Random Unslotted Time-Frequency ALOHA: Theory and Application to IoT UNB Networks

Claire Goursaud, Yuqi Mo

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Claire GOURSAUD
Univ Lyon
INSA-Lyon, Inria
CITI
Lyon, F-69621, France
Email: claire.goursaud@insa-lyon.fr

Yuqi MO
Univ Lyon
INSA-Lyon, Inria
CITI
Lyon, F-69621, France
SIGFOX Company,
Labège, F-31670, France
Email: mo.yuqi@insa-lyon.fr

Abstract—**The ALOHA protocol is regaining interest in the context of the Internet of Things (IoT), especially for Ultra Narrow Band (UNB) signals. In this case, the classical assumption of channelization is not verified anymore, modifying the ALOHA performances. Indeed, UNB signals suffer from a lack of precision on the actual transmission carrier frequency, leading to a behavior similar to a frequency unslotted random access. In this paper, the success probability and throughput of ALOHA is generalized to further describe frequency-unslotted systems such as UNB. The main contribution of this paper is the derivation of a generalized expression of the throughput for the random time-frequency ALOHA systems. Besides, this study permits to highlight the duality of ALOHA in time and frequency domain.**

I. Introduction

Internet of Things (IoT) is a new concept gaining more and more interest from researchers and industries [1][2]. The objective is to provide connectivity to a very large number of heterogeneous devices such as RFID tags, actuators, and sensors. The collected data can then be exploited in several application areas, such as smart cities, home automation, industrial applications, among others. The main common feature between these devices is that, contrary to human-driven communications, object-driven communications usually require low transmission rates. Indeed these objects occasionally send and/or receive a small amount of data.

For the communication point of view, the main challenge is not the individual rate, but rather having a very low energy consumption. With contemporary transmission schemes such as LTE [3], the nodes' emission power can be reduced by using small cells, but this leads to a high number of base stations with small load, but high infrastructure costs. Another option is to consider other transmission schemes that provide sufficient coverage for limited emission power. During the last decade, 4 dedicated transmission schemes have emerged for the uplink of such Low Power Wide Area Networks (LPWAN). They are based on new radio technologies [4]: LoRa (Long Range), Weigthless, RPMA (Random Phase Multiple Access) and UNB-based (Ultra Narrow Band). LoRa and RPMA use spread spectrum techniques, UNB occupies a very narrow band (typically 100 Hz), while Weigthless proposes either UNB or spread spectrum techniques depending on the targeted application. The main advantage of UNB transmissions is that it does not rely on access protocols to handle multiple access [5], while limited signaling is an important issue for IoT networks [6]. Indeed, the hand-shake protocol overheads and the payload packets have almost the same size due to the small amount of data to transfer. Thus, if a reservation packet is successfully received, there is a high probability that a data packet sent in the same conditions would be correctly received. Therefore, the hand-shake protocol overhead should be avoided. Furthermore, it was shown in [7] that the subdivision of the available frequency band, leading to smaller transmission bandwidth, permits to reduce the overall M2M network energy consumption. Thus, it is expected that UNB will be more energy efficient than spread spectrum techniques. This paper focuses on the case of UNB.

In addition to energy, the node cost is an important issue for IoT deployment. Simple modulation schemes such as BPSK (with data rate of 100 bps) are compliant with UNB. A limited node cost means that the use of off the shelf low cost oscillators for the carrier generation is mandatory. This leads to the main specificity of UNB transmissions: the carrier generation imprecision. As a matter of fact, no oscillator is perfect. This lack of precision leads to an offset between the targeted frequency and the actual generated one. For UNB systems, the uncertainty is higher than the signal bandwidth. At such scale, the network performance is insensitive to an additional increase of the carrier frequency uncertainty. This allows the use of unprecise oscillators, without any loss of performances. Nonetheless, with any uncertainty above the signal bandwidth, contrarily to narrow band systems, it is not possible to obtain non-overlapping frequency channels with reasonable frequential guard intervals [8]. Therefore, classical transmission schemes implicitly based on frequency channelization [9] are not pertinent anymore. This leads to a new paradigm for the multiple access scheme.

In this paper, we take into account the carrier frequency uncertainty to evaluate the ALOHA protocol behavior. The Base Station (BS) collects signals that occur randomly, generated in an unslotted way both in the time and the frequency domains (RFTMA: Random Frequency and Time Multiple Access). This can be viewed as a more general case of the well-known ALOHA medium access [10]. The main difference is that, for

UNB systems, interference occurs only *in a portion of the frequency band*. In this case, the interference cannot be easily processed, neither by transmitter cooperation or signal post-processing at the receiver as in literature [11]. The goal of this paper is to introduce this new RFTMA scheme and provide its theoretical characterization.

Previous works on ALOHA-based schemes do not consider full and continuous randomness in frequency domain. Usually, the band is divided into several perfectly orthogonal channels, and frequency hopping (FH) is considered [12]. However, few works consider the frequency offset. In [13], its impact on the relative phase between considered symbols is taken into account. In [14], the authors consider that most of the frequency errors are within the signal bandwidth, thus they do not consider UNB. In [15], the authors consider K nodes with random frequency offsets relative to a common carrier frequency, and focus on the use of a wide-band receiver to take advantage of the jitters. We complement these studies by considering a uniform and continuous randomness of the carrier frequencies to characterize an UNB-based ALOHA uplink.

The organization of the paper is as follows. In Section II we define a general model of the network. In Section III we develop a theoretical analysis of UNB success probability. In Section IV we present numerical results and applications for a realistic UNB networks. Section V concludes the paper.

II. UNB System Model

We consider a BS that collects data from $N+1$ nodes. The nodes are uniformly deployed in the cell, which can be modeled by a Poisson point process.

All nodes are considered to be perceived with the same power level at the BS. This case corresponds to the worst one. Indeed, when colliding packets are received at the same power level, both are lost. On the contrary, if the received powers are sufficiently unbalanced, the capture effect may permit to decode at least the strongest one [16]. We thus characterize in this paper the lower bound of the network success probability.

Assume that each node transmits a message of duration τ seconds, at a randomly chosen time, every D_p seconds on average. Thus, $p_t = \tau/D_p$ is the expected temporal generation rate for a single user. Besides, transmissions are performed inside a dedicated band, which has bandwidth B. One may note that, each transmission occupies a bandwidth b which represents a very small fraction $p_f = b/B$ of the total channel bandwidth B. Thus, p_f is the frequential occupancy ratio for a single active user. These notations are reported in Table I.

From the base-station point of view, the dedicated band contains from time to time, transmitted ultra narrow signals at random carrier frequencies. For each detected transmission, the BS extracts the signal at the estimated frequency of interest and decodes the packet. Such a detection and estimation can be done as described in a Sigfox patent [17]. They are currently deployed in their commercial network, and do not fall in the scope of this paper.

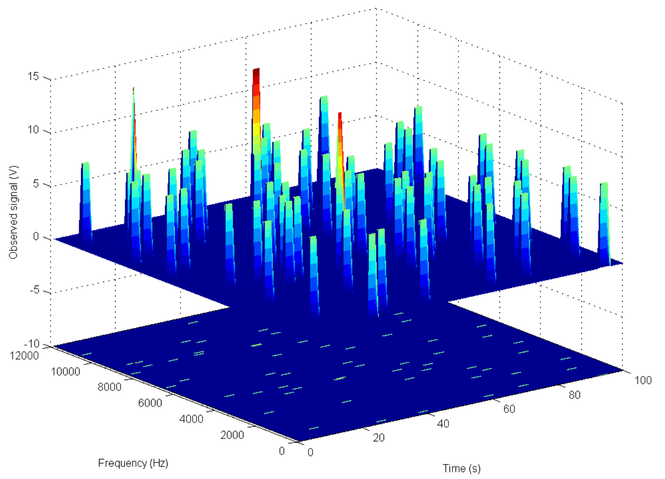

Fig. 1. Example of a time-frequency UNB realization, for $b = 116$Hz, $W = 12000$Hz, $\tau = 2$s, during 100s.

TABLE I
TABLE OF NOTATIONS

N	Number of potentially interfering nodes
τ	Packet duration
D_p	Average transmission period
b	Signal spectrum occupancy
B	Bandwidth of the available channel
p_t	Expected temporal generation rate $= \frac{\tau}{D_p}$
p_f	Frequential occupancy ratio $= \frac{b}{B}$
α_t	$= \begin{cases} 1, & \text{for time-slotted ALOHA} \\ 2, & \text{for time-unslotted ALOHA} \end{cases}$
α_f	$= \begin{cases} 1, & \text{for frequency-slotted ALOHA} \\ 2, & \text{for frequency-unslotted ALOHA} \end{cases}$
G_{tf}	Time-frequency load in the network $= Np_tp_f$
f_i	Carrier frequency of user $i \in \{1; N\}$

An example of channel occupancy realization is presented on Fig.1. We can observe the sparse time-frequency occupations of signals. The three highest peaks (in red) correspond to transmissions that experienced collisions, while the others are interference-free. One may note that the widths (both in time and frequency domain) differ among the collision peaks. This is due to the frequency-unslotted and time-unslotted selection, which induce partial collision in time and frequency domain.

Finally, one may note that a key advantage of UNB is that the noise floor $N_0 = k \cdot T \cdot b$ (with k Boltzmann constant, and T the temperature in Kelvin) which is proportionnal to the bandwidth is highly reduced compared to classical systems. This permits to cover an exceptionally large area with each BS (up to 50km). As noise is negligible, interference is the main limitation of the system performance. We consider in this paper, the interference due to the collisions issuing from uncontrolled random medium access, both in time and frequency domain.

III. Error Probability Theoretical Analysis

In the original ALOHA protocol, all packets are sent on the same frequency channel. Thus, only the transmission time is random.

If the total number of generated packets (retransmissions included) in the network is Poisson distributed, and if even partial-time collision leads to the packet loss, it was shown in [10], that the probability of success of a given user is:

$$P_{\text{1D}} = e^{-\alpha_t G_t} \tag{1}$$

with $\alpha_t = 1$ for time-slotted ALOHA (TS), $\alpha_t = 2$ for time-unslotted ALOHA (TU), and $G_t = Np_t$ the average number of packets generated by all the other users during the considered packet transmission.

In addition, it was shown in [18], that for frequency hopping systems with $\frac{1}{p_f} = B/b$ frequency channels having the same bandwidth, eq.(1) becomes

$$P_{\text{FH}} = e^{-\alpha_t p_f G_t} \tag{2}$$

In the following, we extend these equations to the unslotted frequency selection, and provide a new expression that describes all cases.

Theorem 1. The ALOHA success probability with slotted or unslotted time, and slotted or unslotted frequency, and uniform distribution in time and frequency domain, is given by:

$$P_{\text{2D}} = e^{-\alpha_t \alpha_f G_{tf}} \tag{3}$$

with $G_{tf} = Np_t p_f = \dfrac{N\,\tau\,b}{D_p\,B}$, and $\alpha_f = 2$ (resp.1) for frequency-unslotted (FU) (resp. frequency slotted (FS)) ALOHA.

Proof. For the slotted frequency selection case, i.e., for $\alpha_f = 1$, Theorem 1 simplifies to eq.(2).

The proof now focuses on the unslotted frequency selection $\alpha_f = 2$. To do so, we need to consider the pulse shaping filter of UNB signals. The steep filter edges that can be observed in Fig.1, allow us to approximate the transmitted signal spectrum by a rectangular function with bandwidth b centered at the actual carrier frequency. We use this model to derive the theoretical throughput.

Any desired packet will be correctly received if it does not experience collision. A collision occurs when there is an overlap both in time and frequency domain between the desired and an interfering packet. In the frequency domain, this implies that their respective frequencies f_0 and f_i verify: $|f_0 - f_i| \le b$.

We consider that the transmitting nodes' carrier frequencies are uniformly distributed in the total available bandwidth B. Thus, in the continuous case, spectral collision occurs when at least one undesired user chooses a frequency in the vulnerable band $[f_0 - b, f_0 + b]$. This happens with probability $2 \cdot b/B$. Thus, the spectral collision probability for a unique potential interferer is $\alpha_f \cdot p_f$ with $\alpha_f = 2$. Therefore the probability that a packet is successfully received, given k additionnal simultaneous transmissions is given by:

$$P_{s/k} = (1 - \alpha_f p_f)^k \,. \tag{4}$$

As the total number of packets are generated according to a Poisson point process with rate G_t, the probability to have k users transmitting during (partly or totally) the desired packet is $\frac{(\alpha_t G_t)^k}{k!} \cdot e^{-\alpha_t G_t}$. Therefore, the probability of success for the targeted packet is :

$$\begin{aligned} P_s &= \sum_{k=0}^{\infty} P_{s/k} \cdot \frac{(\alpha_t\ G_t)^k}{k!} \cdot e^{-\alpha_t G_t} \\ &= e^{-\alpha_t G_t} \cdot \sum_{k=0}^{\infty} \frac{(\alpha_t G_t \cdot (1 - \alpha_f p_f))^k}{k!} \\ &= e^{-\alpha_t G_t} \cdot e^{(\alpha_t G_t \cdot (1 - \alpha_f p_f))} \\ &= e^{-\alpha_t \alpha_f G_{tf}} . \end{aligned} \tag{5}$$

This completes the proof. □

In practice, the network behaves as if we use the classical ALOHA protocol restricted to the p_f portion of the undesired users. As a consequence, the number of packets created during a time period and in the frequency collision interval is also Poisson distributed, with an average generation rate of potential time-colliding packets $G_{tf} = G_t \cdot p_f$.

Corollary 1. *Time and frequency domain are dual in the ALOHA protocol.*

The relevance of Corollary 1 lies on the fact that the time (α_t and p_t) and frequency (α_f and p_f) parameters can be indifferently interchanged in eq.(3).

IV. Generalized ALOHA Expression Validation and Results

A. Validation

We present simulation results for the frequency-unslotted case ($\alpha_f = 2$). To be as realistic as possible, we consider typical values used in SigFox's network. Transmissions are performed in the 868 MHz ISM band, and each individual signal occupies a bandwidth $b = 116$ Hz during $\tau = 2$s. With current available good oscillators (0.25 ppm) [19], the frequency uncertainty (217 Hz) would be close to the signal bandwidth. So, with cheaper oscillators, (more suitable for massive deployment), the uncertainty is bigger than the signal bandwidth. These features comply with the UNB definition.

Based on these realistics values, Monte Carlo simulations were conducted for both time-slotted ($\alpha_t = 1$) and unslotted ($\alpha_t = 2$) case. Besides, we tested two kind of signal shapes, depending on the considered filter. The first one is the rectangular filter: this exactly corresponds to the case treated in the theoretical section. The second one is a realistic filter as used in SigFox (with a realistic signal spectrum shape obtained by a 1255th order lowpass FIR filter, with a 100 Hz cut-off

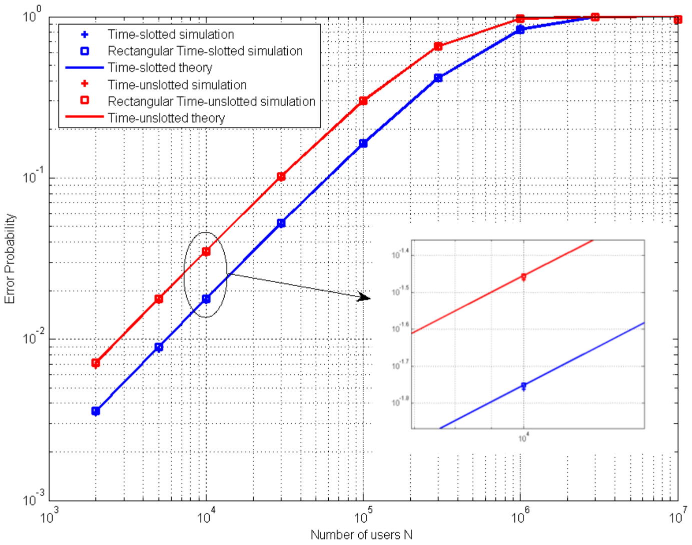

Fig. 2. Simulated and theoretical success probability as a function of the number of undesired users for the time slotted and unslotted case, for $b = 116$Hz, $W = 12000$Hz, $\tau = 2$s, and $D_p = 12$ hours.

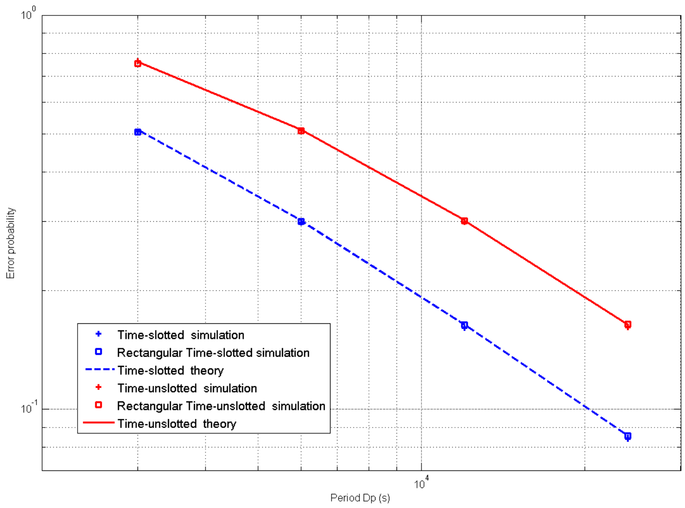

Fig. 3. Simulated and theoretical success probability as a function of the total availaible bandwidth B for the time slotted and unslotted case, for $b = 116$Hz, $N = 1000000$, $\tau = 2$s, and $D_p = 12$ hours.

frequency), to evaluate the accuracy of the rectangular model, and the validity of the theorem. These results are compared to the theoretical ones eq.(3).

We present on Fig.2-4 the comparison of the theoretical and simulation error probability $(1 - P_s)$ as a function of the three unconstrained parameters of the network: total of users $N+1$, total available bandwidth B, and individual packet period D_p.

As expected, we can first note that the error probability increases with the number of users, while it decreases with the available bandwidth and the temporal generation period. More importantly, we verify on all these figures the accuracy of the theoretical analysis. Indeed, both theoretical expression and

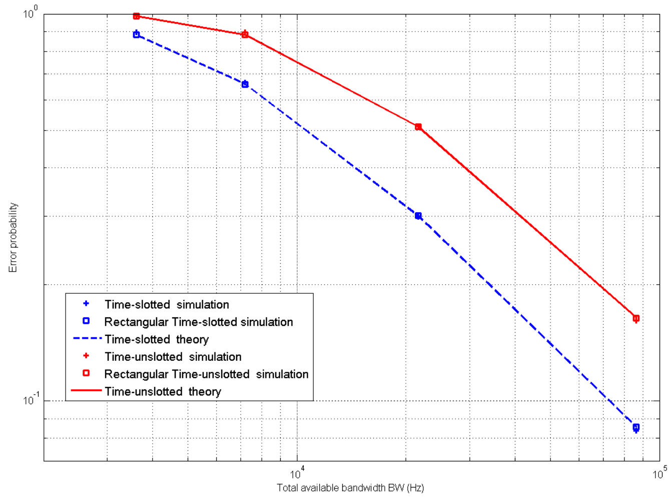

Fig. 4. Simulated and theoretical success probability as a function of the temporal generation period D_p for the time slotted and unslotted case, for $b = 116$Hz, $B = 12000$Hz, $\tau = 2$s, and $N = 100000$.

rectangular model simulation curves coincide, for the time-slotted and time-unslotted cases. Besides, the realistic filter simulation also seems to perfectly coincide. However, if we get more precision (zoom on Fig.2), we can observe that the rectangular model actually overestimate a little the real case. This is due to the fact that the rectangular model slightly overestimates the interference level in the collision band.

To conclude, the proposed theoretical eq.(3) provides a tight upperbound on the realistic frequency-unslotted case.

B. *Throughput derivation and analysis*

To further analyse the network behavior, we use eq.(3) to deduce the throughput of the network T, as a function of G_{tf} the average total load per time-frequency resource:

$$T = G_{tf} e^{-\alpha_t \alpha_f G_{tf}} = N p_t p_f e^{-N \alpha_t p_t \alpha_f p_f} \tag{6}$$

We plot on Fig.5 the throughput as a function of the load G_{tf}. As time and frequency can be independantly slotted or unslotted, there are 4 possible scenarios : frequency-slotted (resp. unslotted) time-slotted : FSTS (resp. FUTS), and frequency-slotted (resp. unslotted) time-unslotted : FSTU (resp. FUTU). We can first note that the FSTS case is the best one as the time-frequency space is divided into orthogonal channels, thus minimizing the probability of collision. On the opposite, FUTU is the worst one as partial overlapping is possible both in time and frequency domain. Finally, FSTU and FUTS coincide due to the time-frequency duality as stated in Corollary (1). Thus, there are in fact only 3 distinct curves, according to the possible values of the product $\alpha_t \cdot \alpha_f$.

One may note that the time-frequency duality brings flexibility. Indeed, if both precise frequency and timing are difficult to handle simultaneously, FSTS can not be achieved. The duality between FSTU and FUTS allows to decide which constraint to relax, independantly of the impact on the performances, but based for example, on the network deployment cost.

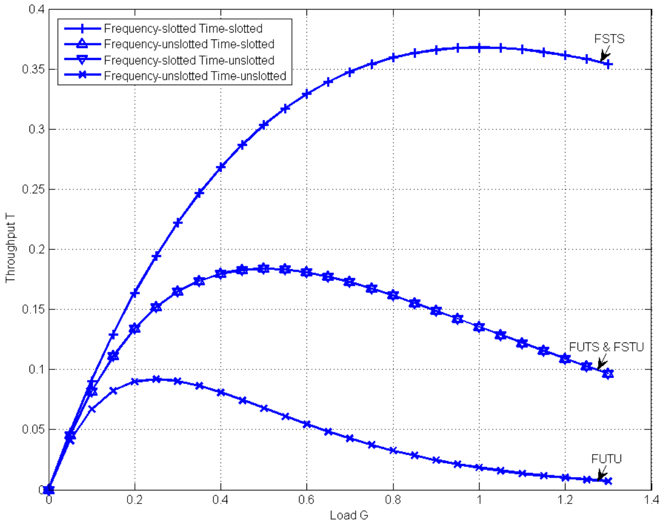

Fig. 5. Network throughput as a function of the load for all (α_t, α_f).

Furthermore, the best achievable load can be evaluated. We derive eq.(6) with respect to G_{tf}. The maximum throughput is obtained for $G_{tf} = \frac{1}{\alpha_t \alpha_f}$, and can be expressed :

$$T = \frac{1}{\alpha_t \cdot \alpha_f} \cdot e^{-1}. \tag{7}$$

As can be verified on Fig.5, there are in practice 3 optimum throughputs depending on the values of α_f and α_t: $T = 1/e$ (for FSTS), $1/2e$ (obtained for either FSTU or FUTS), or $1/4e$ (for FUTU).

Finally, the constraint for the optimum throughput can be written as

$$\frac{N}{B} = \frac{D_p}{\alpha_t \cdot \alpha_f \cdot \tau \cdot b}. \tag{8}$$

So, for a given configuration induced by the targeted application and the targeted rate (i.e. τ, D_p and b fixed), and a given configuration for $\alpha_t \cdot \alpha_f$, the ratio N/B is a constant. It is thus straightforward to dimension the network transmission band for a targeted number of users.

V. Conclusion

In this paper, we have evaluated the extension of the ALOHA scheme to the case of time-frequency random access, as experienced for example with UNB transmissions in IoT networks. We have derived and validated the theoretical expression of the success probability for all the configurations (time slotted or unslotted, and frequency slotted or unslotted). We have exploited the theoretical expression to derive the throughput. Besides, we have also highlighted that frequency randomness and time randomness identically affect the throughput, and that they can be interchanged without loss of performances. This duality is promising as it opens the field to transposition of all existing results on ALOHA to unslotted-frequency networks.

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