Constant Delivery Delay Protocol Sequences for the Collision Channel Without Feedback

Lou Salaun, Chung Shue Chen, Yi Chen, Wing Shing Wong

Constant Delivery Delay Protocol Sequences for the Collision Channel Without Feedback

Lou Salaün, Chung Shue Chen
Bell Labs, Nokia
Centre de Villarceaux, 91620 Nozay, France
{lou.salaun, chung_shue.chen}@nokia.com

Yi Chen, Wing Shing Wong
Department of Information Engineering
The Chinese University of Hong Kong, Shatin, Hong Kong
{yichen, wswong}@ie.cuhk.edu.hk

***Abstract*—We consider a collision channel model without feedback based on a time-slotted communication channel shared by K users. In this model, packets transmitted in the same time slot collide with each other and are unrecoverable. Each user accesses the channel according to an internal periodical pattern called protocol sequence. Due to the lack of feedback, users cannot synchronize their protocol sequences, leading to unavoidable collisions and varying throughput. Protocol sequences that provide constant throughput regardless of delay offsets between users are called shift-invariant (SI), they have been studied and characterized in previous work. We propose a new class of SI sequences: Constant Individual Delivery Delay (CIDD) sequences which ensure that the delay between two successfully delivered packets is constant for each user. We present a characterization of CIDD sequences. We also prove that CIDD sequences can achieve the lower bound of SI sequences period but not the optimal throughput.**

***Keywords*—Collision channel without feedback, protocol sequences, wireless ad-hoc networks, multiple access.**

I. Introduction

Many research activities have been conducted in the context of multiple access communications in which several users access a shared channel. Without synchronization, users might transmit packets at the same time causing collisions and the loss of packets. A possible solution is to use protocol that achieves collision-free transmission such as the well known *time division multiple access* (TDMA). However, in communication systems such as impulse radio [1], wireless sensor networks [2] and ad hoc mobile networks [3], devices have constrained resource, limited communication power and need a flexible transmission scheme. Collision-free protocols such as TDMA may not be practical for these systems, since it requires stringent time synchronization. Other contention based random access protocols such as IEEE 802.11 CSMA/CA [4] can provide a more flexible transmission scheme. However, they still require some backoff algorithms and a feedback link which may not be practical for these low resource devices. These systems require simple multiple access protocol with no stringent time synchronization, frequent channel monitoring and feedback link.

Such a *collision channel without feedback* model was introduced by Massey and Mathys in [5]. In this model without feedback mechanism, senders cannot synchronize their transmission schedule, implying relative time offsets between users are unavoidable and unknown for them. They also proposed the idea of using *protocol sequences* as medium access control (MAC) schemes. Each user transmission schedule follows their protocol sequence. Many protocol sequences have been studied in the literature. One can consider slotted ALOHA [6] as a MAC using *probabilistic* protocol sequences, where each user transmits a packet with probability p independently. Other protocol sequences are deterministic, such as constant-weight cyclically permutable codes [7], [8], linear congruence sequences [9], prime sequences [10], [11], wobbling sequences [12] and CRT sequences [13]. See [14] for a survey on these coding techniques.

Important features of protocol sequences are the sequence length (which should be as low as possible, *e.g.*, for devices with memory limitations) and the resulting throughput. Some sequences have non-zero throughput regardless of relative delay offsets between users, this property was studied in [15] as *user unsuppressibility*. Moreover, an important class of protocol sequences ensure constant throughput, independent of any relative delay offsets. This class is called *shift-invariant protocol sequences* (SI), they have been introduced in [5] and studied in [16]–[18]. Even if this class enjoys a zero-variance shift-invariant throughput, its main disadvantage is that its sequence length grows exponentially with the number of users.

In this paper, we propose a new class of SI sequences called *Constant Individual Delivery Delay* (CIDD) protocol sequences which ensure not only a constant throughput but also a constant delay between each successfully delivered packet of each user. After setting up the channel model and required notations in Section II, we show two important properties of SI sequences in Section III. In Section IV, we develop some properties of CIDD protocol sequences and prove its characterization theorem. We show that this class contains only a small number of instances. Furthermore, it does not achieve the optimal throughput of SI sequences except for the two users case. In Section V, we introduce *Uniform Individual Delay* (UID) protocol sequences, and we prove that they are equivalent to CIDD sequences. Finally, a brief conclusion is presented in Section VI.

II. Channel Model and Notations

We consider the collision channel without feedback model introduced in [5] in which K users share the same communication channel. Time is divided into discrete time slots and is denoted by $t \in \mathbb{N}$. We suppose that each user $i \in [\![1;K]\!]$ is aware of time slots boundaries and chooses to transmit following a periodic binary sequence $\boldsymbol{s}_i(t)$, namely *protocol sequence*.

$$\boldsymbol{s}_i(t) = \begin{cases} 1 \text{ , user } i \text{ transmits a packet at time slot } t \\ 0 \text{ , user } i \text{ keeps silent at time slot } t \end{cases}$$

Note that it is possible to remove the slot synchronized condition and have user transmitting in continuous time. Some approaches and discussions are presented in [5], [19], [20]. However, we will not address it in this paper.

At any time slot t, if more than one user transmit, a collision will occur and all packets transmitted in this time slot are lost. Since this model suppose no feedback mechanism, users are not aware of the packets lost.

Let $\boldsymbol{s}_1, \ldots, \boldsymbol{s}_K$ be binary sequences of least common period L. Since users cannot synchronize their protocol sequences, a realization will have delay offsets $\tau_1, \ldots, \tau_K \in [\![0; L-1]\!]$ between users. We call *cyclic shift* of $\boldsymbol{s}_i$ by τ_i the sequence defined as

$$\forall\, t \in \mathbb{N},\ \boldsymbol{s}_i^{(\tau_i)}(t) = s_i(t \oplus \tau_i)$$

where $\oplus$ represents addition modulo L. The following example illustrates the above notations used in this paper.

Example 1. *In this example, the protocol sequence $\boldsymbol{s}_i$ has period $L = 6$, where $\boldsymbol{s}_i(0) = \boldsymbol{s}_i(5) = 1$, $\boldsymbol{s}_i(3) = 0$ and $*$ denotes either 0 or 1 (value not specified, used for examples). $s_i^{(1)}$ and $s_i^{(2)}$ have $\tau_i = 1$ and $\tau_i = 2$ with respect to $s_i^{(0)}$.*

$$\begin{array}{cccccccc} \boldsymbol{s}_i^{(0)}: & 1 & * & * & 0 & * & 1 \\ \boldsymbol{s}_i^{(1)}: & 1 & 1 & * & * & 0 & * \\ \boldsymbol{s}_i^{(2)}: & * & 1 & 1 & * & * & 0 \end{array}$$

Given the delay offsets $\tau_1, \ldots, \tau_K$, we say that user i successfully delivers a packet at time t, if and only if

$$\boldsymbol{s}_i^{(\tau_i)}(t) \prod_{j \neq i} (1 - s_j^{(\tau_j)}(t)) = 1.$$

In the next section, we will define shift-invariant protocol sequences.

III. Properties of Shift-Invariant Protocol Sequences

Definition 2 (Shift Invariant Protocol Sequences)**.** *Let $\boldsymbol{s}_1, \ldots, \boldsymbol{s}_K$ be binary sequences of least common period L. $\{\boldsymbol{s}_1, \ldots, \boldsymbol{s}_K\}$ is said to be shift-invariant if each of its user's throughput is constant for every choice of offsets $\tau_1, \ldots, \tau_K$.*

$$\forall\, i \in [\![1;K]\!],\ \exists\, r_i > 0,\ \forall\, \tau_1, \ldots, \tau_K,$$
$$\frac{1}{L} \sum_{t=0}^{L-1} \boldsymbol{s}_i^{(\tau_i)}(t) \prod_{j \neq i} (1 - s_j^{(\tau_j)}(t)) = r_i$$

where r_i denotes user i's throughput.

Definition 3 (Duty Factor)**.** *Let $\{\boldsymbol{s}_1, \ldots, \boldsymbol{s}_K\}$ be a SI sequence set of least common period L. The duty factor f_i is the fraction of time slots in which user i transmits.*

$$f_i = \frac{n_i}{d_i} = \frac{1}{L} \sum_{t=0}^{L} \boldsymbol{s}_i(t)$$

where $n_i \geq 0,\ d_i \geq 0,\ n_i \leq d_j$, so that $f_i \in [0;1] \cap \mathbb{Q}$.

In this paper, we will suppose $n_i \geq 1,\ d_i \geq 2$ and $n_i < d_j$ to remove the trivial cases where either a user does not transmit at all or transmits in every time slot.

The following Theorem 4 and Theorem 5 are from [17], they describe the achievable throughput and the least common period of SI sequences. Their proofs can be found in [17].

Theorem 4 (Characterization of SI throughput)**.** *Let $\{\boldsymbol{s}_1, \ldots, \boldsymbol{s}_K\}$ be a SI sequence set with duty factors $f_1, \ldots, f_K$ respectively. The throughput of user i is equal to*

$$f_i \prod_{j \neq i} (1 - f_j)\,. \tag{1}$$

When all K users have the same duty factor f, the optimal (maximal) throughput is achieved by setting $f = 1/K$.

Theorem 5 (Characterization of SI period)**.** *For any set of K SI sequences with duty factors $n_1/d_1, \ldots, n_K/d_K$, such that $gcd(n_i, d_i) = 1$ for all i, the least common period L is divisible by $\prod_{i=1}^{K} d_i$. In particular, the minimal period for given $d_1, \ldots, d_K$ is $\prod_{i=1}^{K} d_i$.*

IV. Constant Individual Delivery Delay Sequences

We propose a new class of deterministic protocol sequences called *Constant Individual Delivery Delay* (CIDD) sequences. The main idea is to have not only a constant throughput like SI sequences, but also to ensure a constant delay between each packet delivered by a user, meaning that user i's packets are exactly successfully delivered every D_i time slots, in a periodic manner of period D_i.

Definition 6 (Constant Individual Delivery Delay Sequences)**.** *A binary sequence set $\{\boldsymbol{s}_1, \ldots, \boldsymbol{s}_K\}$ of least common period L has CIDD if each user i's delay between two successive delivered packets is constant equal to a certain value $D_i \in \mathbb{N}^*$.*

$$\forall\, i \in [\![1;K]\!], \forall\, \tau_1, \ldots, \tau_K \in \mathbb{N}, \exists\, t \in \mathbb{N},$$
$$\forall\, t' \in \{t + qD_i\}_{q \in \mathbb{Z}}\,,\ \boldsymbol{s}_i^{(\tau_i)}(t') \prod_{j \neq i} (1 - s_j^{(\tau_j)}(t')) = 1,$$
$$\forall\, t' \notin \{t + qD_i\}_{q \in \mathbb{Z}}\,,\ \boldsymbol{s}_i^{(\tau_i)}(t') \prod_{j \neq i} (1 - s_j^{(\tau_j)}(t')) = 0,$$

where D_i is the constant delivery delay of user i. Note that D_i must divide L for all i.

Let $\boldsymbol{s}_1, \ldots, \boldsymbol{s}_K$ be binary sequences of least common period L. Suppose that $\{\boldsymbol{s}_1, \ldots, \boldsymbol{s}_K\}$ has constant individual delivery delays of D_i for each user $i \in [\![1;K]\!]$. We first derive the following properties, then prove characterization Theorem 10.

Lemma 7. *$\{\boldsymbol{s}_1, \ldots, \boldsymbol{s}_K\}$ is SI with throughput of $1/D_i$ for each user $i \in [\![1;K]\!]$.*

Proof: By definition of CIDD, each user $i \in [\![1;K]\!]$ delivers one packet exactly one time per D_i time slots. Thus their throughputs are respectively $1/D_i$ and are shift invariant. □

Since $\{\boldsymbol{s}_1, \ldots, \boldsymbol{s}_K\}$ is SI with throughput $1/D_i > 0$, in the following proofs we will use the fact that for each user $i \in [\![1;K]\!]$, there exists offsets $\tau_1, \ldots, \tau_K$ such that user i delivers a packet at time 0, *i.e.*, $\boldsymbol{s}_i^{(\tau_i)}(0) = 1$ and $\forall\, j \neq i, \boldsymbol{s}_j^{(\tau_j)}(0) = 0$.

Lemma 8. *Each $\boldsymbol{s}_i$ is periodic with period D_i.*

Proof: Let $i \in [\![1;K]\!]$. Suppose that $\tau_1, \ldots, \tau_K$ are chosen such that user i delivers a packet at time 0. That is

$$\boldsymbol{s}_i^{(\tau_i)}(0) = 1,$$
$$\forall\, j \neq i,\ \boldsymbol{s}_j^{(\tau_j)}(0) = 0.$$

Since $\boldsymbol{s}_i$ is CIDD, we have, for all positive integer q

$$\boldsymbol{s}_i^{(\tau_i)}(qD_i) = 1,$$
$$\forall\, j \neq i,\ \boldsymbol{s}_j^{(\tau_j)}(qD_i) = 0.$$

Suppose for the sake of contradiction that $\boldsymbol{s}_i$ is not D_i-periodic. Then

$$\exists\, \tau_i' \in [\![0; L-1]\!], \exists\, q' \in [\![1; L/D_i - 1]\!],$$
$$\boldsymbol{s}_i^{(\tau_i')}(0) = 1,$$
$$\boldsymbol{s}_i^{(\tau_i')}(q'D_i) = 0.$$

Under the offsets $\tau_1, \ldots, \tau_{i-1}, \tau_i', \tau_{i+1}, \ldots, \tau_K$, we have

$$\boldsymbol{s}_i^{(\tau_i')}(0) = 1 \text{ and } \forall\, j \neq i, \boldsymbol{s}_j^{(\tau_j)}(0) = 0, \tag{2}$$
$$\boldsymbol{s}_i^{(\tau_i')}(q'D_i) = 0. \tag{3}$$

User i delivers a packet at time 0 but fails to deliver a packet at time $q'D_i$, see Eqn. (2) and (3), respectively. This contradicts the fact that $\boldsymbol{s}_i$ has constant delivery delay of D_i. Thus, $\boldsymbol{s}_i$ is D_i-periodic. □

The following example (4) shows a case of the proof by contradiction with 3 users i, j, k, where $D_i = 6$ and the non-periodic pattern is at $\tau_i' = 2$ and $q = 1$.

$$\begin{aligned}
\boldsymbol{s}_i^{(0)} &: 1\ *\ *\ *\ 0\ *\ 1\ *\ *\ *\ 1\ * \\
\boldsymbol{s}_j^{(0)} &: 0\ *\ *\ *\ *\ *\ 0\ *\ *\ *\ *\ * \\
\boldsymbol{s}_k^{(0)} &: 0\ *\ *\ *\ *\ *\ 0\ *\ *\ *\ *\ * \\
\\
\boldsymbol{s}_i^{(\tau_i')} &: 1\ *\ 1\ *\ *\ *\ 0\ *\ 1\ *\ *\ * \\
\boldsymbol{s}_j^{(0)} &: 0\ *\ *\ *\ *\ *\ 0\ *\ *\ *\ *\ * \\
\boldsymbol{s}_k^{(0)} &: 0\ *\ *\ *\ *\ *\ 0\ *\ *\ *\ *\ *
\end{aligned} \tag{4}$$

Note that D_i is not necessarily the fundamental period of $\boldsymbol{s}_i$. Its fundamental period is a divisor of D_i.

Lemma 9. *Each D_i is a common period of the sequence set $\{\boldsymbol{s}_1, \ldots, \boldsymbol{s}_K\}$.*

Proof: Let $i, j \in [\![1;K]\!]$, $i \neq j$. Suppose that $\tau_1, \ldots, \tau_K$ are chosen such that user i delivers a packet at time 0. Since $\boldsymbol{s}_i$ has constant delivery delay of D_i, we know that

$$\forall\, q \in [\![0; L/D_i - 1]\!],\ \boldsymbol{s}_i^{(\tau_i)}(qD_i) = 1,$$
$$\forall\, l \neq i,\ \boldsymbol{s}_l^{(\tau_l)}(qD_i) = 0.$$

Using the same argument as in Lemma 8, we will show that user j is also D_i-periodic. Suppose for the sake of contradiction that $\boldsymbol{s}_j$ is not D_i-periodic. Then

$$\exists\, \tau_j' \in [\![0; L-1]\!], \exists\, q' \in [\![1; L/D_i - 1]\!],$$
$$\boldsymbol{s}_j^{(\tau_j')}(0) = 1,$$
$$\boldsymbol{s}_j^{(\tau_j')}(q'D_i) = 0.$$

Therefore, under the offsets $\tau_1, \ldots, \tau_j', \ldots, \tau_K$, user i delivers a packet at time $q'D_i$ but not at time 0. This contradicts the constant delivery delay property of user i.

We prove that for all $i, j \in [\![1;K]\!]$, user j is D_i-periodic. It follows that each D_i is a common period of the sequence set $\{\boldsymbol{s}_1, \ldots, \boldsymbol{s}_K\}$. □

Theorem 10 (Characterization of CIDD Sequences)**.** *Let $\boldsymbol{s}_1, \ldots, \boldsymbol{s}_K$ be binary sequences of least common period L. The following conditions are equivalent.*

1) *The sequence set $\{\boldsymbol{s}_1, \ldots, \boldsymbol{s}_K\}$ has constant individual delivery delay (CIDD) of D_i for each user $i \in [\![1;K]\!]$.*
2) *The sequence set $\{\boldsymbol{s}_1, \ldots, \boldsymbol{s}_K\}$ is SI, such that $L = 2^K$ and $\forall\, i \in [\![1;K]\!]$, $f_i = 1/2$, $D_i = L$.*

Proof:

1) ⇒ 2). Since L is the least common period of $\{\boldsymbol{s}_1, \ldots, \boldsymbol{s}_K\}$ and each D_i is a common period (from Lemma 9), we know that L divides D_i. We also know that, by definition, D_i divides L. Hence, $D_i = L$ for all i.

Eqn. (5) follows from Lemma 7 and Eqn. (1) in Theorem 4.

$$\forall\, i \in [\![1;K]\!],\ \frac{n_i}{d_i} \prod_{j \neq i} \left(1 - \frac{n_j}{d_j}\right) = \frac{1}{D_i} = \frac{1}{L}. \tag{5}$$

Using Theorem 5, we know that $\prod_{i=1}^{K} d_i$ divides L. Therefore, (5) is equivalent to (6), with $\alpha_i \in \mathbb{N}^*$.

$$\forall\, i \in [\![1;K]\!],\ n_i \prod_{j \neq i} (d_j - n_j) = \frac{1}{\alpha_i}. \tag{6}$$

The left hand side of (6) is a positive integer, so $\alpha_i = 1$ for each $i \in [\![1;K]\!]$. It follows that the unique solution is $n_i = 1,\ d_i = 2$ for each $i \in [\![1;K]\!]$.

From this solution and Lemma 7, we conclude that $\{\boldsymbol{s}_1, \ldots, \boldsymbol{s}_K\}$ is SI with duty factor $f_i = n_i/d_i = 1/2$ and delivery delay $D_i = L = \alpha_i \prod_{i=1}^{K} d_i = 2^K$ for each user i.

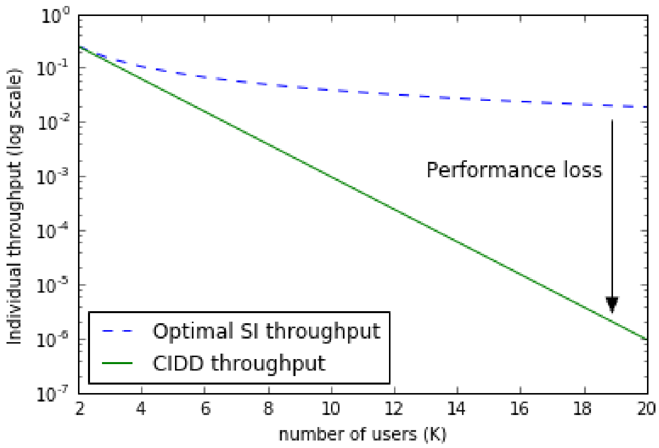

Figure 1. Optimal SI throughput and CIDD throughput comparison in a symmetric system (each user has the same duty factor) as a function of the number of users K.

2)$\Rightarrow$1). From (1) the throughput of user i is $f_i \prod_{j\neq i}(1-f_j) = 1/2^K$. By definition of SI sequences, each user i will send exactly one packet per period $L = 2^K$ regardless of the offsets. Since L is the least common period, user i's delivery delay is constant and $D_i = L$. □

Theorem 10 shows that the CIDD sequences class is limited to a small number of instances. Indeed, for a K-users system with CIDD protocol sequences, each user throughput is $1/2^K$, and their period (sequence length) is 2^K. According to Theorem 5, CIDD sequences achieve the minimal SI sequences period. However, according to Theorem 4, SI sequences optimal throughput is $(K-1)^{K-1}/K^K$ which is achieved by duty factor $f = 1/K$ while CIDD sequences throughput is $1/2^K$. Hence, CIDD sequences throughput is optimal only for $K = 2$ and its performance drop increases exponentially with K, as we can see on Fig. 1. In Section VI, we will discuss some special structures that allow to reduce the system complexity K in order to have better throughput performance.

In this paper, we have not dealt with explicitly constructing CIDD instances. However, [17] proposed simple methods to generate minimal period SI sequences for given duty factors and number of users. In particular, using duty factors $f = 1/2$, it follows from Theorem 10 that it is possible to generate every possible CIDD sequences.

V. Uniform Individual Delay Protocol Sequences

User i's *individual delay* is the duration between $t = 0$ and its first delivered packet time [21]. Suppose that user offsets $\tau_1, \ldots, \tau_K$ are random variables following an independent discrete uniform distribution $\sim U(0, L-1)$, then individual delays follow a non-increasing distribution. In this section we study a particular class of protocol sequences that has *uniform individual delay* (UID) over all possible offsets. We also define UID sequences to be user unsuppressible, *i.e.*, each user delivers at least one packet regardless of offsets. Thus, excluding trivial cases of all-ones or all-zeros sequences. We first introduce some notations and combinatorial properties, then prove that UID sequences are equivalent to CIDD sequences in Theorem 13.

The idea is to count the number of realizations of each individual delays over all the possible offsets. For that, we will reduce all $\tau_1, \ldots, \tau_K$ combinations to equivalence classes. Let $N_i(delay = t)$ be the number of realizations in which user i has its individual delay equal to t. We define the following equivalence relation

$$\forall\, (\tau_i)_{i=1..K}, (\tau_i')_{i=1..K},$$
$$(\tau_i)_i \sim (\tau_i')_i \iff \exists\, t \in [\![0; L-1]\!], (\tau_i)_i = (\tau_i' \oplus t)_i. \quad (7)$$

The equivalence relation (7) induces several equivalence classes

$$[(\tau_i)_i] = \{(\tau_i')_i \,|\, (\tau_i)_i \sim (\tau_i')_i\} = \{(\tau_i \oplus t)_i,\, t \in [\![0; L-1]\!]\} \quad (8)$$

which represents all offsets that are equal to $(\tau_i)_i$ ignoring cyclic shift (cyclic shift of the same value t over all offsets: $\tau_1 \oplus t, \ldots, \tau_K \oplus t$). We then gather these equivalence classes (8) in a quotient set $X/\!\sim$. Since τ_i takes a finite number of values $[\![0; L-1]\!]$, $X/\!\sim$ is also finite. Hence, simplifying the calculation of delays counting with

$$N_i(delay = t) = \sum_{[(\tau_i)_i] \in X/\sim} N_i(delay = t \,|\, [(\tau_i)_i])$$

where $N_i(delay = t \,|\, [(\tau_i)_i])$ is the number of realizations of $delay = t$ over every offsets of $[(\tau_i)_i]$ equivalence class.

Definition 11 (Uniform Individual Delay Protocol Sequences)**.** *A binary sequence set $\{\boldsymbol{s}_1, \ldots, \boldsymbol{s}_K\}$ has UID if, and only if, for all user $i \in [\![1; K]\!]$, there exists a threshold $T > 0$ such that*

$$\forall\, t < T,\ N_i(delay = t) = N_i(delay = 0)$$

and

$$\forall\, t \geq T,\ N_i(delay = t) = 0.$$

Lemma 12. *Let $\{\boldsymbol{s}_1, \ldots, \boldsymbol{s}_K\}$ be a set of protocol sequences with user unsuppressible property (i.e., each user delivers at least one packet regardless of offsets). Let $D_{max}(i, [(\tau_l)_l])$ and $D_{min}(i, [(\tau_l)_l])$ be respectively the maximum and minimum delivery delay (as defined in Section IV) of user i with offsets equivalence class $[(\tau_l)_l]$. For simplicity, we use a shortened notation: D_{max}, D_{min}. They are well-defined since we suppose $\{\boldsymbol{s}_1, \ldots, \boldsymbol{s}_K\}$ is user unsuppressible and periodic. For all $i \in [\![1; K]\!]$, $[(\tau_l)_l] \in X/\!\sim$, we have the following properties:*

(a) $N_i(delay = t \,|\, [(\tau_l)_l])$ is a non-increasing function of t.

$$\forall\, t,\ N_i(delay = t+1 \,|\, [(\tau_l)_l]) \leq N_i(delay = t \,|\, [(\tau_l)_l])$$

(b) Its first D_{min} values for $t = 0\ \ldots\ D_{min}-1$ are equal, expressible as (b'), and its value at $t = D_{min}$ is different, expressible as (b'').

(b'):

$$\forall\, t < D_{min},\ N_i(delay = t \,|\, [(\tau_l)_l]) = N_i(delay = 0 \,|\, [(\tau_l)_l])$$

(b″):

$$N_i(delay = D_{min} \mid [(\tau_l)_l]) < N_i(delay = D_{min} - 1 \mid [(\tau_l)_l])$$

(c) Its values for $t \geq D_{max}$ are equal to zero, expressible as (c′). For $t < D_{max}$ its values are not equal to zero, expressible as (c″).

$$(c'):\quad \forall\, t \geq D_{max},\ N_i(delay = t \mid [(\tau_l)_l]) = 0$$

$$(c''):\quad N_i(delay = D_{max} - 1 \mid [(\tau_l)_l]) \neq 0$$

Proof: Let L be the least common period of $\{\boldsymbol{s}_1, \ldots, \boldsymbol{s}_K\}$. Let $i \in [\![1; K]\!]$, $[(\tau_l)_l] \in X/\!\sim$. Each element of $[(\tau_l)_l]$ is a cyclic shift of $(\tau_l)_l$, so they all have the same delivery pattern ignoring cyclic shift. We define $\boldsymbol{O}_t^{[(\tau_l)_l]}$ as the subset of $[(\tau_l)_l]$ that achieves individual delay of t for user i. As a consequence,

$$N_i(delay = t \mid [(\tau_i)_i]) = |\boldsymbol{O}_t^{[(\tau_l)_l]}|$$

where $|\,.\,|$ denotes the cardinality of a finite set.

Proof of (a): Every offsets in $[(\tau_l)_l]$ give the same delivery pattern ignoring cyclic shift. Let $(\tau_l + \tau + 1)_l \in X/\!\sim$ be such that user i's delay is $t_{delay} + 1 > 0$. That is

$$\begin{aligned} &\boldsymbol{s}_i^{(\tau_i+\tau+1)}(t_{delay}+1) \prod_{j\neq i}(1 - s_j^{(\tau_j+\tau+1)}(t_{delay}+1)) = 1, \\ &\forall\, t < t_{delay}+1,\ \boldsymbol{s}_i^{(\tau_i+\tau+1)}(t) \prod_{j\neq i}(1 - s_j^{(\tau_j+\tau+1)}(t)) = 0. \end{aligned} \tag{9}$$

Then, if we consider offsets $(\tau_l + \tau)_l \in X/\!\sim$, we have

$$\begin{aligned} &\boldsymbol{s}_i^{(\tau_i+\tau)}(t_{delay}) \prod_{j\neq i}(1 - s_j^{(\tau_j+\tau)}(t_{delay})) = 1, \\ &\forall\, t < t_{delay},\ \boldsymbol{s}_i^{(\tau_i+\tau)}(t) \prod_{j\neq i}(1 - s_j^{(\tau_j+\tau)}(t)) = 0. \end{aligned} \tag{10}$$

Thus, user i's delay with offsets $(\tau_l + \tau)_l$ is t_{delay}. In sum, we have proven that for every user i, for any $(\tau_l + \tau + 1)_l \in X/\!\sim$ with delay $t_{delay} + 1 > 0$, there is $(\tau_l + \tau)_l \in X/\!\sim$ with delay t_{delay} (the converse (10) $\Rightarrow$ (9) is not necessarily true due to the pattern's cyclic shift).

Moreover, if we take two different offsets $(\tau_l + \tau + 1)_l$ and $(\tau_l + \tau' + 1)_l$ in $X/\!\sim$ which realize delay $t_{delay} + 1$, then $(\tau_l + \tau)_l$ and $(\tau_l + \tau')_l$ are also different. We infer that there exists an injective function from $\boldsymbol{O}_{t+1}^{[(\tau_l)_l]}$ to $\boldsymbol{O}_t^{[(\tau_l)_l]}$. As a consequence, $N_i(delay = t_{delay} + 1 \mid [(\tau_l)_l]) \leq N_i(delay = t_{delay} \mid [(\tau_l)_l])$ is proven.

Proof of (b′): Suppose that $D_{min} = 1$, then *(b′)* is true. Now, suppose that $D_{min} > 1$, we will show that the converse of the previous relation is true: (10) $\Rightarrow$ (9) for $t_{delay} < D_{min} - 1$. It is direct to see that (10) implies

$$\begin{aligned} &\boldsymbol{s}_i^{(\tau_i+\tau+1)}(t_{delay}+1) \prod_{j\neq i}(1 - s_j^{(\tau_j+\tau+1)}(t_{delay}+1)) = 1, \\ &\forall\, 0 < t < t_{delay}+1, \boldsymbol{s}_i^{(\tau_i+\tau+1)}(t) \prod_{j\neq i}(1 - s_j^{(\tau_j+\tau+1)}(t)) = 0. \end{aligned} \tag{11}$$

It remains to prove that (11) is also true for $t = 0$. Since the duration between $t = t_{delay} + 1$ and $t = 0$ is less than D_{min}, only one delivery in this duration is possible, *i.e.*, $s_i^{(\tau_i+\tau+1)}(0) \prod_{j\neq i}(1 - s_j^{(\tau_j+\tau+1)}(0) = 0$. Hence, (10) $\iff$ (9) for $t_{delay} < D_{min} - 1$. Using the same argument as previously, we infer there exists a bijection that maps $\boldsymbol{O}_{t+1}^{[(\tau_l)_l]}$ to $\boldsymbol{O}_t^{[(\tau_l)_l]}$ for any $t_{delay} < D_{min} - 1$. Thus, *(b′)* is proven.

Proof of (b″): Proof of *(a)* shows that there exists an injective function

$$\begin{array}{cccc} f & : & \boldsymbol{O}_{t+1}^{[(\tau_l)_l]} & \to & \boldsymbol{O}_t^{[(\tau_l)_l]} \\ & & (\tau_l \oplus 1)_l & \to & (\tau_l)_l \end{array}$$

Let $(\tau_l^*)_l \in [(\tau_l)_l]$ be such that user i delivers a packet at time $t_1 = -1$ ($\equiv L - 1 \pmod L$) and $t_2 = D_{min} - 1$. Since $t_2 - t_1 = D_{min}$ is the minimum delay, there is no delivery between t_1 and t_2, so these offsets $(\tau_l^*)_l$ are a realization of $delay = D_{min} - 1$. That is, $(\tau_l^*)_l \in \boldsymbol{O}_{D_{min}-1}^{[(\tau_l)_l]}$.

However, $(\tau_l^* \oplus 1)_l$ achieves $delay = 0$, that is $(\tau_l^* \oplus 1)_l \in \boldsymbol{O}_0^{[(\tau_l)_l]}$. Hence, $(\tau_l^*)_l$ has no inverse image by f. We infer that $N_i(delay = D_{min} \mid [(\tau_l)_l]) < N_i(delay = D_{min} - 1 \mid [(\tau_l)_l])$.

Proof of (c′): Since user i's delivery delay is at most D_{max}, its first delivered packet can only be sent at $t < D_{max}$. Therefore, *(c′)* is proven.

Proof of (c″): Let's consider offsets $(\tau_l)_l$. By definition of D_{max}, there exists two successive delivered packets at t_1 and t_2 such that $t_2 - t_1 = D_{max}$. By cyclically shifting these offsets, it is possible to shift t_1 to position $L - 1$ in the sequence and shift t_2 to position $D_{max} - 1$. Hence, $N_i(delay = D_{max} - 1 \mid [(\tau_l)_l]) \neq 0$. □

Theorem 13 (Characterization of UID Sequences)**.** *Let $\boldsymbol{s}_1, \ldots, \boldsymbol{s}_K$ be binary sequences. The following conditions are equivalent.*

1) *The sequence set $\{\boldsymbol{s}_1, \ldots, \boldsymbol{s}_K\}$ has UID.*
2) *The sequence set $\{\boldsymbol{s}_1, \ldots, \boldsymbol{s}_K\}$ has CIDD.*

Proof: Let $\{\boldsymbol{s}_1, \ldots, \boldsymbol{s}_K\}$ be a set of protocol sequences. By Definition 11, $\{\boldsymbol{s}_1, \ldots, \boldsymbol{s}_K\}$ has UID if, and only if, for all user i, there exists a threshold $T > 0$ such that

$$\forall\, t < T,\ N_i(delay = t) = N_i(delay = 0)$$

and

$$\forall\, t \geq T,\ N_i(delay = t) = 0. \tag{12}$$

Lemma 12(a) shows that for all $[(\tau_l)_l] \in X/\!\sim$, $N_i(delay = t \mid [(\tau_l)_l])$ is a non-increasing function of t, hence condition (12) is equivalent to: For all user i, for all $[(\tau_l)_l] \in X/\!\sim$,

$$\forall\, t < T,\ N_i(delay = t \mid [(\tau_l)_l]) = N_i(delay = 0 \mid [(\tau_l)_l])$$

and

$$\forall\, t \geq T,\ N_i(delay = t \mid [(\tau_l)_l]) = 0.$$

It follows from Lemma 12(b) and 12(c) that for each user i, $D_{max}(i) = D_{min}(i) = T$ (note that this value does not depend on the choice of $[(\tau_l)_l] \in X/\sim$ in this case). That is, for each user i the delivery delay is constant regardless of any relative offsets between users. $\square$

VI. Conclusion

In this paper, we investigate constant individual delivery delay (CIDD) protocol sequences as a multiple access scheme for collision channel without feedback [5]. These sequences have strong properties of constant throughput and periodic delivery for each user in the system.

We first proved that they are equivalent to a small family of shift-invariant (SI) protocol sequences. Then, we infer that they achieve SI sequences minimal length. However, they do not achieve optimal SI throughput; indeed, Fig. 1 shows its performance drop increasing exponentially in the number of users. To overcome this issue of throughput performance while preserving their periodic delivery property, one possible solution would be to reduce the number of users communicating in the same channel. For example, in a graph network where nodes represent users and edges their communication links (or shared channels), the number of sequences needed to preserve SI and CIDD properties is the graph's chromatic number χ. In particular, the chromatic number of a planar graph is at most 4 (see the *four color theorem* in [22]–[24] for more information about planar graph coloring), therefore only 4 sequences are needed in this kind of structure. So, the performance drop would be approximately 41% (see Fig. 1). In an even more restricted grid structure with $\chi = 3$, performance drop would be approximately 16%. Practical applications of this protocol to wireless sensor networks or ad hoc mobile networks is an interesting subject for future exploration.

Note that we also proved that CIDD sequences are equivalent to another class of protocol sequences called uniform individual delay (UID) protocol sequences.

Previous work shows that there is a tradeoff between the length of protocol sequences and the variance of throughput performance. SI sequences represent the extreme case where the throughput of each user is not affected by any relative delay offsets. In turn, the length of SI sequences must be exponential in the number of users. The CIDD property we developed in this paper, ensures very strong delivery stability and deterministic quality-of-service as a tradeoff for throughput performance drop which increases exponentially with the number of users.

Acknowledgement

The authors would like to thank Kenneth W. Shum for offering his software implementation of protocol sequences and valuable comments. A part of the work was carried out at Laboratory of Information, Networking and Communication Sciences (www.lincs.fr).

References

[1] M. Z. Win and R. A. Scholtz, "Impulse radio: how it works," *IEEE Communications Letters*, vol. 2, no. 2, pp. 36–38, 1998.

[2] Y. Li, C. S. Chen, Y.-Q. Song, and Z. Wang, "Real-time QoS support in wireless sensor networks: a survey," in *7th IFAC International Conference on Fieldbuses & Networks in Industrial & Embedded Systems (FeT'2007)*, 2007, pp. 1–8.

[3] E. M. Royer and C.-K. Toh, "A review of current routing protocols for ad hoc mobile wireless networks," *IEEE Personal Communications*, vol. 6, no. 2, pp. 46–55, Apr 1999.

[4] *Wireless LAN Medium Access Control (MAC) and Physical Layer (PHY) Specifications*. IEEE-SA Standards, BoardIEEE Std 802.11, 1999.

[5] J. L. Massey and P. Mathys, "The collision channel without feedback," *IEEE Trans. Information Theory*, vol. 31, no. 2, pp. 192–204, 1985.

[6] V. Anantharam, "The stability region of the finite-user slotted ALOHA protocol," *IEEE Trans. Information Theory*, vol. 37, no. 3, pp. 535–540, 1991.

[7] S. Bitan and T. Etzion, "Constructions for optimal constant weight cyclically permutable codes and difference families," *IEEE Trans. Information Theory*, vol. 41, no. 1, pp. 77–87, 1995.

[8] N. Q. A, L. Györfi, and J. L. Massey, "Constructions of binary constant-weight cyclic codes and cyclically permutable codes," *IEEE Trans. Information Theory*, vol. 38, no. 3, pp. 940–949, 1992.

[9] E. L. Titlebaum, "Time-frequency hop signals part I: Coding based upon the theory of linear congruences," *IEEE Transactions on Aerospace and Electronic Systems*, vol. AES-17, no. 4, pp. 490–493, July 1981.

[10] A. A. Shaar, M. Gharib, and P. A. Davies, "Collision resolution in contention access local area networks using concatenated prime sequences," *IEE Proceedings - Communications*, vol. 149, no. 56, pp. 249–256, 2002.

[11] A. A. Shaar and P. A. Davies, "Prime sequences: quasi-optimal sequences for or channel code division multiplexing," *Electronics Letters*, vol. 19, no. 21, pp. 888–890, October 1983.

[12] W. S. Wong, "New protocol sequences for random-access channels without feedback," *IEEE Trans. Information Theory*, vol. 53, no. 6, pp. 2060–2071, 2007.

[13] K. W. Shum and W. S. Wong, "Construction and applications of CRT sequences," *IEEE Trans. Information Theory*, vol. 56, no. 11, pp. 5780–5795, 2010.

[14] L. Györfi and S. Györi, "Coding for multiple access collision channel without feedback," in *Multiple Access Channels: Theory and Practice*. E. Biglieri, L. Gyrfi, (Eds.), IOS Press, 2007, pp. 299–326.

[15] C. S. Chen, K. W. Shum, C. W. Sung, W. S. Wong, and G. E. Oien, "User unsuppressible protocol sequences for collision channel without feedback," in *International Symposium on Information Theory and Its Applications*, Dec 2008, pp. 1–6.

[16] C. S. Chen, W. S. Wong, and Y. Song, "Constructions of robust protocol sequences for wireless sensor and ad hoc networks," *IEEE Trans. Vehicular Technology*, vol. 57, no. 5, pp. 3053–3063, 2008.

[17] K. W. Shum, C. S. Chen, C. W. Sung, and W. S. Wong, "Shift-invariant protocol sequences for the collision channel without feedback," *IEEE Trans. Information Theory*, vol. 55, no. 7, pp. 3312–3322, 2009.

[18] Y. Zhang, Y. Lo, W. S. Wong, and F. Shu, "Protocol sequences for the multiple-packet reception channel without feedback," *IEEE Trans. Communications*, vol. 64, no. 4, pp. 1687–1698, 2016.

[19] J. Y. N. Hui, "Multiple accessing for the collision channel without feedback," *IEEE Journal on Selected Areas in Communications*, vol. 2, no. 4, pp. 575–582, 1984.

[20] G. Thomas, "Capacity of the wireless packet collision channel without feedback," *IEEE Trans. Information Theory*, vol. 46, no. 3, pp. 1141–1144, 2000.

[21] Y. Wu, K. W. Shum, W. S. Wong, and L. Shen, "Safety-message broadcast in vehicular ad hoc networks based on protocol sequences," *IEEE Transactions on Vehicular Technology*, vol. 63, no. 3, pp. 1467–1479, March 2014.

[22] P. Cayley, "On the colouring of maps," *Proceedings of the Royal Geographical Society and Monthly Record of Geography*, vol. 1, no. 4, pp. 259–261, 1879.

[23] K. Appel and W. Haken, "Every planar map is four colorable. part I: Discharging," *Illinois J. Math.*, vol. 21, no. 3, pp. 429–490, Sep 1977.

[24] K. Appel, W. Haken, and J. Koch, "Every planar map is four colorable. part II: Reducibility," *Illinois J. Math.*, vol. 21, no. 3, pp. 491–567, Sep 1977.