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Abstract. Considering a multi-echelon system, the bullwhip effect is recognized as a significant factor with regards to the inventory management. This paper focuses on the effect of stochastic dependent lead time on the bullwhip effect. Simulation based approach is used to quantify the bullwhip effect with different demand and lead time distributions. The experiment results show that the dependent lead time has much effect on the 2nd echelon (from the downstream to the upstream) and bullwhip effect decrease significantly if the variance of this echelon decreases.

Keywords: bullwhip effect, multi-echelon supply chain, dependent lead time, reorder point, inventory management

1 Introduction

The bullwhip effect occurs when the demand variance in the supply chain is amplified as demand prorogates up the supply chain (Lee et al., 1997). The variance of demand increases upstream in the supply chain. In general, to deal with the variance of demand, safety stock is used. The level of safety stock is proportional to the demand variance and the service level. This means that the safety stock increases toward the upstream of supply chain due to the amplification of demand variance. Higher level of safety stock has to be carried with consequently more investment, extra production capacity, and increased storage space (Chatfield et al., 2004). Therefore, bullwhip effect reduction can bring benefit to the supply chain.

Several studies have been focused on the variation of demand. Nielsen et al. (2010b) considered the time dependent demand and the interdependent of demands of multiple products and developed a method to model the time dependent demand rate profiles by using the estimations of multivariate density distributions, and then to evaluate the interdependence of demand rates of products. Many studies have been investigated on the bullwhip effect and a number of these studies identify the causes of bullwhip effect, for example Lee et al. (1997). Some studies analyze the bullwhip effect by using simulation and show the causes of variance amplification (Forrester, 1958 and Bhaskaran, 1998). Croson and Donohue (2003) concluded that the bullwhip effect can be decreased by information sharing. Several papers focus on quantifying the bullwhip effect in a more analytical framework. Chen et al. (2000) quantified the bullwhip effect of a multi-stage supply chain with constant lead time and stochastic demand and concluded that the variance of orders of a retailer will be greater than variance of demand when that retailer updates the mean and variance of demand based on observed customer demand data. Moreover, they also pointed out that the customer demand information sharing can significantly reduce the bullwhip effect. The impact of lead time is considered in some studies. Wickner et al. (1991) noted that a twenty percent reduction in peak amplification can be achieved if lead time is reduced. Metters (1997) identified the magnitude of the bullwhip effect by establishing an empirical lower bound on the profitability impact of the bullwhip effect and found that eliminating the bullwhip effect can increase product profitability by 10–30% and significantly save inventories and other costs. Chen et al. (2000) showed that the increase in lead time will increase the variability of orders from retailer to manufacturer.

Nielsen et al. (2010a) offers an approach to improve supply chain planning through the use of RFID technology to track and thus be able to provide higher quality information, while Sitek and Wikarek (2013) offers approaches for improving planning through optimization. The main issue in applying optimization methods for supply chain management is the quality of the information available for the methods.

Most research to date has considered deterministic lead time and two echelons systems. A few papers have been investigated on stochastic lead time and multi-echelon system, for example Chatfield et al. (2004). Nielsen et al. (2013) considered the reorder point inventory management models sensitivity to demand distributions, demand dependencies and lead time distribution with four different versions of service level and concluded that the skew of demand distribution is the most significant with regards to the service level. However, no paper has been focused on the effect of stochastic lead time of the upstream echelon to that of the downstream one. All papers assumed that the lead times of echelons are independent. This is not practical because the variance of lead time of a downstream echelon can be longer due to the variance of lead time of an upstream echelon. Moreover, the variance of lead time affects the amount of safety stock of an echelon. Therefore, the reorder point (ROP) of an echelon is adjusted based on the historical data of lead time.

In this paper, the impact of stochastic lead time on the bullwhip effect of a multi-echelon system using an inventory policy is investigated. This paper differs from the previous studies that the dependence of lead times of echelons is considered. The problem description is presented in Section 2. Numerical experiment is shown in Section 3 and Section 4 points out the conclusions.

2 Problem description and methodology

2.1 Problem description

A multi-echelon system is considered as in Figure 1. Li denotes the lead time of the echelon of order i. The variance of lead time is assumed unchanged. The lower echelon will place an order to the next upper echelon when the inventory level lower than or equal the ROP. The ROP is computed as the sum of expected demand during expected lead time and the safety stock. The safety stock is calculated based on the variance of lead time and service level. It is assumed that the continuous review policy is applied for the inventory system of each echelon. The ROP is updated when a new lead time is recorded. Order-up-to is used to determine the order quantity. This means that when the inventory level downs to the ROP, the order quantity can full fill the inventory level up to a desired value. The incoming replenishment and total demand which is not satisfied are considered when review the inventory level. In details, an order will be made when

current inventory level + total incoming replenishment – total unsatisfied demand < ROP

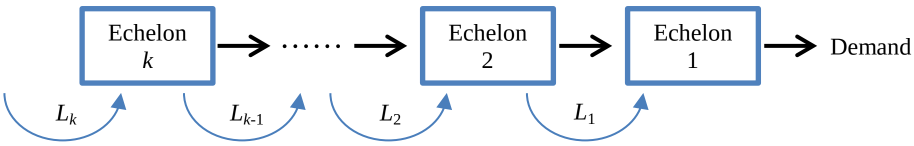

Fig. 1. the description of considered system

If the left hand side of the above inequality is denoted as the relative inventory level, then the order quantity will be

order quantity = desired up-to-value – relative inventory level (1)

To decide when an order should be placed, the manager of a lower echelon needs to know the ROP. However, the ROP is calculated based on the expected lead time. If the expected lead time is higher, the ROP is higher. Unfortunately, in practice, the manager cannot know exactly the expected lead time of the next order. Therefore, the manager estimates the expected lead time based on historical data of lead times of previous orders. In our experiment the expected lead time is estimated using a moving average with n = 3. This means that the expected lead time is the average of actual lead time of three consecutive orders which is closest to the time point to estimate the expected lead time. For example, at a certain time in planning horizon, an echelon placed N orders to the next upper echelon. The actual lead time of order *i* is denoted as LA_i. The expected lead time is estimated by using the following formula

$$\text{expected lead time} = (\mathrm{LA}_N + \mathrm{LA}_{N-1} + \mathrm{LA}_{N-2}) / 3 \quad (2)$$

Because the expected lead time is changed when a new order is made, the ROP of an echelon is also changed. As a result, the order quantity also changes in each order of an echelon. Due to the variance of order quantity, bullwhip effect will be expected to occur. The aim of this paper is to answer two questions. First, is there a bullwhip effect when the lead time of a lower echelon depends of that of the next upper echelon? In this paper, the bullwhip effect is defined as the ratio between the variance of order of the upper echelon and that of the next lower echelon. If all ratios are greater than 1, meaning that the variance of order of the upper echelon is higher than that of the next lower echelon, then the bullwhip effect exits. Secondly, if the bullwhip effect exists what is its size? Furthermore, through quantifying the bullwhip effect, we can see the effect of variance of lead time on the bullwhip effects of each echelon and the supply chain as a whole.

2.2 Methodology

To answer the two mentioned questions, Monte Carlo simulation is implemented on this multi-echelon system. Firstly, the possible events for each echelon are analyzed. The events at an echelon include:

- Order placing: when the relative inventory level is lower than or equal the ROP, an order is placed at the next upper echelon. The order quantity is calculated by equation (1). The information sent to the next upper echelon includes order ID and order quantity. Following this, information about the lead time of this order is sent back from the next upper echelon. The information about this order is recorded including order ID, release date, lead time, and order quantity.
- Order receipt: when the echelon receives an order from the next lower echelon. The lead time for this order is generated and sent back to the next lower echelon. Information about this order is recorded including order ID, order quantity, and delivery date.
- Order delivering: an order is considered to be delivered when its delivery date comes or is over and the current inventory level is higher than the quantity of this order. The upper echelon will sent information about order ID and actual delivery date. Current inventory level is updated and the order is removed out of the list of order receipt.
- Order replenishment: when the lower echelon receives the information about order delivery from the next upper echelon, the replenishment quantity is added to the current inventory. The lead time of this order is updated (in case it is different from the estimated lead time). Following this, the expected lead time is updated.
- Inventory review: investigates whether the relative inventory level is less than or equal to the ROP or not.

- Delivery check: to check whether an order in the list of order receipt is delivered or not. The condition for an order to be delivered includes delivery date of this order comes or is over and the current inventory level is higher than the quantity of this order.

For practical purposes there are some differences for the first and last echelon.

- Order receipt at the first echelon (echelon 1 in Figure 1): when the demand is received, the information is recorded to the list of order receipt following which the order delivery happens instantaneously.
- Order placement at the last echelon (echelon *k* in Figure 1): the lead time is generated when placing the order.
- Order replenishment at the last echelon: the order is replenished at the delivery date. This means that the order is replenished at the time which equals release date plus lead time

The simulation is terminated after the last day of planning horizon has been considered. After that, the variance of order of each echelon is calculated. Finally, the bullwhip effect of each echelon is computed.

3 Numerical experiment

Some scenarios are conducted in the numerical experiment. Four echelons are considered. The scenarios are the combination of the different cases of demand and lead time shown in Table 1. It is assumed that all echelons have the same lead time. The target service level is 90%. All echelon will order up to 1000 units. The initial inventory level follows a uniform distribution U(0,500). The planning horizon is 365 days. There are 30 runs for each scenario. The result of the experiment is shown in Tables 2a, 2b, and 2c.

Table 1. The cases of demand and lead time

	Constant	Normal	Exponential	Uniform
Demand	D = 50	N(50, 10^2)	-	U(40,60)
Lead time	L = 2	N(2,1)	Expo(2)	U(1,3)

Table 2. a. Average and std. deviation of bullwhip affect when demand is constant

	L = 2 days		L ~ N(2,1)		L ~ Expo(2)		L ~U(1,3)	
	Avg	Std	Avg	Std	Avg	Std	Avg	Std
Echelon 2	219.1721	0.8834	141.1904	120.0914	4.1726	0.9078	161.6278	127.3034
Echelon 3	4.4194	0.0011	1.9649	0.4869	1.8175	0.3010	2.4348	1.2051
Echelon 4	2.1398	0.0001	1.7096	0.6015	1.4620	0.2013	1.7175	0.4353

Table 2. b. Average and std. deviation of bullwhip affect when demand follows normal distribution

	L = 2 days		L ~ N (2,1)		L ~ Expo(2)		L ~U(1,3)	
	Avg	Std	Avg	Std	Avg	Std	Avg	Std
Echelon 2	107.6928	52.0421	121.3375	65.2603	5.0435	3.5167	93.0544	67.9523
Echelon 3	6.6632	16.3278	2.2906	1.6281	2.1476	1.0915	2.6482	2.3497
Echelon 4	2.2414	0.6832	1.7408	0.4810	1.5531	0.3856	1.8278	0.6581

Table 2. c. Average and std. deviation of bullwhip affect when demand follows uniform distribution

	L = 2 days		L ~ N (2,1)		L ~ Expo(2)		L ~U(1,3)	
	Avg	Std	Avg	Std	Avg	Std	Avg	Std
Echelon 2	88.4235	57.4673	97.0613	80.6057	5.1346	3.4074	155.8594	102.6469
Echelon 3	28.9540	63.5392	2.4675	1.2501	2.0744	0.7872	1.8746	0.6705
Echelon 4	2.3754	0.8439	1.6630	0.4460	1.7812	0.5060	1.6923	0.4614

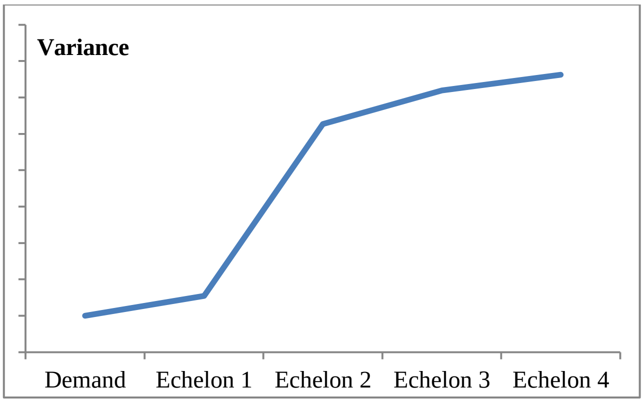

Fig. 2. Variance of order of each echelon

It can be seen from the experiment result that the bullwhip effect exists when lead time is stochastic and dependent. The different cases of demand do not affect the decrement in bullwhip effect when moving to the upstream (upper echelon) of the multi-echelon system. It shows that the second echelon has big bullwhip effect. The bullwhip effect becomes smaller when moving to the upper echelon. This shows that the bullwhip effect tends to have the value of 1 when more echelons are considered. Figure 2 shows the variance of order of each echelon

Figure 2 shows an interesting result. In echelon 1, the echelon closest to the demand and hence the natural demand, variance is very low. This seems reasonable as at this echelon the demand should be easy to observe and thus estimate. However, the orders sent to echelon 2 has an effect on the variance of order of echelon 2. Due to the amplification of demand variance, the upper echelon will have higher variance on demand. Figure 3 shows the direction of the increase of demand variance for a 4-echelon system where echelons 1 and 2 are considered as retailer and manufacturer and other echelons are the subsequent material suppliers. On the other hand, the cumulative effect of the lead time variance seems to directly impact the upper echelons, specifically the manufacturer. This is shown as the increase of lead time variability in Figure 3. The result is that the manufacturer experiences a very large bullwhip effect, much more so than any other echelon to echelon combination. The conclusion must be that the manufacturer especially absorbs the variance effect due to both the variance of lead time and demand. This underlines that the bullwhip effect of this system can be significantly reduced if the inventory policy of the manufacturer is carefully investigated. Furthermore, it can be concluded that the effect of lead time variation dominate that of demand variance in a multi-echelon system with dependent lead time. This means that decision making relate to inventory management is much different when lead time is uncertain and dependent. The study on inventory management with dependent lead time is very promising and can be put in future research.

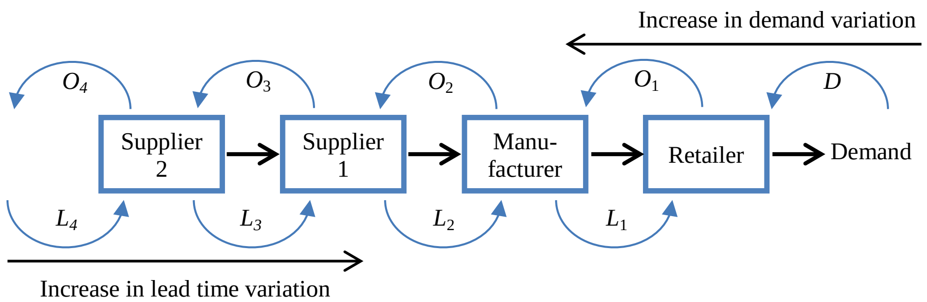

Fig. 3. Effect of the variances of lead time and demand on a 4-echelon system

4 Conclusion

In this paper, the stochastic and dependent lead time is considered in a multi-echelon system. The lead time of downstream (lower) echelon is assumed to be affected by that of upstream (upper) echelons. A simulation based experiment is run to determine and quantify the bullwhip effect in this type of system. Some scenarios are proposed for numerical experiments. The result of experiments shows that there is bullwhip effect in the system and the bullwhip effect is significantly decreased if the bullwhip effect of the 2nd echelon decreases. Several issues can be investigated for further study. Studying an optimal inventory policy for the 2nd echelon with considering stochastic and dependent lead time is very promising to decrease bullwhip effect of the whole system. Information sharing in the system is another issue which can show how much bullwhip effect can be reduced. The study also underlines that lead times and their variations are in fact critical for supply chains, especially if the lead times to some extend are dependent on each other.

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