hat{\Gamma}_E \quad (3)$$



*Simplified view of geometrical boundaries and pseudo-boundaries took into account in the MFS formulation [2]*

- We can remark that:
- The contribution of the submatrix $B_1$ and $B_0$ in the whole solution is related to their respectives norms.
- Due to the meshing of the torso, non-phisiological conditions are applied to the top, bottom of the torso and the arms.
- Accurate computation of the normals at torso surface are required.

- These remarks drove us to the question: Do we need to enforce the homogeneous Neumann conditions in the MFS inverse problem?

- To solve this question:
- We introduced a weighting coefficient in the cost function (*Method 1*), so as to change the balance between the contributions of the Neumann and Dirichlet conditions.
- We proposed to consider the Neumann conditions as a constraint (*Method 2*), so that it is resolved exactly.

## Methods

1. **Minimization with weighed boundary conditions**

- To evaluate the contribution of both boundary conditions:

$$J_\lambda(a)=\frac{1}{2}\left((1-\lambda)^2\|B_0a-\phi_T\|_2^2+\lambda\|B_1a\|_2^2+\alpha\|a\|_2^2\right),\ \lambda\in[0,1] \quad (4)$$

The solution of (4) was found by solving the following system using Tikhonov regularization

$$\begin{pmatrix}(1-\lambda)B_0\\ \lambda B_1\end{pmatrix}a=\begin{pmatrix}\lambda\phi_T\\ 0\end{pmatrix} \quad (5)$$

- Choosing:
  - $\lambda=0.5$ corresponds to standard MFS.
  - $\lambda=0$ corresponds to MFS without Neumann conditions.
  - $\lambda=\frac{1}{1+\varepsilon}$, where $\varepsilon=\|B_1\|/\|B_0\|$ provides a balance between both submatrices.

2. **Minimization with the Neumann conditions as constraint**

- The Neumann conditions can be introduced as a homogeneous equality constraint. In this case, we look for $a\in\mathbb{R}^{1+N_S}$ that minimizes

$$J_0(a)=\frac{1}{2}\left(\|B_0a-\phi_T\|_2^2+\alpha\|a\|_2^2\right), \text{s.t. } B_1a=0 \text{ in } \mathbb{R}^{N_T} \quad (6)$$

This is a saddle-point problem and we can introduce its Lagrangian

$$L(a,\mu)=\frac{1}{2}\|B_0a-\phi_T\|_2^2+\frac{1}{2}\alpha\|a\|_2^2+\mu^TB_1a,\ \mu\in\mathbb{R}^{N_T} \quad (7)$$

The solution is found by solving the block linear system

$$\begin{pmatrix}B_0^TB_0+\alpha Id & B_1^T\\ B_1 & 0\end{pmatrix}\begin{pmatrix}a\\ \mu\end{pmatrix}=\begin{pmatrix}B_0^T\phi_T\\ 0\end{pmatrix} \quad (8)$$

We solve this problem with SDPT3, a primal-dual path-following algorithm available on CVX package for convex optimization [3].

- **Comparisons**

- Five **activation patterns** were simulated : *1 single site pacing* (LV lateral endocardial – lasts 500ms), and *4 single spiral waves* with different conductivity coefficients (lasts 3000ms). Simulations were conducted using [4]:

i. The propagation of electrical activity in a membrane-based **realistic-geometry** computer model, using the governing **monodomain reaction-diffusion equation**. The simulations provide both the theoretical, *in-silico* $\phi_T$ and $\phi_E$ every 1ms.

ii. The torso model used had **heterogeneous conductivity**.

- Correlation coefficients (CC) and relative errors (RE) for activation times and electrograms were calculated for solutions of *method 1* with $\lambda=0$, $\lambda=0.5$, $\lambda=\frac{1}{1+\varepsilon}$ where $\varepsilon=\|B_1\|/\|B_0\|$, and *method 2*.

## Results

- **Respective contribution of B0 and B1**

- The $\varepsilon=\|B_1\|/\|B_0\|$ depends only on the geometry and location of sources (for which we used a fixed rule defined in [2]). We found $\varepsilon=0.0013$, indicating predominance of $B_0$ compared to $B_1$ in the standard MFS.
- Choosing $\lambda=1/(1+\varepsilon)=0.9987$ in equation (4) leads to a balance between the contribution of the two submatrices.

- **Comparison between the reconstructions**

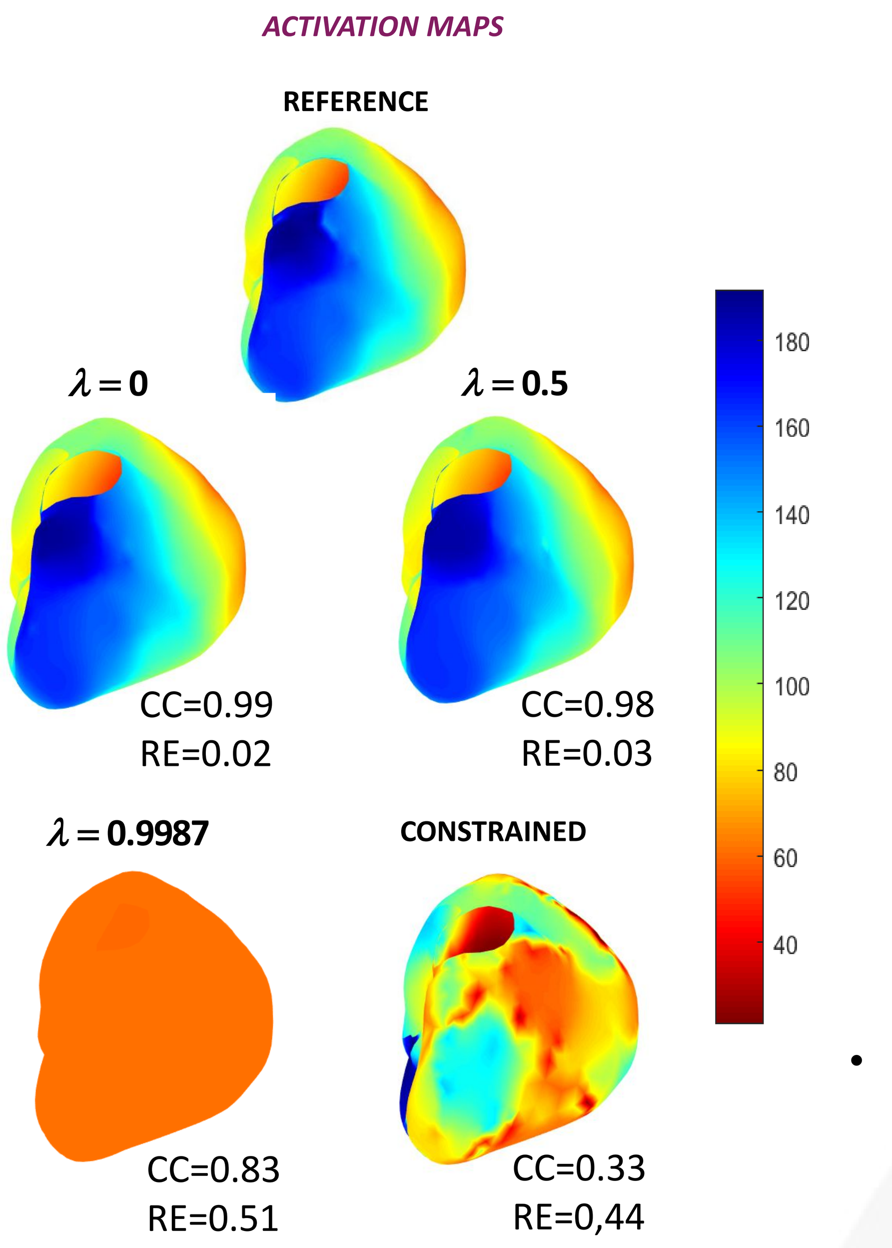


*Figure 1. Activation maps for electrograms, single site pacing.*

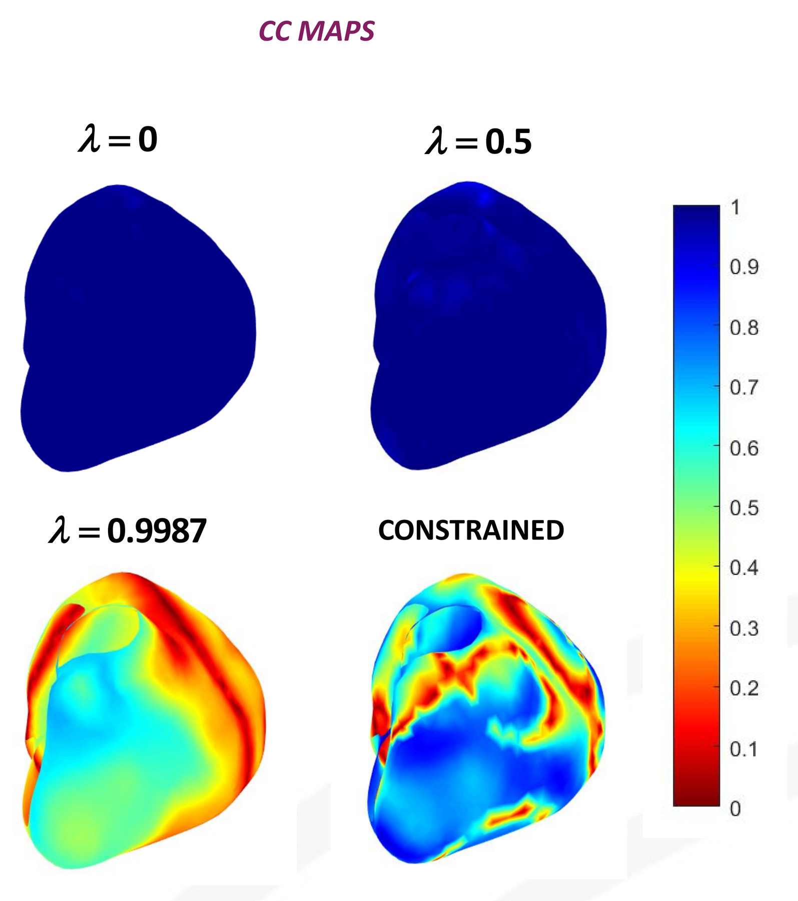


*Figure 2. CC maps for electrograms, single site pacing.*

- The best activation time and CC maps were achieved by $\lambda=0$ (MFS without Neumann conditions) and $\lambda=0.5$ (Standard MFS).

| COMPARISON OF ELECTROGRAMS | | | | | | |
|---|---|---|---|---|---|---|
| | CC | | | RE | | |
| | $Q_1$ | Median | $Q_3$ | $Q_1$ | Median | $Q_3$ |
| $\lambda=0$ | 0,999 | 0,999 | 0,999 | 0,020 | 0,034 | 0,054 |
| $\lambda=0.5$ | 0,989 | 0,995 | 0,998 | 0,071 | 0,107 | 0,152 |
| $\lambda=0.9987$ | 0,258 | 0,435 | 0,58 | 0,997 | 0,999 | 1,001 |
| Constrained | 0,472 | 0,688 | 0,803 | 0,762 | 0,981 | 1,358 |

*Table 2. CC and RE for electrograms, five simulations.*

## Conclusions

- The results of MFS without Neumann condition are in better agreement with the original data. Furthermore, removing the Neumann conditions reduces the forward and inverse computational cost, since the size of the linear system is divided by 2 and the normal vectors do not need to be computed. The ill-posedness of the problem is reduced and consequently the problem is less sensible to the regularization.

- The Lagrange multipliers of *method 2* were significantly higher on the electrodes on the front of the torso than on the ones on the back. This seems to indicate that partial or weighed Neumann conditions (depending on the position of the electrodes) can be explored further in order to refine the inverse problem and improve the resolution of the ECGI problem.

- Note that the results presented here only apply to the MFS method. However, preliminary work seems indicate that homogeneous Neumann conditions at the torso surface have a positive impact on Boundary element method.

- Finally, we used only 5 ventricular datasets obtained by a numerical model. The case of complex atrial activations, and clinical data must be considered for a complete evaluation of the techniques proposed.

## Acknowledgements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