Personal Income Tax Reforms: A Genetic Algorithm Approach

Matteo Morini, Pellegrino Simone

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Abstract

Given a settled reduction in the present level of tax revenue, and by exploring a very large combinatorial space of tax structures, in this paper we employ a genetic algorithm in order to determine the 'best' structure of a real world personal income tax that allows for the maximization of the redistributive effect of the tax, while preventing all taxpayers being worse off than with the present tax structure. We take Italy as a case study.

JEL Codes: C63, C81, H23, H24

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1. Introduction

Personal income tax (hereafter, PIT) is characterized around the world by several parameters that define its structure: marginal tax rates, upper limits of thresholds, allowances and deductions, as well as tax credits. Applied to the distribution of income observed in a specific country, the PIT structure of that country determines a given tax revenue and a given redistributive effect, as well as influences economic efficiency, first of all work incentives and tax compliance.

The existing economic literature represents a fundamental tool for the PIT design and for the need of balancing equity and efficiency, as well as social preferences for redistribution. This literature first focused on axioms that are required in order to equally apportion the burden of taxation among citizens (Mill, 1848; Samuelson, 1947). However, starting with the seminal paper by Mirrlees (1971), the theoretical literature has mainly focused on the equity-efficiency trade-off in optimum taxation. This is a difficult task, since many empirical simulation studies have, in fact, shown that in the short-run it is almost impossible to find a tax reform, which does not decrease efficiency or equity and, at the same time, is still financially and politically feasible. Moreover, if applied to a real-world tax system, most of the results of the economic literature would imply considerable modifications of the present tax structure and would certainly affect the tax revenue, which is one of the most important concerns that policymakers have to face. In other words, governments are certainly interested in setting up a tax system by implementing the literature's results, at least in the long-run. In the short-run, they are undoubtedly subject to budget constraints and to the political feasibility of a reform.

Despite these arguments, the PIT structure is subject to continuous evolution around the world. Peter, Buttrick and Duncan (2010) study and categorize the trends in the PIT structure over the period of 1981 to 2005 in 189 countries. They show that many governments substantially and/or frequently change the PIT structure; according to their analysis, about 45 per cent of governments changed at least one parameter of the PIT every year. They also emphasize that '*The high frequency of changes may also be due to the gradual enactment of tax reforms that get implemented over several years, fiscal policy responses to the business cycle, or continuous experimentation and search for the best tax structure.*'.

This paper focus on these key issues by considering a recently proposed real-world tax cut; it evaluates a feasible tax reform by optimizing the government's target and, in the meantime, by exactly complying with the government's budget constraint. The solution for this problem can be obtained by employing a genetic algorithm (hereafter, GA): a search heuristic inspired by natural selection, well-suited to the identification of the most promising solution to the problem under consideration. To our knowledge, no previous attempts at employing GAs for PIT structure optimization exist. The unique applications to tax systems deal with other and simpler aspects (Brooks, 2000; Chen and Lee, 1997).

The tax cut, which we study, has recently been implemented by the Italian government. In order to increase the purchasing power of 'poor' PIT taxpayers, as well as taxpayers belonging to the 'middle class' (a proxy of the redistributive effect maximization), the Italian government recently reduced the PIT revenue by 9.324 billion euros[3] (about 6 per cent of the PIT tax revenue) by introducing a cash transfer of 80 euros per month only for employees with a PIT gross income in the range of 8-26 thousand euros (about 10.9 million taxpayers).[4]

Considering this reform, two questions arise: is this tax cut allocation the best one the government could have considered? Or, given this settled amount of the tax cut, which is the best way to reform the whole PIT structure in order to achieve the highest redistributive effect, whilst leaving no taxpayers worse-off with respect to the present tax structure? The GA is an appropriate tool, and Italy is a perfect case study, since the Italian PIT is very complicated and its structure incorporates more than thirty parameters. Our results show that a more general, better, and more equity-oriented reform is possible; moreover, this methodology can be applied to any other specific target.

The solution of the problem discussed here faces an equity-efficiency trade-off: in order for the redistributive effect to be its highest, the efficiency of the tax (i.e. the level of the

[3] According to the official statistics made available by the Department of Finance of the Ministry of Finance (2016), the tax cut amounts to 9.1 billion euros and affects 11.3 million taxpayers.

[4] There is a political explanation behind this choice. The government announced that it would reduce PIT liability by 80 euros per month for all taxpayers belonging to the middle class. Of course, such an announcement proved to be too expensive; moreover, it would have been very difficult to be reached through a PIT structure reform since it is hard to reform such a complex tax structure and, in the meantime, ensure an equal tax reduction of 80 euros for all these taxpayers. In order to at least partially meet its commitment, the government decided to only apply the 80 euros pledge to a group of taxpayers, given the revenue constraint. In order to not modify the PIT structure and ensure exactly 80 euros, it chose the cash transfer instrument.

effective marginal tax rates) can worsen. In this first paper we just focus on the equity side of the problem. This does not imply that we forget about efficiency; we suggest a few constraints to the allowable parameters of tax structure in order to not arrive at both trivial and inefficient solutions.

Even if by employing the 'best' tax structure (almost) no taxpayer is worse-off, its actual applicability could face political resistance since all parameters of the tax change and, consequently, taxpayers could hardly believe that no one is worse-off. We do not discuss these political economy inconveniences. Finally, it has to be noted that we also do not consider taxpayers' responses to the new parameters of the tax structure; it is a 'short run' solution that can help policy makers when they think of PIT reform. In order to consider taxpayers' responses further research can be done regarding a long-run perspective: for example by modelling the equity-efficiency trade-off in a genetic algorithm framework or by employing agent-based models. This is the baseline of our further research.

The structure of the paper is as follows. Section 2 briefly reviews the existing economic literature on the design of the tax system and further explains the motivation of this paper. Section 3 describes in greater detail the 2010 structure of the Italian personal income tax, the baseline for our analysis. Section 4 presents how tax progressivity and the redistributive effect exerted by the tax can be measured. Section 5 shows the data and peculiarities of the static micro-simulation model employed for simulations. Section 6 first describes how genetic algorithms work and it then presents the implementation used in this work. Section 7 shows the results, whilst section 8 offers a conclusion.

2. Literature Review and Purpose of the Paper

Given its progressive nature, the PIT is a globally fundamental tool through which the redistributive effect of the whole tax system of a country is achieved, even if a large variability, in terms of both tax revenue and redistributive effect, is observed around the world (Verbist and Figari, 2014; Wagstaff et al., 1999). The two key reasons for this variability deal with the role played by social preferences for redistribution (Lefranc, Pistolesi and Trannoy, 2008) and the equity-efficiency trade-off of taxation (Tuomala, 2016; Saez, 2001; Feldstein, 1976; Sandmo, 1981; Stern, 1976).

Starting from Mill's (1848) approach, economic theory has first elaborated precise axioms to equally apportion the burden of taxation among citizens. The principle of equal sacrifice can thus give normative and positive contents to the ability-to-pay principle and justify the tax progressivity. The resulting degree of progressivity depends on the amount of tax revenue to be raised, as well as the social welfare function characterizing preferences of the society. Within this framework, Young (1990) proposed a theoretical strategy in order to test the possibility of stating that a country's lawmaker adopts a precise criterion of distributive justice and a particular social welfare function when he determines vertical equity and modifications in the PIT structure (e.g., Pellegrino (2008) for an application to the Italian case). Young's (1990) framework does not consider efficiency and incentive effects, so his methodology favours high marginal tax rates on higher incomes. Beliant and Gouveia (1993) introduce incentive effects within Young's (1990) methodology, finding conditions for progressive taxation similar to the standard ones elaborated by Samuelson (1947).

On the other hand, Mirrlees (1971) introduces the theory of optimal direct taxation, which also deals with the equity-efficiency trade-off. As the real-world PIT systems are very complex, empirical applications of the optimal income tax theory are based on stylized tax, which involve only a few tax parameters. Within this framework, the most effective estimation difficulties deal with the economic behaviour modelling since the labour supply responses (Bargain et al., 2014; Aaberge and Colombino, 2013; Blundell and Shephard, 2011) and the tax base responses to tax changes have to be introduced and parameterized (Saez, 2001; Saez, Slemrod and Giertz, 2012).

On the contrary, governments have to face PIT structures composed by several parameters and, most importantly, the PIT structure observed in a country (in a given year) is the result of several and partial adjustments that have occurred over the previous years. Focusing on those aspects, this paper differs from the existing literature for several reasons. First of all, it has a different the point of view from which the PIT reform is evaluated. Usually, the existing empirical literature evaluates the effects of a tax reform *after* the government introduces it. On the contrary, in this paper, we consider the government's point of view *before* such a reform is introduced.

Let us consider a generic government. Every year, this government plans finance law, which sets the annual adjustments on the level of the overall value of public spending and tax revenue to be obtained by the current legislation, in order to achieve some specific goals (such as the level of the government deficit) set in the long-term budget. One possible adjustment is the level of the PIT revenue. Starting from the actual PIT structure, the government may then want to cut PIT revenue in order to increase the purchasing power of taxpayers; conversely, it may want to increase the redistributive effect of the tax leaving the tax revenue unchanged; or, it may want to increase tax revenue by letting the richest taxpayers face the whole tax increase, or it may want to modify the PIT structure in order to reduce the inefficiencies of the tax. These are, of course, only some explanatory perspectives. Whatever the target, which parameters of the tax should be changed in order to optimize it? How much should they change? Or, more generally, how should the whole PIT structure change? Given this target, it is not the case that policy-makers consider these questions when thinking of such a PIT reform. In order to implement a reform, the government usually changes some parameters of the tax compatible with its revenue constraint. Whether such a tax structure change is aimed at achieving the best way to obtain the specific target is debatable. The perspective discussed here can therefore be useful in setting a short-run tax reform. Apart from exceptional cases, the tax cuts represent a small percentage of the overall revenue of the tax; on one hand, results of the economic literature can be only guidelines for the implementation of such a reduction; on the other hand, the government is primarily interested in correctly forecasting the tax reduction, given its balanced budget constraint.

3. The Personal Income Tax in the 2010 Fiscal Year: Technical Details

Let x_i be the personal gross income of taxpayer i $\left(i = 1, 2, ..., n\right)$. The 2010 Italian tax law considers two different kinds of deductions: d_i^1 is deduction for the main residence cadastral income; d_i^2 is the sum of deductions for social security contributions and alimonies as well as donations. The taxable income y_i is evaluated as:

$$y_i = \begin{cases} x_i - d_i^1 - d_i^2 & if \quad d_i^1 + d_i^2 < x_i \\ 0 & if \quad d_i^1 + d_i^2 \geq x_i \end{cases} \tag{1}$$

From 2007 onwards the rate schedule $S(y_i)$ contemplates 5 thresholds as reported in Table 1. The upper limits $UL_j = LL_{j+1}$ $(j = 1,2,3,4)$ of thresholds are 15, 28, 55, 75 thousand euros, being the first lower limit $LL_1 = 0$; tax rates t_j range between 23 and 43 per cent.

TABLE 1 AROUND HERE

By applying the rate schedule to the tax base the gross tax liability GT_i is obtained.

In order to determine the net tax liability T_i, tax law admits three distinct kinds of effective tax credits. They are: tax credits for earned income $c_i^1(x_i^{MR})$, tax credits for dependent individuals within the household $c_i^2(x_i^{MR})$, and tax credits for items of expenditure c_i^3, where $x_i^{MR} = x_i - d_i^1$.

The net tax liability T_i is then evaluated as:

$$T_i = \begin{cases} GT_i - c_i^1(x_i^{MR}) - c_i^2(x_i^{MR}) - c_i^3 & if \quad GT_i > c_i^1(x_i^{MR}) + c_i^2(x_i^{MR}) + c_i^3 \\ 0 & if \quad GT_i \leq c_i^1(x_i^{MR}) + c_i^2(x_i^{MR}) + c_i^3 \end{cases} \tag{2}$$

In what follows we do not consider regional and municipal surtaxes and then we evaluate taxpayer *i*'s net income as $z_i = x_i - T_i$.

Focusing on tax credits for employees and pensioners as well as self-employed,

$$c_i^1(x_i^{MR}) = \begin{cases} t_1 m_r & if \;\; x_i^{MR} \leq m_r \\ (t_1 m_r - a_r) + a_r\left(\dfrac{LL_2 - x_i^{MR}}{LL_2 - m_r}\right) & if \;\; m_r < x_i^{MR} \leq LL_2 \\ (t_1 m_r - a_r)\left(\dfrac{LL_4 - x_i^{MR}}{LL_4 - LL_2}\right) + b & if \;\; LL_2 < x_i^{MR} \leq LL_4 \\ 0 & if \;\; x_i^{MR} > LL_4 \end{cases} \tag{3}$$

where t_1 is the lowest marginal tax rate (23 per cent); m_r with $r = (1,2,3,4)$ (the level of x_i^{MR} below which taxpayers have a nil net tax liability) is equal to 8,000 euros for employees (m_1), 7,500 for pensioners younger than 75 (m_2), 7,750 for pensioners older than 75 (m_3), 4,800 for the self-employed (m_4), and zero for non-working taxpayers; a_r is equal to 502 euros for employees (a_1), 470 for pensioners younger than 75 (a_2), 486 for pensioners older than 75 (a_3), zero for self-employed (a_4); b, that ranges from 10 to 40 euros in the bandwidth 23-28 thousand euros, is applied only to employees (as discussed later, we always set $b = 0$ in simulations). Non-working taxpayers have no tax credit for earned incomes. Finally, this tax credit decreases from zero to m_4, and from m_4 to LL_4 only for self-employed taxpayers.

Four different tax credits for type of relationship are allowed: tax credit for dependent children $c_i^{2H}\left(x_i^{MR}\right)$, further tax credit for households with more than three children c_i^{2HF}, tax credit for dependent spouse $c_i^{2S}\left(x_i^{MR}\right)$, and tax credit for other household components $c_i^{2O}\left(x_i^{MR}\right)$. The overall value for $c_i^{2}\left(x_i^{MR}\right)$ is then $c_i^{2}\left(x_i^{MR}\right) = c_i^{2H}\left(x_i^{MR}\right) + c_i^{2HF} + c_i^{2S}\left(x_i^{MR}\right) + c_i^{2O}\left(x_i^{MR}\right)$. In particular,

$$c_i^{2H}\left(x_i^{MR}\right) = \begin{cases} c_i^{2Hp}\dfrac{q+(f-1)e-x_i^{MR}}{q+(f-1)e} & if\ \ 0 < x_i^{MR} \le q+(f-1)e \\ 0 & if\ \ x_i^{MR} > q+(f-1)e \end{cases} \tag{4}$$

where $f = \sum_{l=1}^{4} f_l$ is the overall number of dependent children; f_1 is the number of dependent children older than 3 years if the dependent children within the household are 3 or less; f_2 is the number of dependent children younger than 3 years if the dependent children within the household are 3 or less; f_3 is the number of dependent children older than 3 years if the dependent children within the household are more than 3; f_4 is the number of dependent children younger than 3 years if the dependent children within the household are more than 3; e is equal to 15,000 euros; q is equal to 95,000; $c_i^{2Hp} = \sum_{l=1}^{4} f_l c_i^{2Hpl}$; the present values for the potential tax credits are: $c_i^{2Hp1} = 800$, $c_i^{2Hp2} = 900$, $c_i^{2Hp3} = 1{,}000$, $c_i^{2Hp4} = 1{,}100$ euros.

Moreover, whenever $c_i^{2H}\left(x_i^{MR}\right)>0$ and the dependent children within the households are more than 3, the tax law admits a further tax credit c_i^{2HF} equal to 1,200 euros for all beneficiaries. The tax credits for dependent children have to be split between spouses whenever both of them have a positive gross income. Finally,

$$c_i^{2S}\left(x_i^{MR}\right)=\begin{cases} c_i^{2Sp}-u\dfrac{x_i^{MR}}{LL_2} & if \ \ x_i^{MR}\leq LL_2 \\ c_i^{2Sp}-u & if \ \ LL_2<x_i^{MR}\leq w \\ \left(c_i^{2Sp}-u\right)\dfrac{k-x_i^{MR}}{k-w} & if \ \ w<x_i^{MR}\leq k \\ 0 & if \ \ x_i^{MR}>k \end{cases} \tag{5}$$

and

$$c_i^{2O}\left(x_i^{MR}\right)=\begin{cases} c_i^{2Op}\dfrac{k-x_i^{MR}}{k} & if \ \ x_i^{MR}\leq k \\ 0 & if \ \ x_i^{MR}>k \end{cases} \tag{6}$$

where u is equal to 110 euros, w is equal to 40 thousand euros, k is equal to 80 thousand euros, c_i^{2Sp} is equal to 800 euros and c_i^{2Op} is equal to 750 euros. The present tax code considers higher values than $c_i^{2Sp}-u$ in the income range 29,000-35,200 euros. Instead of 690 euros, in this income range values ranging from 700 to 720 euros are applied. We do not consider these differences in simulations, always letting $c_i^{2Sp}-u$ be equal to the same value.

Tax credits for items of expenditures c_i^3 can be classified in two groups according to the percentage of the expense the tax law admits as a tax credit. There are expenses that allow a tax credit of 19 per cent and 36 per cent, respectively.[5] The 19 per cent tax credits (we label this variable *expenditure$_1$*) are very large, 19 different cases, such as expenses for health care, mortgage interests, etc.; 36 per cent tax credits (*expenditure$_2$*) are allowed for home restructuring-related expenses. All together, tax law admits 30 different tax credits for items of expenditure. Finally, tax law admits a tax credit for tenants; it is 300 euros if $x_i \leq 15{,}494$ (we label this variable *tenants$_1$*); 150 if

[5] The tax code considers also a 55 per cent tax credit for interventions for energy saving and a 20 per cent tax credit for purchasing of a washing machine. Because of the low number of taxpayers interested in these two kinds of tax credits, we did not considered them in the micro-simulation model.

$15{,}494 < x_i \leq 30{,}987$ (*tenants2*); 992 euros if $x_i \leq 15{,}494$ and if the taxpayers are younger than 30 (*tenants3*).

4. Distribution of Income and Personal Income Tax Progressivity

Let $x_1, x_2, ..., x_n$ be the pre-tax income levels associated to *n* income units. The corresponding post-tax income levels and tax levels are $z_1, z_2, ..., z_n$ and $T_1, T_2, ..., T_n$, respectively. We denote the pre-tax and the post-tax income distribution as well as the tax distribution by *X*, *Z* and *T*, respectively.

As is well known, inequality among pre- and post-tax income levels as well as tax levels can be evaluated by the Gini coefficient. Let G_X, G_Z and G_T be the corresponding Gini coefficient for pre-tax income, post-tax incomes and taxes, respectively. Then,

$$G_\varepsilon = \frac{2\operatorname{cov}[\varepsilon, F(\varepsilon)]}{\mu_\varepsilon} \tag{7}$$

where $\varepsilon = X, Z, T$, μ_ε is the average value for pre-tax and post-tax incomes and taxes, *cov* represents the covariance and $F(\varepsilon)$ is the cumulative distribution function.

After the tax, it is not guaranteed that post-tax ordering be equal to the pre-tax income one. Indeed, it is most likely that these two orderings differ because of the re-ranking due to the tax. Therefore, the inequality of *Z* and *T* can be evaluated once these distributions are ordered according to the corresponding pre-tax incomes, ranked in a non-decreasing order. For what concerns post-tax incomes and taxes, the corresponding concentration coefficient can then be evaluated as follows:

$$C_\varepsilon = \frac{2\operatorname{cov}[\varepsilon, F(X)]}{\mu_\varepsilon} \tag{8}$$

Progressive taxation produces two different effects on the distribution of pre-tax incomes: post-tax income inequality is lower than that measured on pre-tax income distribution, whilst tax inequality is greater. The first effect is known as the redistributive effect of the tax and the second one as departure from proportionality of the progressive taxation (Lambert, 2001). The overall redistributive effect of the tax *RE* can be evaluated as

$$RE = G_X - G_Z = (G_X - C_Z) - (G_Z - C_Z) = RS - R^{APK} \tag{9}$$

where $RS = G_X - C_Z$ is the Reynolds-Smolensky index, whilst $R^{APK} = G_Z - C_Z$ is the Atkinson-Plotnick-Kakwani index. The more the tax is progressive, the greater *RE* and *RS*; the more the tax causes re-ranking, the greater the negative contribution of re-ranking to the overall redistributive effect. Note that if the tax does not cause re-ranking $R^{APK} = 0$, then $RE = RS$.

The departure from proportionality of the progressive taxation can instead be evaluated by the Kakwani index $K = C_T - G_X$. The Kakwani and the Reynolds-Smolensky indexes are linked by the overall average tax rate θ, namely

$$\theta = \frac{\sum_{i=1}^{n} T_i}{\sum_{i=1}^{n} x_i} \tag{10}$$

As a consequence, $RS = \frac{\theta}{1-\theta} K$. This formula tells us that the Reynolds-Smolensky index has two determinants: the overall average tax rate and the Kakwani index.

In what follows we focus only on the Reynolds-Smolensky index and then we are interested in finding the 'best' tax structure able to determine a given tax revenue (smaller than the present one) and to yield to the greatest *RS* while getting no loser taxpayers. Since we impose a reduction of the tax revenue, note that the value of θ will be smaller than the present one. Note also that the simulated *K* will be greater than the present one in order for *RS* to be the highest.

5. The Data and the Static Micro-simulation Model

In order to design the tax reform, we rely on a static micro-simulation model written in STATA (technical details are available in Pellegrino *et al.* (2011)) that employs, as input data, those provided by the Bank of Italy in its Survey on Household Income and Wealth (hereafter, BI-SHIW), published in 2012 with regard to the 2010 fiscal year. It estimates the most important taxes and contributions which characterise the Italian fiscal system. Here we employ the micro-simulation model module concerning the PIT.

The BI-SHIW survey contains information on household income and wealth in the year 2010, covering 7,951 households and 19,836 individuals (Bank of Italy, 2012). The sample is representative of the Italian population, composed of about 24 million households and 60 million individuals.

The BI-SHIW survey provides information only on each individual's disposable income, which considers items of income that are taxed within the PIT or that can be exempt from the tax, as well as can be taxed under a separate regime. Therefore, the micro-simulation model first distinguishes all incomes included in the PIT taxable income definition, incomes exempt from any taxes and incomes taxed under a separate regime. Then the PIT gross income distribution is evaluated, starting from the PIT net income distribution. The transition from the post- to the pre-tax personal income of each individual has been computed by applying the algorithm proposed by Immervoll and O'Donoghue (2001).

Using original sample weights, the grossing-up procedure simply proportions the sum of individuals' sample weights to the dimension of the population as estimated by the National Statistical Office (ISTAT). Then the grossed-up number of PIT taxpayers has been obtained by considering individuals with a positive gross income within the sample (13,791 taxpayers), corresponding to about 40 million in the population.

Considering the income units, results concerning the PIT gross income distribution are very close to the Ministry of Finance's (2011) official statistics, both considering the gross income distribution by income classes and the composition of PIT income units by work status, as well as by their mean gross income. In addition, the overall tax revenue resulting from the micro-simulation model (148.748 billion euros) is very close to that showed in the official statistics. As a consequence, this instrument is suitable for the type of empirical analysis we propose.

Considering all individual taxpayers, Figure 1 compares the frequency density function obtained with the micro-simulation model and the one obtained using the Ministry of Finance's official data by income classes. Similar pictures emerge considering the frequency density function for pensioners and employees, as well as the self-employed.

FIGURE 1 AROUND HERE

The column 'Present value' of Table 2 shows the inequality indices for individual taxpayers in the 2010 fiscal year, which is our reference situation for the Reynolds-Smolensky index maximization. The Gini coefficient for the gross income distribution is 0.44338, whilst that for the net income distribution is 0.39138. The overall redistributive effect *RE* is 0.05200. The concentration coefficient for the net income distribution is 0.39076, whilst that on the net tax liability distribution is 0.67215; therefore, the Reynolds-Smolensky *RS* index is equal to 0.05262 and the Kakwani index *K* is 0.22877. The overall average tax rate is 18.70 per cent, whilst the Atkinson-Plotnick-Kakwani R^{APK} index is equal to 0.00062.

TABLE 2 AROUND HERE

Table 2 (column 'With cash transfer') also shows the overall redistributive impact of PIT joined with the cash transfer Φ_i. Φ_i favours only employees with a PIT gross income in the range of 8-26 thousand euros (about 10.9 million taxpayers), as follows:

$$\Phi_i = \begin{cases} 960 & if \;\; m_1 < x_i \leq 24{,}000 \;\; \& \;\; GT_i - c_i^1(x_i^{MR}) > 0 \\ 960\dfrac{26{,}000 - x_i}{2{,}000} & if \;\; 24{,}000 < x_i \leq 26{,}000 \;\; \& \;\; GT_i - c_i^1(x_i^{MR}) > 0 \\ 0 & if \;\; x_i > 26{,}000 \;\; \& \;\; GT_i - c_i^1(x_i^{MR}) > 0 \end{cases} \qquad (11)$$

It does not modify the PIT structure at all. Beneficiaries obtain Φ_i, and continue to pay the same amount of PIT net tax liability T_i. The net effect $z_i^* = y_i - T_i + \Phi_i$ is an increase in these taxpayers' disposable income by 960 euros per year, in the income range 8-24 thousands euros and a decreasing amount up to 26 thousand euros. Note also that if $T_i < \Phi_i$ beneficiaries obtain a subsidy. Taxpayers other than employees, on the contrary, do not gain from this cash transfer. According to our microsimulation model, the cost of this measure is 9.324 billion euros.

Even if only one taxpayer out of four obtains the cash transfer, the redistributive effect of the PIT joined with the transfer considerably improves: the Reynolds-Smolensky *RS* index increases by 8.46 per cent; on the contrary, the overall redistributive effect *RE* increases only by 7.92 per cent, since the Atkinson-Plotnik-Kakwani R^{APK} index worsens by 54 per cent. On the efficiency side, the effective marginal tax rate resulting in the income range of 24-26 thousand euros increases by up to 80 per cent; for all other income levels, on the contrary, the effective marginal tax rates do not change with respect to the present tax structure.

By employing a GA, in the next sections, we show that a more equity-oriented reform is possible. From the methodological point of view, the specific measure employed by the Italian government is not particularly interesting; it refers only to employees and then, a few parameters of the tax could be simultaneously changed in a GA framework, i.e. parameters defining the structure of the tax credit for employees. We discuss an overall reform, which considers all the parameters of the tax are able to be trimmed. Since we split the tax cut among all taxpayers, both the average reduction of the net tax liability for all taxpayers and the value of R^{APK} are smaller than that guaranteed to employees by the government's cash transfer.

6. Genetic Algorithm

6.1. General Overview

Genetic algorithms (GAs) are a heuristic search which belong to the field of evolutionary algorithms, a subfield of artificial intelligence. Since their inception (Holland, 1975), GAs found a wealth of applications in the most varied research disciplines, beyond computational science, mathematics, physics, bioinformatics, etc. Applications in economics also exist, broadly including game theory, public economics (Brooks, 2000; Chen and Lee, 1997), finance related works (Sermpinis et al., 2015), schedule optimization and whenever some sort of learning mechanism is needed; historically, the first attempt at employing GAs in economics is due to Miller's (1986) research on adaptive behaviour.

Up to date, there is not an application employing GAs for a tax system optimization; a similar structure of the GA here developed has been however applied in other branch of economics, as in the paper by Lin and Liu (2008). This is a previous example application of GAs to, at large, economics-related problems, portfolio optimization in particular. As it is often the case, GAs are chosen as a last-resort strategy when other optimization strategies fail, or are infeasible, due to the impossibility to describe the problem in a closed form.

The huge solutions search space, which is the aftermath of the combinatorial effect of very many parameters, posing a serious challenge to traditional optimization techniques; brute force methods are out of the question, just like iterative methods (cfr. Newton's); GAs appear as an obviously appropriate choice.

Candidate solutions, which in the GA are internally represented as 'individuals' (each of them is characterized by her own 'genome', a vector of 'chromosomes')[6], are generated as an initial 'population' at random. Evolutionary operators iteratively select, cross-breed and mutate the best (most 'fit', according to an objective function called 'fitness function') 'individuals', in order to produce an offspring of 'individuals' – the subsequent 'generation' – that will enter a new reproduction step. The 'individuals'' average fitness increases after every generation, until a satisfactory solution is found. The stopping criterion normally employed is related to the 'population' homogeneity: as the search process becomes closer to an optimum, the 'individuals' become more and more similar among them.

Detailing the trimming of the GAs technical parameters is out of the scope of this work; suffice it to say, as agreed upon by a vast literature, it is an ad-hoc process, to be performed mostly by trial and error, on every specific search domain.

6.2. The Structure of the Genetic Algorithm We Employ

The static micro-simulation model had to be re-implemented in a more versatile way, in order for the kind of analysis we are interested in to be feasible. We rewrote the modules of the micro-simulation model evaluating the Italian PIT in Python, a very mainstream language that conveniently allows for the use of parallel computing techniques, distributed across multiple nodes. Python also offers an excellent compromise between agility in programming – providing the developers with several libraries optimized for numerical calculations – and computational performance.

The GA implementation employed in this work is based on Python's open-source Pyevolve library (Perone, 2009). The 'population' selection mechanism across generations is the standard roulette wheel (fitness proportional) with elitism selection (the best 5 individuals in each generation are kept unaltered and carried over across

[6] In the early GA implementations (BCGA, binary-coded genetic algorithms), the solutions space had to be coded in binary numbers; RCGA, real-coded genetic algorithms, allow working with variables in continuous domains; cfr. Herrera et al. (1998).

successive generations), while the evolutionary crossover operator is a standard one-point and is set equal to 0.85. For what concerns the mutation operator, we employed a random gaussian operator, replacing (with a 0.03 rate) values *x* with $x + N(0, \sigma)$, where $\sigma = 2.0$ has been determined to be the best. Therefore, a low mutation rate value and a high crossover rate have been utilized, in order to let the search process converge reasonably quickly on solutions, whilst maintaining the ability to escape local maxima. The "population size vs. number of generations" trade-off has been tackled and solved, favouring a small population vs. numerous evolutionary steps.

As the starting point, we let the GA set up a 'population' of 200 different tax structures ('individuals' from the GA point of view) and then we let it evolve them for 10,000 'generations'. Consequently, the GA has to evaluate as much as 2 million candidate solutions, applying all these different tax structures to the same pre-tax income distribution, composed of 13,791 taxpayers.

6.2.1. Setting Up the Problem and the Choice of the Fitness Function

We are interested in coming up with a reasonable tax structure that inhibits both trivial and inefficient solutions. The GA then has to be provided with a few specific constraints that have to be obeyed, in terms of some parameters of the tax structure. If this was not the case, problematic solutions would appear. For example, having to find the highest redistributive effect with no constraints at all, the GA would certainly impose excessively high marginal tax rates on higher income earners and a zero marginal tax rate on too many of the poorest taxpayers; as a result, a polarization of tax rates and bandwidths of thresholds would appear and the tax revenue would consequently be too high. Or the GA would disproportionately favour high levels of some peculiar tax credits, simply because they are enjoyed by a small group of taxpayers, resulting in a negligible impact on the tax revenue, but in awkward preferential treatment for some income groups.

In order to avoid these unpleasant outcomes, we imposed two constraints.

First, we impose a condition that no taxpayers should be worse off as a result of the tax reform; that is, all taxpayers must pay a lower (or, at most, equal) amount of taxes than the present one. Therefore, since the Italian personal income tax does not allow for negative income taxation, and as we are looking for a tax reform with no losing taxpayers, we let the ‘no tax area’ be greater, or at least equal, to the present one. We also require the highest marginal tax rate as well as the lower limit of the top threshold, to be lower, or at most equal, to the present values, and the upper limit of the bottom threshold to be greater, or at most equal, to the present value.

Second, we keep the rank applied by the present tax structure to certain kinds of tax credits unchanged: for example, the present tax credit applied to employees is greater than the one applied to pensioners, and the one applied to pensioners is greater than that which is applied to self-employed taxpayers; similarly, the tax credit for tenants is greater with regard to younger ones.

In doing so, we do not allow the GA to run free with too ‘imaginative’ solutions.

Then, we have to define our target: we are interested in obtaining the highest possible redistributive effect of the tax. To measure it, we refer to the Reynolds-Smolensky *RS* index, given by the difference between the Gini coefficient for the pre-tax income distribution and the corresponding concentration coefficient for the post-tax distribution. Instead of the equivalent household gross and net income distributions, we refer to the taxpayers’ ones. The main reasons are twofold: the cash transfer introduced by the Italian government favours taxpayers; this is the first exercise that employs a genetic algorithm for a tax system optimization, so that by observing the composition of the tax cut by income classes, we can ensure that our result is among the ‘best’ (by referring to the equivalent household income distribution, this check could be much harder to assess). The sensitivity analysis we made on the GA ‘best’ result confirms the optimality of the solution discussed here; results are available upon request.

For each tax structure and each taxpayer, the GA computes all the relevant tax variables in the transition from the pre- to the post-tax income. For each tax structure, it then computes the overall tax revenue, the share of loser taxpayers by considering each taxpayer’s actual net tax liability, the average loss for the loser taxpayers, as well as the Reynolds-Smolensky *RS* index (these are the four parts of the objective fitness function to maximize, see below); it then saves the resulting values on a dump file. We employ a computer powerful enough to evaluate 2 million runs in about 4 days; the duration of our average run is then 0.18 seconds.

The GA has to maximize a fitness function. We employed

$$fitness = e^{\frac{\alpha RS - \beta\Delta - \delta\Pi - \lambda\Omega}{10}} \quad (12)$$

where Δ is the percentage deviation of the computed tax revenue of each run from the target one (139.424 billion euros, 9.324 billion euros less than the present tax revenue), whilst Π is the share of taxpayers losing with the simulated tax structure, Ω is the average loss (in euros) for the loser taxpayers, α, β, δ and λ are all positive parameters. We fix $\alpha = 580$, $\beta = 53$, $\delta = 20$ and $\lambda = 150^{-1}$. We made several attempts, with different parameters and different functional forms; this fitness function has proved to be the most effective.

The exponential form of the fitness function helps the GA to always converge to the 'best' solution, since generation after generation, more-than-linearly high scores are assigned to the most promising candidate solutions.

The first term of the exponent shows the part of the fitness function depending on *RS*. We highly favour *RS* with respect to Δ, Π, and Ω, since we are interested in obtaining the highest Reynolds-Smolensky index. The second and third, as well as the fourth terms of the exponent of the fitness function, show the 'penalties' we impose on the fitness value when Δ, Π and Ω became too large. As a result, the smaller Δ, Π and Ω, the more the fitness value increases. The parameter λ is crucial for the convergence of the GA: there are combinatorial spaces where *RS* increases even if Π increases; a low value for λ allows the GA to attribute low scores to those candidate solutions.

When the GA has evolved for a reasonable number of generations, we obtain $\Delta \approx 0$, $\Pi \approx 0$, the smallest Ω, and the highest *RS*. The parameters of the 'best' structure of the tax can then be observed.

Figure 2 shows the maximum value of the fitness function applied in this work for each generation. As can be noted, the maximum value tends to increase generation after generation. The highest fitness value is 26.93.

FIGURE 2 AROUND HERE

6.2.2. The Choice of Chromosomes

Each candidate tax structure is characterized by 33 different parameters, each of them related to a specific parameter of the Italian PIT structure described in Section 2. In order to evaluate each of the 33 parameters, the GA couples 36 'chromosomes' that are values ranging from zero to 1.

We now turn to describing how we let the GA trim each 'chromosome'.

First of all, we let the GA choose five marginal tax rates as in the actual tax code. Given the constraints we impose, we know that the top marginal tax rate cannot be too much higher than the present one (since we impose a 'no loser' taxpayers' constraint); moreover, we do not want it to be higher than the present value. Conversely, we do not know the minimum allowable value of the bottom marginal tax rate; we set the lowest marginal tax rate at not lower than 17 per cent (being the present value equal to 23 per cent).

In particular, the GA randomly sets a group of six chromosomes serving for the definition of the five marginal tax rates. It then adds them up in order to obtain a normalization value as follows: $norm_t = \frac{0.43-0.17}{\sum_{\varphi=1}^{6} chromosome_\varphi}$.

The GA finally chooses the five tax rates t_λ with $\lambda = (1,2,3,4,5)$ as follows:

$$t_\lambda = 0.17 + \sum_{\varphi=1}^{\lambda} chromosome_\varphi * norm_t .$$

We then set a second group of 5 chromosomes (7-11) defining the four upper limits $UL_j = LL_{j+1}$ of the thresholds, being $LL_1 = 0$ by definition and $UL_j < UL_{j+1}$. We applied an empirical strategy similar to that employed for the definition of the marginal tax rates and we impose $UL_1 \geq 15{,}000.1$ (see on for the choice of this value) and we let the highest value of UL_4 be 75 thousand euros (as in the present tax structure). As a consequence,

$$norm_{UL} = \frac{75{,}000 - 15{,}000.1}{\sum_{\varphi=7}^{11} chromosome_\varphi}$$

and, for $j = 1,2,3,4$,

$$UL_j = 15{,}000.1 + \sum_{\varphi=1}^{j} chromosome_{\varphi} * norm_{UL}\ .$$

Afterwards, we define 25 chromosomes related to the tax credits' structure. Starting at the tax credit for employees (Equation (3)), we let the GA choose the no tax area m_1 applied to employees, that is the limit of pre-tax income below which these taxpayers face a zero net tax liability, between 8 thousand euros (the present value) and $\psi * UL_1$: $m_1 = 8{,}000 + chromosome_{12} * (\psi * UL_1 - 8{,}000)$. We set $\psi = \frac{8}{15}$, equal to the present value. Note that $UL_1 = 15{,}000.1$ ensures m_1 to be properly set.

The choice of the parameter ψ influences the equity-efficiency trade-off. The higher the value chosen by the GA, the more likely the slope of the effective marginal tax credit in the income level $m_r - LL_2$ is high; therefore, the higher the effective marginal tax rate for this income bandwidth.[7] The constraint we impose on the share of loser taxpayers lets the GA choose the highest admissible value for m_1 and at the same time keep under control the level of the effective marginal tax rates (see on).

The GA then choose m_2, m_3, m_4 as follows:

$$\begin{aligned} m_2 &= (0.9375 * m_1) - (chromosome_{13} * 0.2 * 0.9375 * m_1) \\ m_3 &= m_2 + chromosome_{14} * (0.96875 * m_1 - m_2) \\ m_4 &= 0.6 * m_1 - chromosome_{15} * 0.3 * m_1 \end{aligned}$$

where 0.2 and 0.3 are arbitrarily chosen, whilst $0.9375 = \frac{m_2}{m_1}$, $0.96875 = \frac{m_3}{m_1}$, and $0.6 = \frac{m_4}{m_1}$ are the corresponding values according to the present tax structure. In so doing, we let the GA choose m_r with $r = (1,2,3,4)$ in a large combinatorial space preserving the present rank of m_r.

[7] If we let the GA to run free with respect to this parameter, for example by letting the GA choose it up to 1, a lower than 1 value would be chosen (given the constraints we impose), a higher redistributive effect would be obtained, as well as a confiscatory effective tax rate for incomes just above the 'no tax area'. The reason is clear: *ceteris paribus*, having to maximize the Reynolds-Smolensky index given a revenue constraint, the larger the share of taxpayers with a nil net tax liability, the higher the Reynolds-Smolensky index (and the higher the tax rates and narrower the upper limit of each threshold in order to obtain the target tax revenue). As a consequence, the larger the 'no tax area', the narrower the income bandwidth between the 'no tax area' and the upper limit of the bottom threshold, and then the more sharply the subsequent reduction of the effective tax credit.

Having the GA chosen t_1 and m_r, note that the potential tax credits $t_1 m_r$ are automatically defined. We then let the GA choose also the parameters a_r with $r = (1,2,3)$ in the range $\left(0 - t_1 m_r\right)$ by defining chromosomes 16, 17, and 18 (as described in Section 2, according to the actual tax code the parameter a_4 is equal to zero and we keep it unchanged). In so doing, a very large combination of tax credits for earned incomes is allowable.

Finally, we always set the parameter *b* equal to zero. In 2010 fiscal year, it is applied only to employees and its values range from 10 to 40 euros for levels of x_i^{MR} belonging to the threshold 23-28 thousand euros. We prefer this parameter to be fixed at zero since, if it were positive, it would not let the tax credit under discussion be a continuous function for all levels of x_i^{MR}. Finally, note that we do not let the effective tax credits for earned income be piecewise decreasing with respect to limits others than those observed in the rate schedule. If $c_i^1\left(x_i^{MR}\right)$ were piecewise decreasing with respect to other thresholds, the number and the level of effective marginal tax rates would not be under control, leading to unpleasant and inefficient outcomes.

We continue by defining specific chromosomes, in order to set the combinatory space for the three tax credits for dependent individuals within the household: $c_i^{2H}\left(x_i^{MR}\right)$, $c_i^{2S}\left(x_i^{MR}\right)$ and $c_i^{2O}\left(x_i^{MR}\right)$.

Starting with the tax credits for dependent children $c_i^{2H}\left(x_i^{MR}\right)$, we let the GA choose the potential level of the tax credits c_i^{2Hpl} in the range 600-5,000 euros (being the present values ranging between 800 and 1,100 euros). Similar to the choice of the tax rates and the upper limits of the thresholds, we then define 5 different chromosomes (from 19 to 23) to set the 4 kinds of tax credits for dependent children. In so doing, we set specific constraints, in order to let the potential tax credit be higher for households with more than 3 children and lower for those with fewer than 3 children, as well as higher for children aged 3 or less and lower for a child aged more than 3. Then we introduce chromosomes 24 and 25 for the choice of parameters *q* and *e*. We let the GA choose *q* between 30 and 200 thousand euros (being the present value equal to 95 thousand), and the parameter *e* between zero and 75 thousand euros (being the present value equal to 15 thousand). Note that these are very large ranges, so that the GA can choose an extremely large set of combinations. Finally, the GA chooses chromosome 26 in order to set c_i^{2HF} between zero and 2,000 euros (being the actual value 1,200 euros).

Turning to the effective tax credit for the spouse $c_i^{2S}\left(x_i^{MR}\right)$, we generate chromosome 27 in order for c_i^{2Sp} to range between 500 and 2,500 euros (being the present value equal to 800 euros), and a further chromosome 28 in order for the parameter *u* to range between zero and c_i^{2Sp}. Chromosomes 27 and 28 let the effective tax credit for the spouse be a non-increasing function with respect to x_i^{MR} and let the GA choose among a very large combination of structures for this tax credit.

Looking at Equation (5), this effective tax credit $c_i^{2S}\left(x_i^{MR}\right)$ is piecewise linearly decreasing with respect to three thresholds: from zero to LL_2, from LL_2 to $LL_3 < w < LL_4$, and from *w* to $k > LL_5$. In order to define *w* and *k*, we introduced chromosomes 29 and 30 as follows:

$$w = LL_2 + chromosome_{29} * \left(LL_4 - LL_2\right)$$

$$k = w + chromosome_{30} * (150{,}000 - w)$$

Finally, concerning the tax credit for other dependent individuals within the household, we introduce chromosome 31 in order for c_i^{2Op} to range between $0.75 * c_i^{2Hp1}$ and $0.95 * c_i^{2Hp1}$, and we impose $c_i^{2O}\left(x_i^{MR}\right)$ to be linearly decreasing between zero and *k*, and to be zero if $x_i^{MR} > k$.

Afterwards, we let the GA choose chromosomes 32, 33 and 34 in order to set tax credits for tenants:

$$tenants_1 = chromosome_{32} * 1{,}500$$

$$tenants_2 = chromosome_{33} * tenants_1$$

$$tenants_3 = tenants_1 + chromosome_{34} * (tenants_1 * 2)$$

According to the present tax code, these tax credits are applied to two income thresholds: 15,494 and 30,987 euros; we consider this aspect by letting them change according to UL_1 and UL_2.

Finally, the GA chooses chromosomes 35 and 36 in order to set the percentage of the expenses the tax law admits as further tax credits for items of expenditure. We let it chose $expenditure_1$ between 0 and 70 per cent (at present equal to 19), and $expenditure_2$ between 20 and 90 per cent (at present equal to 36).

7. Results

Table 3 shows all the parameters of the 'best' tax structure able to maximize the Reynolds-Smolensky *RS* index, given that the tax revenue is exactly 9.324 billion euros lower than the present one and (almost) no taxpayers have to be worse-off due to the tax reform.

TABLE 3 AROUND HERE

As can be noted, the bottom marginal tax rate t_1 significantly decreases from 23 to 21.39 per cent; its reduction lowers the gross tax liability, not only for the poorest taxpayers but also for all the other taxpayers; t_4 also decreases from 41 to 40.70 per cent; as expected, t_5 is set to its maximum admittable value (43 per cent). On the contrary, the other two marginal tax rates increase: t_2 from 27 to 30.66 per cent, and t_3 from 38 to 38.99 per cent.

In terms of the bandwidth of the thresholds, the first one broadens from 0-15,000 to 0-20,815.53 euros, whilst the second narrows from 15,000-28,000 to 20,815.53-27,314.58 euros. No remarkable changes can be observed for the bandwidth of the third and fourth thresholds; the upper limits of the third threshold increases from 55,000 to 55,551.83 euros, whilst that of the fourth decreases from 75,000 to 72,757.14.

The 'no tax' area enlarges for all four kinds of taxpayer: m_1 increases from 8,000 to 11,070.33 euros, m_2 from 7,500 to 10,031.52, m_3 from 7,750 to 10,523.08, whilst m_4 rises from 4,800 to 5,841.36. As expected, $\frac{m_1}{UL_1}$ is set to (almost) its maximum value.

The parameters that define the shape of the effective tax credits for earned income increase considerably: a_1 from 502 to 1,489.72 euros, a_2 from 470 to 1,392.05, and a_3 from 486 to 1,393.99 euros (note that in all cases the GA sets $a_r < t_1 m_r$).

Since we impose the tax credit $c_i^1(x_i^{MR})$ piecewise decreasing with respect to m_r, LL_2 and LL_4, its shape does not change with respect to those originally observed, as shown in Figure 3, for what concerns employees (similar pictures emerge if pensioners and self-employed are considered).

FIGURE 3 AROUND HERE

However, after the reform, the slope of the effective tax credit in the income range $(m_1 - LL_2)$ is higher (in absolute value). Such a change affects the level of the effective marginal tax rates (EMTRs) in this income bandwidth, which increase with respect to the ones observed before the tax reform (Figure 4).

FIGURE 4 AROUND HERE

Therefore, an equity-efficiency trade-off emerges; in order for the Reynolds-Smolensky index to be the highest, we have to agree to higher EMTRs for taxpayers belonging to the income range $m_1 - UL_2$. In order to lower the effective marginal tax rate on the bottom of the income distribution, a lower value for parameter ψ should be set and/or high values for parameter a_r should be denied; RS would necessarily decrease.

The shape of the tax credit for a spouse $c_i^{2S}(x_i^{MR})$ is similar to the present one (Figure 5).

FIGURE 5 AROUND HERE

Both the potential tax credit and the effective tax credit are higher than before (except in the income range $UL_1 - w$); the effective one becomes zero above $k = 116{,}204.00$ euros, whilst parameter *w* decreases from 40,000 euros to 34,994.19.

In terms of tax credits for dependent children, the two tax credits for children, if the dependent children within the household are 3 or less, are very similar to those observed before the tax reform: c_i^{2Hp1} decreases from 800 to 700.48 euros, whilst c_i^{2Hp2} decreases from 900 to 774.79 euros. The other two tax credits are significantly higher: both of them are set equal to 1,218.04 euros. Even if c_i^{2Hp3} and c_i^{2Hp4} are only a little bit higher than before, the GA sets c_i^{2HF} equal to zero.

The income limits above which this tax credit becomes zero substantially change: q is equal to 110,937.33, whilst e is equal to 41,485.32. A similar picture emerges when considering the tax credit for other dependent individuals within a household: the potential tax credit is a little lower than the present value (665.46 euros), and it is positive for income below 116,204.00 euros.

Focusing on the remaining parameters of the tax, the tax credits for tenants are close to those observed before the tax reform: the tax credit for taxpayers with gross income below UL_1 is 439.68 euros as opposed to 300, the tax credit for tenants with income in the range $UL_1 - UL_2$ is set to 187.00 euros instead of 150, whilst the tax credit for younger tenants is set to 732.90 euros instead of 992.

Finally, the percentages of expenses that the tax law admits as a tax credit also remain relatively unchanged: 19.42 per cent instead of 19 per cent, and 41.46 per cent instead of 36 per cent.

Very few taxpayers are worse-off as a result of this tax reform (0.85 per cent and, on average, they lose 26.57 euros per year), whilst 24.48 per cent are unaffected (we consider the taxpayers, for whom the absolute value of the computed net tax liability differs from the present one by, at most, one euro, as unaffected).

The remaining 74.67 per cent of taxpayers gain from the reform. Looking at the composition of the tax cut in terms of income classes, 93.31 per cent of the tax cut favours taxpayers in the income range of 8-28 thousands euros, whilst 1.93 per cent favours taxpayers with lower incomes (Table 4).

TABLE 4 AROUND HERE

This is due to the fact that the Italian personal income tax system does not admit negative income taxation; therefore, taxpayers with a nil net tax liability (almost all taxpayers with income lower than 8 thousand euros) are not affected by the tax reform. If the Italian PIT allowed negative income taxation, the tax reform would show a different distribution in terms of the tax cut among income classes; in particular, there would be lower gain for the top income earners and higher gain for the bottom ones.
Only 3.20 per cent of the tax cut favours taxpayers with incomes in the range of 28-55 thousand euros, whilst the remaining 1.55 per cent favours richer taxpayers. It can be observed that the *RS* could be higher, were the (very low) gains of the richer taxpayers transferred to the poorest ones. Given the structure of 'chromosomes' described in sub-section 6.2, this is not possible or, at most, not likely since the GA has to balance the effects on *RS*, Δ, Π and Ω according to 33 parameters.
Finally, Table 5 compares the inequality indexes for taxpayers according to the present tax structure, and those obtained by applying the new structure of the tax to the same pre-tax income distribution.

TABLE 5 AROUND HERE

As can be noted, the Reynolds-Smolensky index *RS* is 9.08 per cent higher than the present value: since the overall average tax rate decreases from 18.70 per cent to 17.53 per cent, the Kakwani index *K* increases by 18.05 per cent. The GA tax reform positively affects R^{APK}, which decreases by 8.74 per cent. The overall redistributive effect *RE* increases by 9.29 per cent.
The tax reform discussed here also improves redistributive indexes with respect to the reform implemented by the government (Table 6). More precisely, *RS* considerably increases, from 0.05707 to 0.05739.

TABLE 6 AROUND HERE

This is a very large improvement, for two reasons: the two tax structures yield exactly the same tax revenue; and the reform implemented by the government lets some taxpayers face a net income greater than the gross one (this aspect also has a great impact on the reduction of the Gini coefficient for net income distribution). This is not allowed by the GA tax reform. A similar improvement can be observed by comparing the overall redistributive effects *RE*; it is 0.05611 with the reform implemented by the government, and 0.05683 with the reform obtained within the GA framework. Note also that the GA tax reform positively affects the Atkinson-Plotnick-Kakwani index R^{APK}: this index switches from a very high and unreasonable value (0.00096, that is 54 per cent more than the present value) to a normal value (0.00056, that is 10.19 per cent better than the present value).

Finally, in order to ascertain that the GA 'best' tax structure ensures the highest admittable Reynolds-Smolensky index, given the constraints we imposed, we employ a sensitivity analysis for each of the 33 parameters that define the Italian PIT structure. As previously discussed, our fitness function is composed by four elements: the Reynolds-Smolenky index *RS*, the share of loser taxpayers Π, the percentage deviation from the target tax revenue Δ and the average loss Ω for the loser taxpayers. Given the target tax revenue, we are interested in obtaining the highest admittable *RS* with no losing taxpayers.

The highest fitness function value we obtained should therefore be an absolute maximum for our problem. The sensitivity analysis shows that this is the case and it also permits us to evaluate the impact on each of the four parts that compose the fitness function due to a small change of each parameter of the tax (we check for small changes ranging from -1 per cent to +1 per cent by steps of 0.1 per cent). No small change of all 33 parameters increases the fitness value.

The most sensitive parameters (the ones reducing the fitness value by more than one per cent) are the five tax rates, the first two upper limits of the thresholds, and the parameters defining the 'no tax area'. All the other tax parameters determine a small reduction on the fitness function value.

Let us consider the small changes of t_1. If t_1 were increased by 1 per cent, *RS* would increase by 0.09 per cent; such a change would reduce the fitness value by 15.80 per cent, due to the violation on Δ (which would increase by 0.51 per cent) and, of course, the violation on Π (which would increase by 898.98 per cent) as well as that on Ω

(which would decrease by 24.22 per cent). A similar picture emerges when considering a small reduction in the level of t_1: a reduction of t_1 by 1 per cent would reduce the fitness value by 3.73 per cent, *RS* by 0.09, Δ by 0.51, Π by 69.66, and would increase Ω by 107.14 per cent.

8. Concluding Remarks

In this paper, we propose a new methodology to implement a personal income tax reform. In particular, given a settled tax cut decided upon by the government (a similar strategy can be applied if the tax revenue increases), we show how a genetic algorithm can be employed, in order to find out the values of all parameters defining the structure of the personal income tax able to satisfy a specific target. Our methodology can be applied to any other specific target; as an example, in this work our target is the maximization of the redistributive effect of the tax, while preventing all taxpayers being worse off with respect to the present tax structure. We apply this methodology to the Italian personal income taxation system for two reasons: the tax structure is quite complicated, and recently the government decided to reduce tax revenue by 9.324 billion euros. The aim of this tax cut is to increase the purchasing power of 'poor' taxpayers and taxpayers belonging to the 'middle class', and the instrument is the introduction of a cash transfer (not related to the structure of the personal income tax) only for employees with gross incomes in the range 8-26 thousand euros (in order for the yearly gain to be about one thousand euros), whilst all other kinds of taxpayer are not affected by this money transfer. Here we show that a better and more equity-oriented reform is possible. This methodology allows a short run reform, and can help policy makers when they think of a tax reform.

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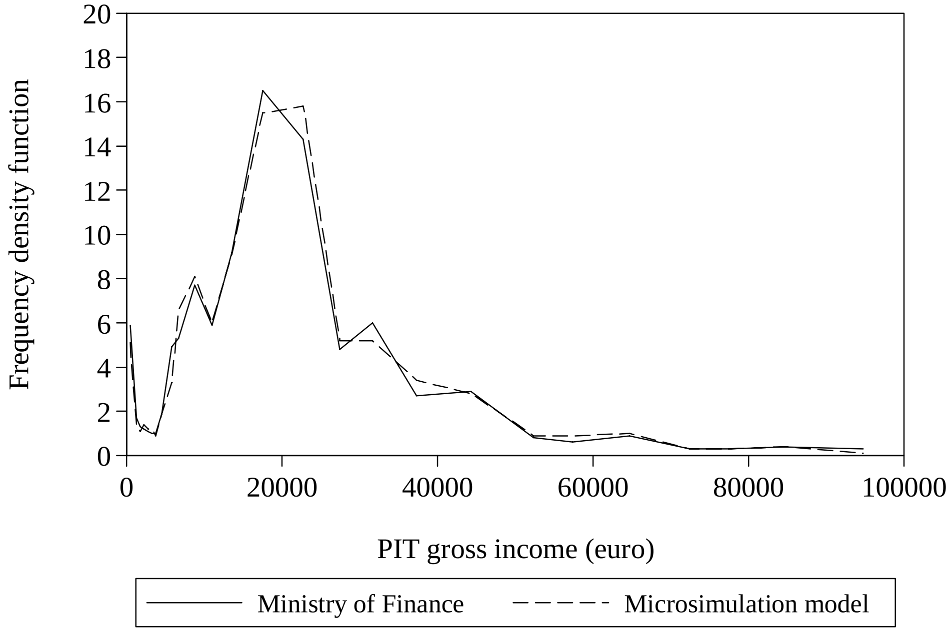

Fig. 1. Frequency density function for all individual taxpayers.

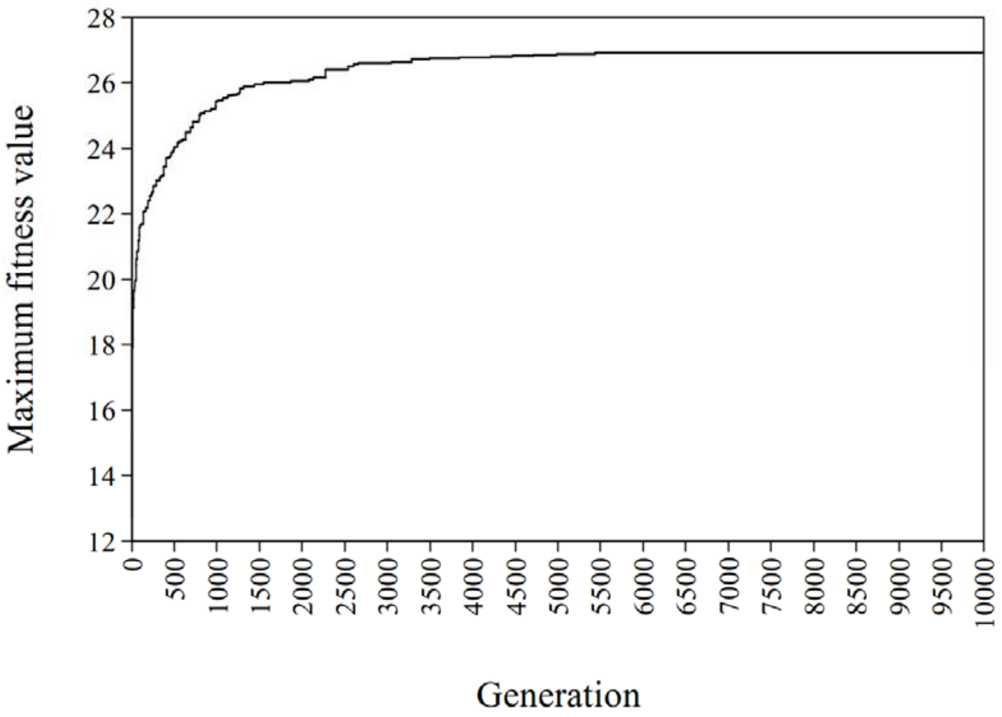

Fig. 2. Maximum values of the fitness function.

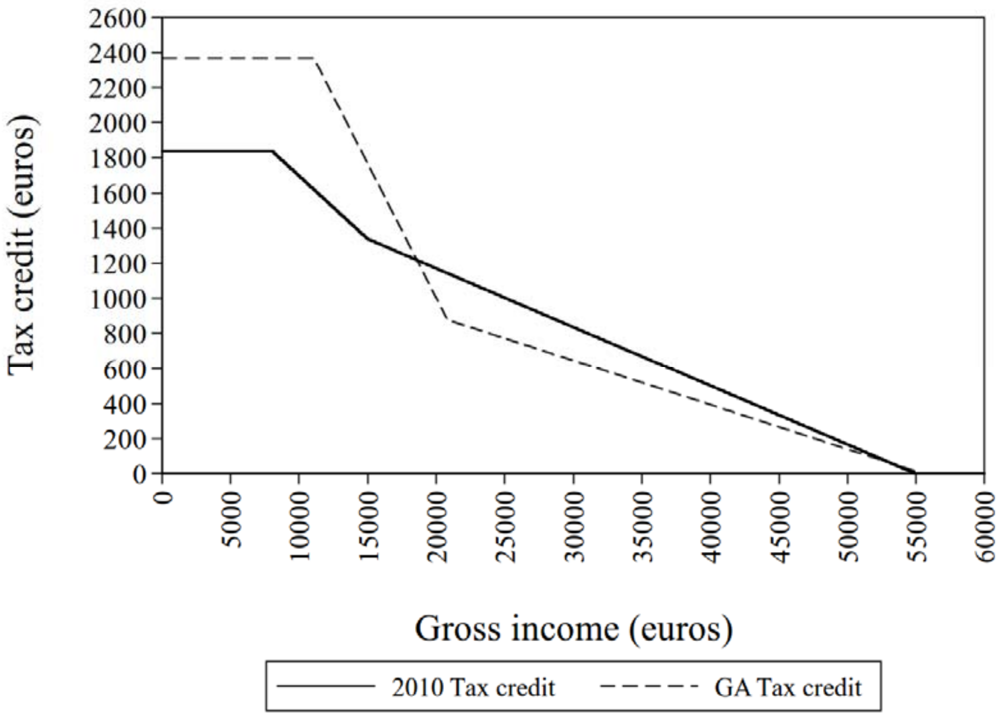

Fig. 3. The effective tax credit for an employee.

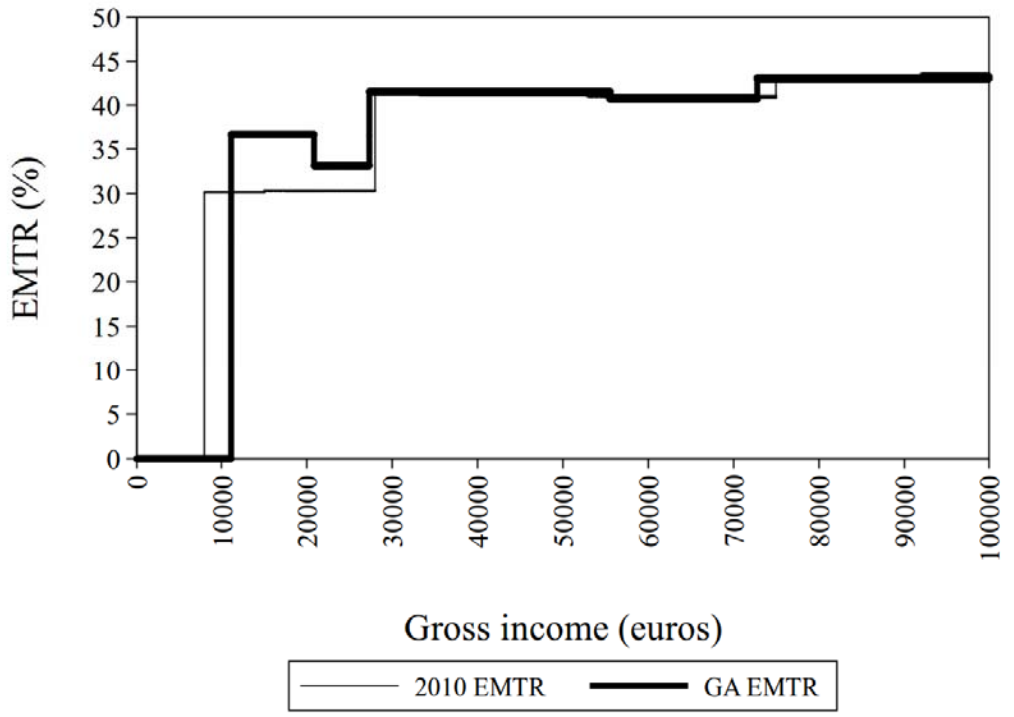

Fig. 4. The effective marginal tax rates for a celibate employee without children.

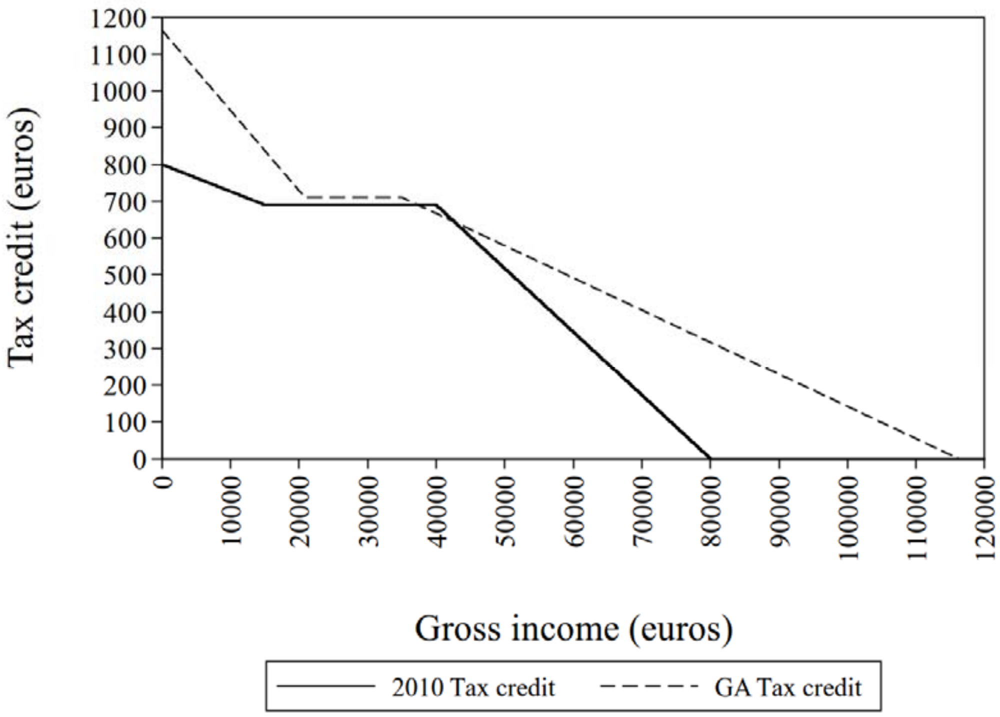

Fig. 5. The effective tax credit for a spouse.

Table 1
Rate schedule.

Taxable income (euros)			
Threshold (*j*)	Lower limit (*LL*)	Upper limit (*UL*)	Tax rate (*t*)
1	0	15,000	0.23
2	15,000	28,000	0.27
3	28,000	55,000	0.38
4	55,000	75,000	0.41
5	75,000	-	0.43

Table 2

Inequality indexes for taxpayers: present values and Government’s reform ones

Index	Present value	With cash transfer	Absolute difference	Percentage difference
Gini coefficient for the gross income	0.44338	0.44338	0.00000	0.00
Gini coefficient for the net income	0.39138	0.38727	-0.00412	-1.05
Concentration coefficient for the net income	0.39076	0.38631	-0.00445	-1.14
Gini coefficient for the net tax liability	0.68150	0.73268	0.05117	7.51
Concentration coefficient for the net tax liability	0.67215	0.71191	0.03976	5.92
Redistributive effect	0.05200	0.05611	0.00412	7.92
Reynolds-Smolensky index	0.05262	0.05707	0.00445	8.46
Kakwani index	0.22877	0.26853	0.03976	17.38
Atkinson-Plotnik-Kakwani index	0.00062	0.00096	0.00033	54.00
Average tax rate (%)	18.70	17.53	-1.17	-6.27

Table 3

Present and computed parameters of the tax.

Parameters	Present value	Best Value
t_1	0.23	0.21390
t_2	0.27	0.30656
t_3	0.38	0.38994
t_4	0.41	0.40698
t_5	0.43	0.43000
UL_1	15,000	20,815.53
UL_2	28,000	27,314.58
UL_3	55,000	55,551.83
UL_4	75,000	72,757.14
m_1	8,000	11,070.33
m_2	7,500	10,031.52
m_3	7,750	10,523.08
m_4	4,800	5,841.36
a_1	502	1,489.72
a_2	470	1,392.05
a_3	486	1,393.99
c_i^{2Sp}	800	1,164.16
u	110	453.38
w	40,000	34,994.19
k	80,000	116,204.00
c_i^{2Op}	750	665.46
c_i^{2Hp1}	800	700.48
c_i^{2Hp2}	900	774.79
c_i^{2Hp3}	1,000	1,218.04
c_i^{2Hp4}	1,100	1,218.04
q	95,000	110,937.33
e	15,000	41,485.32
c_i^{2HF}	1,200	0.00
$tenants_1$	300	439.68
$tenants_2$	150	187.00
$tenants_3$	992	732.90
$expenditures_1$	0.19	0.19422
$expenditures_2$	0.36	0.41461

Table 4

The composition of the tax cut by income classes.

Income class (thousand euros)	Composition of the tax cut (%)	Winner (%)	Indifferent (%)	Loser (%)	Total (%)	Average win (euros)	Average loss (euros)
0-8	1.93	5.94	19.31	0.10	25.35	73.1	12.6
8-15	39.88	17.26	4.68	0.00	21.94	532.6	0.0
15-28	53.43	34.85	0.40	0.10	35.35	343.5	53.1
28-55	3.20	13.03	0.03	0.47	13.52	52.6	33.7
55-75	0.89	2.06	0.03	0.09	2.19	92.5	4.6
above 75	0.67	1.54	0.03	0.09	1.65	86.9	2.1
Total	100.00	74.67	24.48	0.85	100.00	303.23	26.57

Table 5

Inequality indexes for taxpayers: present values and GA ones

Index	Present value	Best value	Absolute difference	Percentage difference
Gini coefficient for the gross income	0.44338	0.44338	0.00000	0.00
Gini coefficient for the net income	0.39138	0.38655	-0.00483	-1.23
Concentration coefficient for the net income	0.39076	0.38599	-0.00478	-1.22
Gini coefficient for the net tax liability	0.68150	0.72235	0.04085	5.99
Concentration coefficient for the net tax liability	0.67215	0.71344	0.04129	6.14
Redistributive effect	0.05200	0.05683	0.00483	9.29
Reynolds-Smolensky index	0.05262	0.05739	0.00478	9.08
Kakwani index	0.22877	0.27006	0.04129	18.05
Atkinson-Plotnik-Kakwani index	0.00062	0.00057	-0.00005	-8.74
Average tax rate	18.70	17.53	-1.17	-6.27

Table 6

Inequality indexes for taxpayers: Government's tax reform and GA tax reform

Index	With cash transfer	Best value	Absolute difference	Percentage difference
Gini coefficient for the gross income	0.44338	0.44338	0.00000	0.00
Gini coefficient for the net income	0.38727	0.38655	-0.00071	-0.18
Concentration coefficient for the net income	0.38631	0.38599	-0.00033	-0.08
Gini coefficient for the net tax liability	0.73268	0.72235	-0.01032	-1.41
Concentration coefficient for the net tax liability	0.71191	0.71344	0.00153	0.22
Redistributive effect	0.05611	0.05683	0.00071	1.27
Reynolds-Smolensky index	0.05707	0.05739	0.00033	0.57
Kakwani index	0.26853	0.27006	0.00153	0.57
Atkinson-Plotnik-Kakwani index	0.00096	0.00057	-0.00039	-40.74
Average tax rate	17.53	17.53	0.00	0.00