Lossy compression of unordered rooted trees

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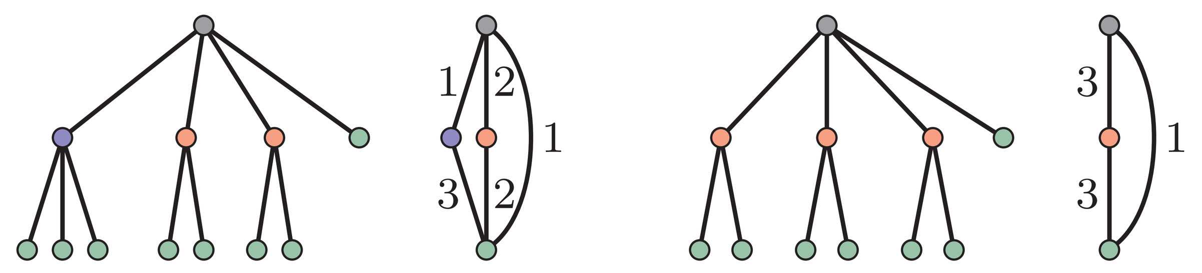
Objectives

DAG compression is a classical technique that consists in building a Directed Acyclic Graph (DAG) that represents an unordered tree without the redundancy of its subtrees.



Only trees with a high level of redundancy are efficiently compressed by this method. We introduce **a lossy compression method that consists in computing the DAG of a self-nested structure** that both presents the highest level of redundancy and **approximates the initial data to compress.**

Edit Distance

This study requires to introduce a distance on the space of unordered trees in order to quantify the quality of a given self-nested estimate of a tree structure. We consider an editing distance defined from the following tree edit operations: **insertion and deletion of a leaf**. An edit script is a sequence of edit operations. The result of applying an edit script s to a tree τ is the tree τ^s obtained by applying the component edit operations to τ in the order they appear in the script. The cost of an edit script is only the number of edit operations. Finally, given two trees τ_1 and τ_2, the edit distance $\delta(\tau_1, \tau_2)$ is the **length of the minimum edit script** that transforms τ_1 into a tree that is isomorphic to τ_2.

Example

The tree data to compress and its DAG version (top) and the solutions provided by the lossy compression algorithms NEST (middle) and LP-RFC (bottom). The NEST solution has too many nodes for looking like the initial tree (107 nodes added to obtain a self-nested tree). The LP-RFC solution is **visually close to the initial tree**, which is confirmed by the error rate of 35%. Both algorithms achieve a compression rate greater than 80%.

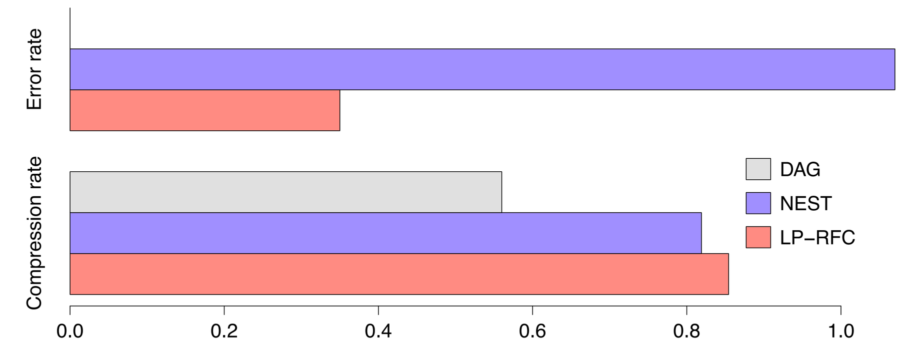
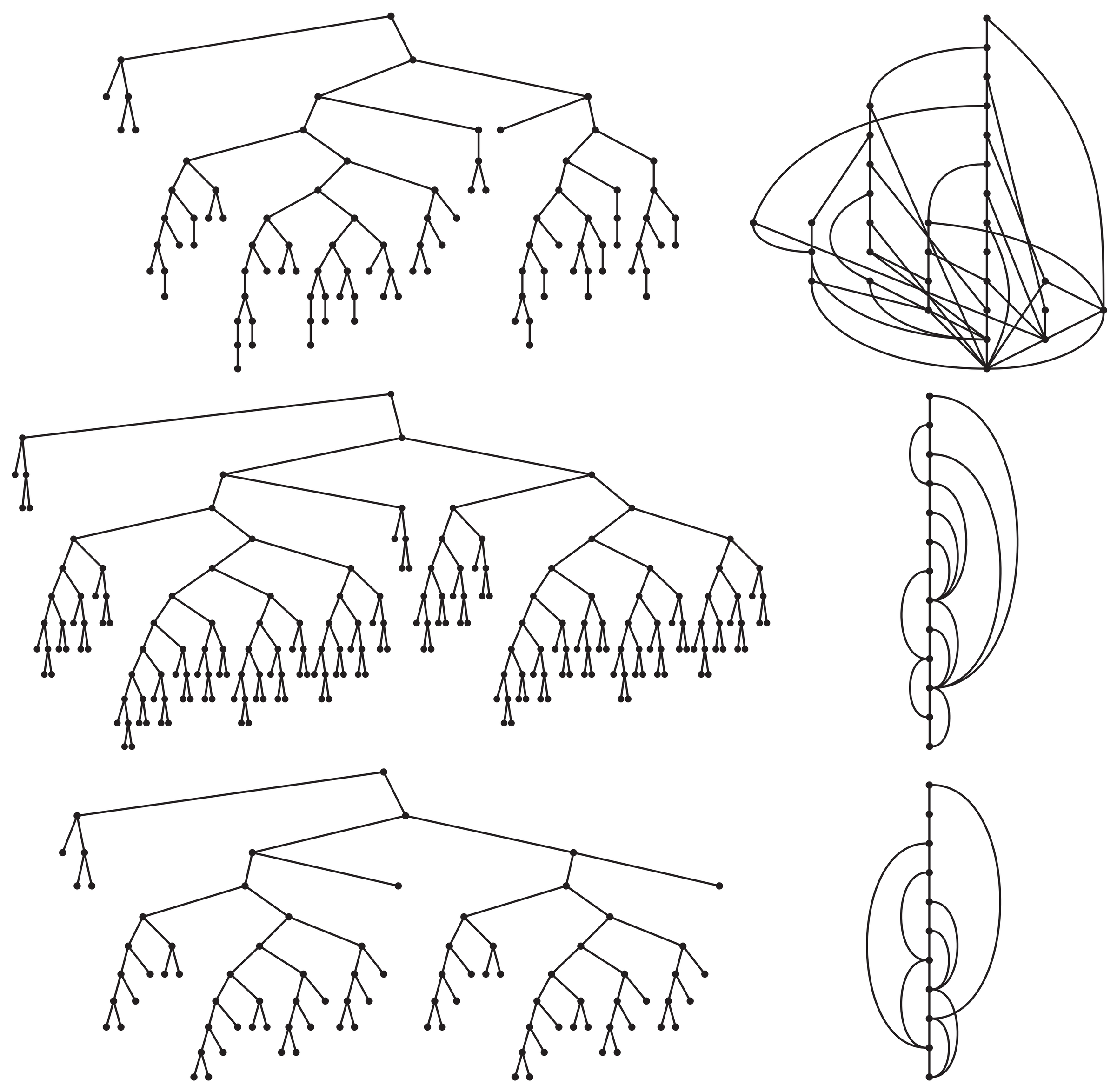


Figure 1: Error and compression rates on the example.



Self-Nested Trees

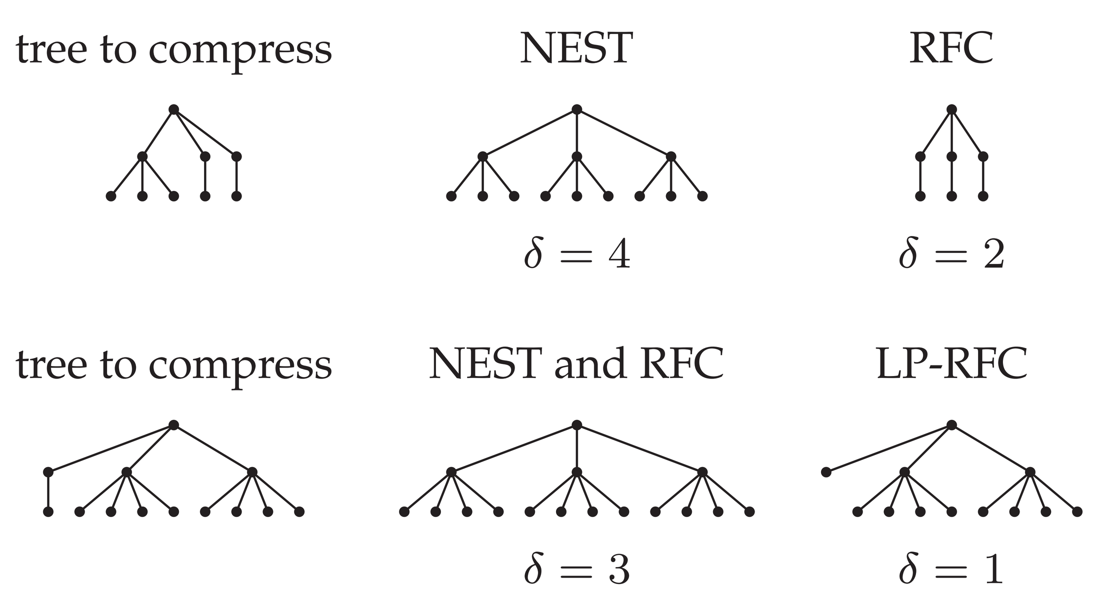
Self-nested trees present remarkable compression properties because of the **systematic repetition of subtrees**. They are defined as trees such that all the subtrees of a given height are isomorphic. The DAG related to a self-nested tree τ is linear (there exists a path going through all its vertices) and has thus height$(\tau)+1$ nodes. In other words, **self-nested trees achieve maximum compression rates**. In addition, self-nested trees are very rare, while still being quite close to any unordered tree.

	$\deg \leq 2$	$\deg \leq 3$	$\deg \leq 4$
$h \leq 2$	0.88	$6.18.10^{-1}$	$3.52.10^{-1}$
$h \leq 3$	0.49	$3.38.10^{-2}$	$7.43.10^{-5}$
$h \leq 4$	0.07	$2.90.10^{-8}$	$4.16.10^{-23}$
$h \leq 5$	$3.36.10^{-4}$	$3.56.10^{-28}$	$1.66.10^{-100}$

Table 1: Frequency of self-nested trees with given maximal height and ramification number with respect to unordered trees under the same constraint.

Algorithm

The preexisting algorithm (NEST) only adds nodes to a tree τ to transform it into a self-nested structure. Instead of adding nodes, we propose to **replace some internal structures of τ by their centroid** (RFC). In particular, this allows us to delete some nodes and thus gives flexibility. These self-nested estimates may be computed in polynomial time. Nevertheless, this procedure can only modify subtrees and not delete them. That is why we introduce **a new algorithm that exploits local pruning** of τ (LP-RFC).



Simulation Study

Our simulations are performed on 500 small binary trees generated from a stochastic model. The three algorithms are equivalent in terms of compression rates. The key parameter is thus **the error rate that is much better for RFC and LP-RFC algorithms** than for NEST procedure (substantial gain of 20%). Local pruning is useful in RFC 24.2% of the time and makes the error decrease of 1.8% (7.3% when useful).

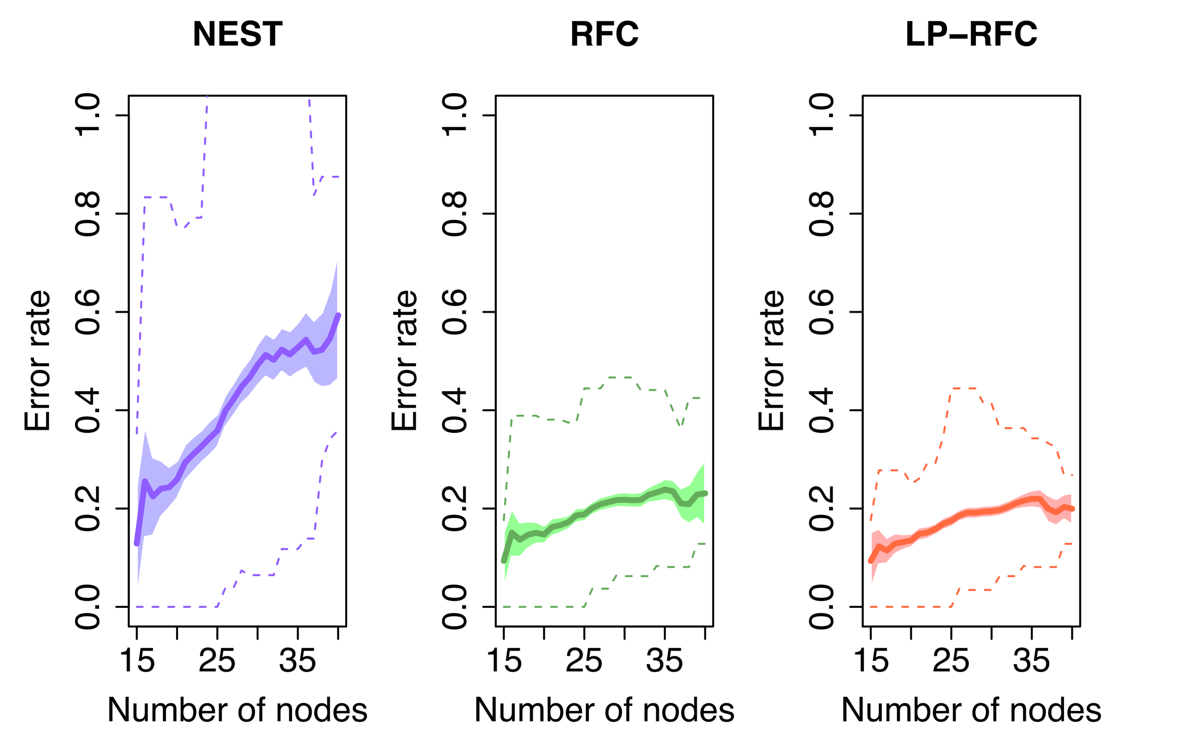


Figure 2: Error rates on the simulated dataset.

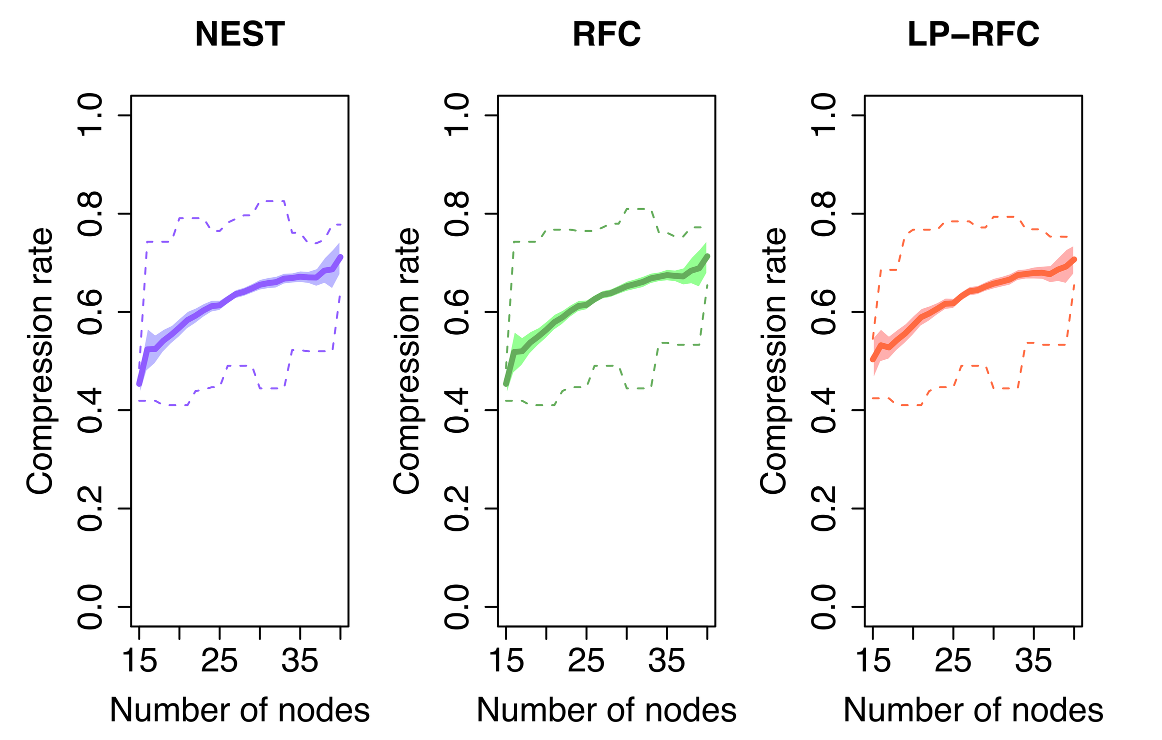


Figure 3: Compression rates on the simulated dataset.

Worst-Case Approximation Error

Among trees of height h and outdegree less than m, the tree that is the farthest to a self-nested tree may be identified. The editing distance to its best self-nested approximation is of order $0.25\,m^h$. The diameter of the space of unordered trees being of order m^h, it follows that **the largest area without any self-nested tree is of relative radius** 0.25. In addition, the error rate for this tree (and thus the worst error rate that must be expected from any lossy compression algorithm) is of order 0.33.

References

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Contact Information

Romain Azaïs

Inria team BIGS – Institut Élie Cartan de Lorraine

Université de Lorraine in Nancy, France

romain.azais@inria.fr