# Optimum Arrangement of Taxi Drivers' Working Hours

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**Abstract.** We propose optimum arrangement of taxi drivers' working hours. In Japan, income of taxi vehicle is decreasing about 11 thousand yen in the past 15 years. Then some taxi companies are investing to gain more customers. But there are many small taxi companies that are difficult to invest with much money. Therefore we have been researching the other method to gain more customers by little investment for small companies. In this paper, we analyze present situation of the Taxi Company which we research, research optimum arrangement of taxi drivers' working hours so as to increase sales amount using mathematical model, and verify validity of our method with numerical calculation.

**Keywords:** optimum arrangement, mathematical model, taxi company, staff scheduling, Weighted Constraint Satisfaction Problem

## 1 Introduction

Financial condition (ex. Income of taxi vehicle per day) of taxi companies in Japan from 1995 to 2011 is shown in Fig.1. The annual number of customer is decreasing by about 800 million people, the annual transportation income is decreasing by about 860 billion yen, the number of taxi vehicle is decreasing by about 13 thousand cars, and income of a taxi vehicle per day is decreasing about 11 thousand yen. It is thought

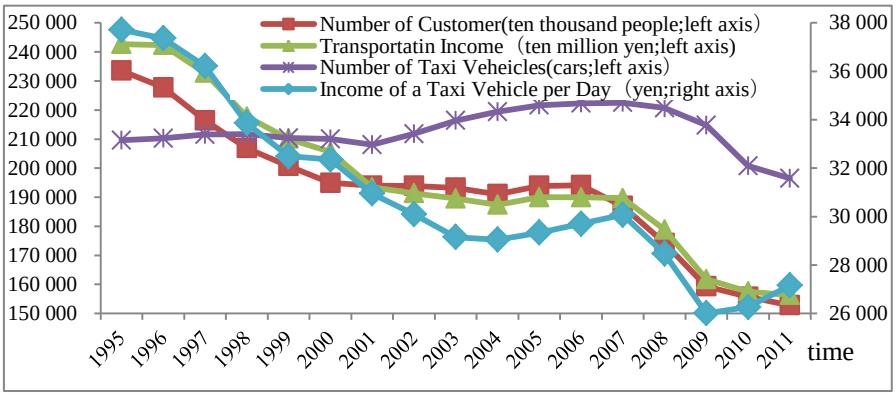


**Fig. 1.** Financial condition of taxi companies [1]

that this factor is in the collapse of the bubble economy and the failure of Lehman Brothers. Then taxi companies introduce various methods to gain more customers. Some companies begin to introduce new taxi operation system using GPS and smartphone to shorten taxi waiting time. But there are many small taxi companies that are difficult to introduce above system. So we have been researching the other method to gain more customers by little investment for small companies as follows [2][3].

- Rearranging taxi drivers' working hours using computer simulation so as to work many taxis with many taxi delivery demands by telephone requests.
- Optimum allocation of standby taxis at taxi stands so as to minimize total millage between taxi stands and the place where customer takes a taxi.

In this paper, we analyze present situation of the Taxi Company which we research (we call Company A in this paper), research optimum arrangement of taxi drivers' working hours so as to increase sales amount using mathematical model, and verify validity of our method with numerical computation results.

## 2 Present Situation of Company A

### 2.1 Taxi Drivers' Working Hours of Company A

Taxi drivers' working hours according to service pattern of Company A is shown in Table 1. Taxi driver's service pattern consists of four groups, such as Shift, Day, Fixed time, and Night. Details of each service pattern are described in [2].

**Table 1.** Working hours

| Service pattern | | Start | End | Total working hours |
|---|---|---|---|---|
| Shift | 1 | 7:00 | 1:00 | 18hours |
| | 2 | 8:00 | 2:00 | 18hours |
| | 3 | 8:00 | 3:00 | 19hours |
| | 4 | 12:00 | 8:00 | 20hours |
| Day | | 7:00 | 19:00 | 12hours |
| Fixed time | | 7:00 | 21:00 | 14hours |
| Night | | 20:00 | 4:00 | 8hours |

### 2.2 The sales amount of Company A

We get sales data between 2011/12 and 2012/11 from Company A. We calculate average sales amount according to time by the day of the week shown in Fig.2, by week shown in Fig.3, respectively. Average sales amount values are normalized so that the minimum value is equal to 1.0. From Fig.2, the characteristics of average sales amount are as follows.

- Fhe sales amount differs by the day of the week. The Sales amount of Friday and Saturnday are high. That of Sunday is low.
- The sales amount according to the week is the almost same.

- The sales amount from 8:00 to 9:00 and from 17:00 to 24:00 is high, from 2:00 to 7:00 is very low, from 10:00 to 11:00 and from from 13:00 to 16:00 is low.

The sales amount is different depend on time and the day of the week. We research optimum arrangement of taxi drivers' working hours so as to increase sales amount using mathematical model.

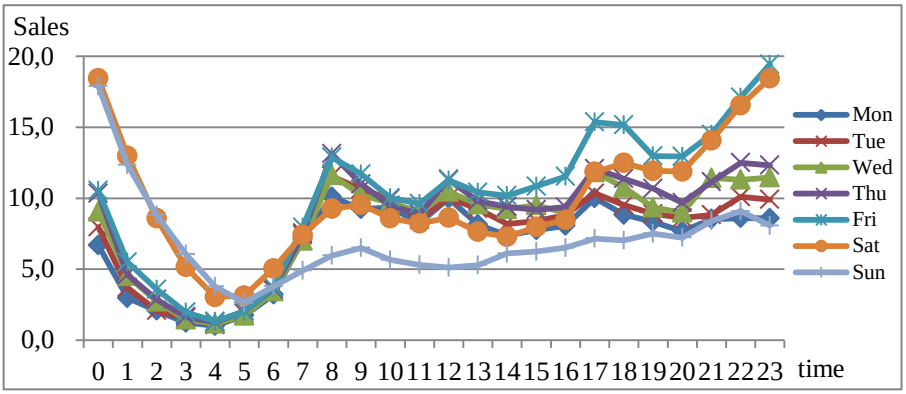


**Fig. 2.** Average sales amount according to time by day of week

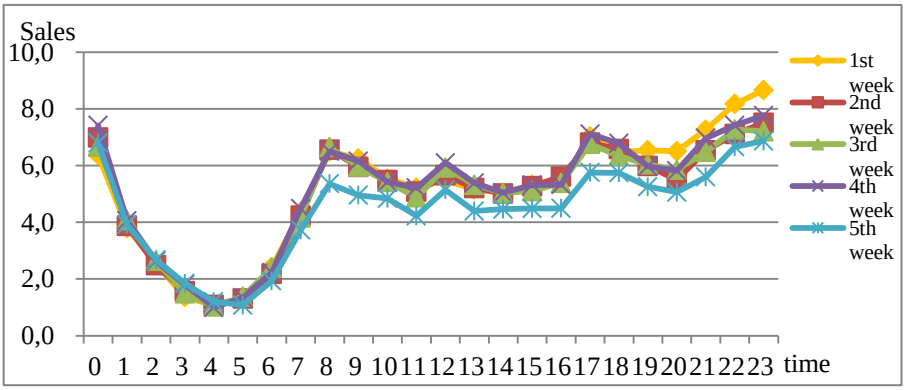


**Fig. 3.** Average sales amount according to time by week

## 3 Formulation

There are differences in the sales amount by time, the day of the week. This reason is because the number of customers and the taxi operation number vary according to time and the day of the week. Therefore we research optimum arrangement of taxi drivers' working hours so as to increase sales amount using mathematical model. Basic idea for arrangement of taxi drivers' working hours is putting the belt of continuous working time in the day under the following constraints (Fig.4).

- Taxi drivers must work working hours for each service pattern continuously.
- Working hours for every service pattern is the same every day.

- There is at least one service pattern “Shift” in every time zone.
- Service pattern “Night” differs in the day of the week which works in odd weeks and even weeks. Therefore we formulate two week model.

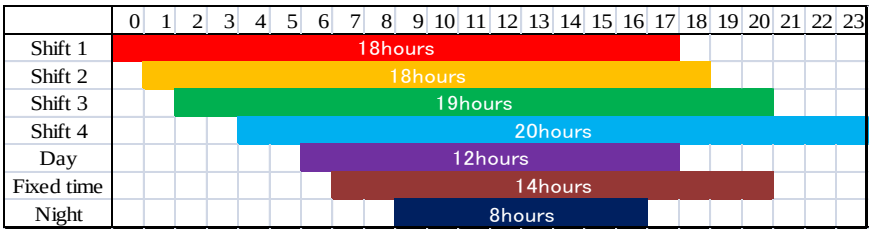


**Fig. 4.** Example of working hours

### 3.1 Notation

The following notations are used to formulate this problem.

i:time zone (i = 0, 1, 2, ⋯, 335)

j:service pattern (j = 0, 1, ⋯, 7)

0; Shift 1, 1;Shift 2, 2;Shift 3, 3;Shift 4, 4;Day,

5;first half of Fixed time, 6;second half of Fixed time, 7;Night

$C_i$:Sales amount in time zone i

$C_i$ is calculated as follows

$$C_i = \frac{\text{Annual sales amount in time zone i}}{\text{Annual days of time zone i}}$$

$N_{ij}$;The number of taxis which can work in time zone i for service pattern j

$T_j$:Continuation working hours for service pattern j

$x_{ij}$:0-1 integer variable which denotes taxi operation in time zone i for service pattern j

$$x_{ij} = \begin{cases} 1 \text{ (working)} \\ 0 \text{ (not working)} \end{cases}$$

### 3.2 Formulation

The objective is to maximize total sales amount. Problem is formulated as follows:

$$\text{Maximize } \sum_{i=0}^{335} \sum_{j=0}^{7} C_i * N_{ij} * x_{ij}. \quad (1)$$

Subject to

$$\sum_{j=0}^{3} x_{ij} \geq 1, \quad (2)$$

$$\sum_{i=0}^{335} x_{ij} = 14 * T_j, \quad (3)$$

$$\sum_{i=1}^{335} x_{ij} x_{i-1,j} = 14 * (T_j - 1), \quad (4)$$

$$x_{ij} = x_{i+24*n,j} \ (n=1,2,\ldots,13). \quad (5)$$

Constraint (2) shows that at least one service pattern "Shift" exits in each time zone. Constraints (3) and (4) shows service pattern j continues $T_j$ hours. Constraint (5) shows taxi operation in each time zone is the same every day.

### 3.3 Solution method

This model is nonlinear programming problem. It is difficult to calculate the optimum solution in short time. We reformulate the problem as Weighted Constraint Satisfaction Problem (WCSP) because variables are 0-1 integer, and solve WCSP by taboo search. WCSP is a problem which assigns a value in order to satisfy important constraints as much as possible [4]. In WCSP, the importance of constraints is set up as weight parameter. Therefore it is possible to calculate solution in which the model maker's intention was reflected about the sufficiency condition of constraints. An optimization problem is formulated as WCSP by changing objective function $f(x_1, x_2, \cdots, x_n)$ into the constraints introducing the target value $\mu$ as follows:

- In case of Minimization problem

$$\mu - f(x_1, x_2, \cdots, x_n) \geq 0$$

- In case of Maximization problem

$$f(x_1, x_2, \cdots, x_n) - \mu \geq 0$$

An objective function is treated as soft restrictions. Although soft restrictions must not necessarily be satisfied, they are satisfied as much as possible. In our problem, objective function (1) is changed constraint (6) in WCSP.

$$\sum_{i=0}^{335} \sum_{j=0}^{7} C_i N_{ij} x_{ij} - \mu \geq 0 \quad (6)$$

WCSP will end calculation, if one of the following conditions is satisfied.

(1)The solution which satisfies all the constraints is solved.
(2)The specified iteration count is exceeded.
(3)The specified computation time is exceeded.

We choose (3) and computation time is made into 3600 seconds. The solution procedure is as follows.

Step 1 Set up of computation time.
Step 2 Set up of target value $\mu$.
Step 3 Solve WCSP using taboo search.
Step 4 When a solution comes out within computation time, a target value is increased and back to Step 3.
Step 5 When it exceeds computation time, calculation is ended. The solution at the time is adopted.

# 4 The application result to Company A

## 4.1 Numerical computation case

We calculate quasi-optimum solution using procedure in chapter 3. We compare the solution result of the problem in chapter 3 and the problem added constraints including heuristics to which sales amount may become high based on data analysis. Company A requests us to study the new service pattern which become the sales amount high not adhering to the present service pattern. Therefore we compare the solution result of the problem added the constraints with new service pattern (Table 2).

- Pattern 0

[Method] Solve problem in chapter 3.
[Purpose] Calculate quasi-optimum solution, and compare with the present condition

- Pattern 1

[Method] Add the constraints which change working hours "Fixed time".
[Purpose] Increase the number of taxi operation after 20:00 with high sales amount.

- Pattern2

[Method] Add the constraints which divide working hours of "Fixed time".
[Purpose] Increase the number of taxi operation after 20:00 and decrease the number of taxi operation between 14:00 and 16:00 with low sales amount.

- Pattern 3

[Method] Add the constraints which change the day of the week of "Fixed time".
[Purpose] Increase the number of taxi operation on Friday and Saturday with high sales amount.

**Table 2.** Numerical computation case

| Pattern | Case | Working hours |
|---|---|---|
| 0 | 0 | No changes |
| 1 | 1 | From 8:00 to 22:00 |
| Change working hours of "Fixed time" | 2 | From 9:00 to 23:00 |
| | 3 | From 10:00 to 0:00 |
| | 4 | From 11:00 to 1:00 |
| | 5 | From 12:00 to 2:00 |
| 2 | 6 | From 7:00 to 14:00, From 17:00 to 0:00 |
| Devide working hours of "Fixed time" | 7 | From 8:00 to 15:00, From 17:00 to 0:00 |
| | 8 | From 8:00 to 14:00, From 18:00 to 1:00 |
| | 9 | From 7:00 to 13:00, From 16:00 to 0:00 |
| | 10 | From 8:00 to 14:00, From 16:00 to 0:00 |
| | 11 | From 8:00 to 14:00, From 17:00 to 1:00 |
| 3 | 12 | A:Mon, Fri, Sat, B:Tue, Fri, Sat |
| Change the day of the week of "Fixed time" | 13 | A:Mon, Fri, Sat, B:Wed, Fri, Sat |
| | 14 | A:Mon, Fri, Sat, B:Thu, Fri, Sat |
| | 15 | A:Tue, Fri, Sat, B:Wed, Fri, Sat |
| | 16 | A:Tue, Fri, Sat, B:Thu, Fri, Sat |
| | 17 | A:Wed, Fri, Sat, B:Thu, Fri, Sat |

### 4.2 Computation results

Numerical computation results using sales data between 2011/12 and 2012/11 from Company A is in Table3. The annual sales amount of case 17 of pattern 3 is the highest. On the whole, the sales amount of the pattern 3 are high. Furthermore, it is a result of the tendency which reduces the number of taxi operation in time zone of low sales amout (9:00 to 16:00) , and increases the number of taxi operation in time zone of high sales amount (18:00 to 0:00). We get some hints which consider new working hours as follows.

- The number of taxi operation from Monday to Thursday is reduced, and it from Friday to Saturday is increased.
- The number of taxi operation from 9:00 to 16:00 is reduced, and it from 18:00 to 0:00 is increased.

**Table 3.** Computation results

| | case | Shift | | | | Day | Fixed | | Night | Sales ammout*) | rank |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | | | | | | |
| | Now | 7:00 | 8:00 | 8:00 | 12:00 | 7:00 | 7:00 | | 20:00 | base | 15 |
| | | 1:00 | 2:00 | 3:00 | 8:00 | 19:00 | 21:00 | | 4:00 | | |
| | 0 | 8:00 | 12:00 | 6:00 | 6:00 | 18:00 | 13:00 | | 20:00 | 106.0% | 5 |
| | | 2:00 | 7:00 | 1:00 | 2:00 | 6:00 | 3:00 | | 4:00 | | |
| pattern 1 | 1 | | | | | | | | | No feasible solution | |
| | 2 | 8:00 | 8:00 | 7:00 | 17:00 | 14:00 | 9:00 | | 17:00 | 102.3% | 13 |
| | | 2:00 | 2:00 | 2:00 | 13:00 | 2:00 | 23:00 | | 1:00 | | |
| | 3 | 3:00 | 7:00 | 7:00 | 8:00 | 14:00 | 10:00 | | 13:00 | 99.2% | 16 |
| | | 21:00 | 1:00 | 2:00 | 4:00 | 2:00 | 0:00 | | 21:00 | | |
| | 4 | 8:00 | 16:00 | 8:00 | 8:00 | 15:00 | 11:00 | | 20:00 | 105.9% | 6 |
| | | 2:00 | 10:00 | 3:00 | 4:00 | 3:00 | 1:00 | | 4:00 | | |
| | 5 | 7:00 | 14:00 | 1:00 | 5:00 | 17:00 | 12:00 | | 23:00 | 105.5% | 9 |
| | | 1:00 | 8:00 | 20:00 | 1:00 | 5:00 | 2:00 | | 7:00 | | |
| pattern 2 | 6 | 19:00 | 7:00 | 8:00 | 7:00 | 15:00 | 7:00 | 17:00 | 19:00 | 105.7% | 8 |
| | | 13:00 | 1:00 | 3:00 | 3:00 | 3:00 | 14:00 | 0:00 | 3:00 | | |
| | 7 | 8:00 | 8:00 | 8:00 | 16:00 | 8:00 | 8:00 | 17:00 | 18:00 | 101.9% | 14 |
| | | 2:00 | 2:00 | 3:00 | 12:00 | 20:00 | 15:00 | 0:00 | 2:00 | | |
| | 8 | | | | | | | | | No feasible solution | |
| | 9 | 8:00 | 8:00 | 7:00 | 13:00 | 13:00 | 7:00 | 16:00 | 19:00 | 103.8% | 11 |
| | | 2:00 | 2:00 | 2:00 | 9:00 | 1:00 | 13:00 | 0:00 | 3:00 | | |
| | 10 | 15:00 | 8:00 | 10:00 | 7:00 | 14:00 | 8:00 | 16:00 | 20:00 | 105.9% | 7 |
| | | 9:00 | 2:00 | 5:00 | 3:00 | 2:00 | 14:00 | 0:00 | 4:00 | | |
| | 11 | 5:00 | 9:00 | 9:00 | 1:00 | 14:00 | 8:00 | 17:00 | 15:00 | 102.4% | 12 |
| | | 23:00 | 3:00 | 4:00 | 21:00 | 2:00 | 14:00 | 1:00 | 23:00 | | |
| pattern 3 | 12 | 2:00 | 20:00 | 8:00 | 16:00 | 20:00 | 20:00 | | 15:00 | 107.7% | 2 |
| | | 20:00 | 14:00 | 3:00 | 12:00 | 8:00 | 10:00 | | 23:00 | | |
| | 13 | 14:00 | 22:00 | 6:00 | 4:00 | 14:00 | 11:00 | | 7:00 | 106.5% | 4 |
| | | 8:00 | 16:00 | 1:00 | 0:00 | 2:00 | 1:00 | | 15:00 | | |
| | 14 | | | | | | | | | No feasible solution | |
| | 15 | 0:00 | 7:00 | 14:00 | 23:00 | 15:00 | 3:00 | | 20:00 | 104.9% | 10 |
| | | 18:00 | 1:00 | 9:00 | 19:00 | 3:00 | 17:00 | | 4:00 | | |
| | 16 | 8:00 | 8:00 | 11:00 | 5:00 | 18:00 | 13:00 | | 20:00 | 106.7% | 3 |
| | | 2:00 | 2:00 | 6:00 | 1:00 | 6:00 | 3:00 | | 4:00 | | |
| | 17 | 7:00 | 10:00 | 21:00 | 6:00 | 16:00 | 12:00 | | 20:00 | 108.7% | 1 |
| | | 1:00 | 4:00 | 16:00 | 2:00 | 4:00 | 2:00 | | 4:00 | | |

*)Contract with " case now"

## 5 Conclusions

We research optimum arrangement of taxi drivers' working hours to gain more customers by little investment for small companies. For the above purpose, we analyze present situation of Taxi Company A using sales data, and formulate mathmatical model so as to increase sales amount. This model is nonlinear programming problem and difficult to calculate the optimum solution in short time. Therefore we reformulate the problem as WCSP, and solve WCSP by taboo search. We verify validity of our method with numerical computation results. As a result, it turns out that it is effective strategy to reduce the number of taxi operation in time zone of low sales amout (9:00 to 16:00) , and increases the number of taxi operation in time zone of high sales amount (18:00 to 0:00).

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