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# Conditional Mean-Variance and Mean-Semivariance models in portfolio optimization

Hanene Ben Salah

*BESTMOD Laboratory, ISG 41 Rue de la Liberté, Cité Bouchoucha 2000 Le Bardo, Tunisie*

*IMAG 34095 Montpellier cedex 05*

*Université de Lyon, Université Claude Bernard Lyon 1, Institut de Science Financière et d'Assurances, LSAF EA2429, F-69366 Lyon, France*

Ali Gannoun

*IMAG 34095 Montpellier cedex 05*

Christian de Peretti

*Université de Lyon, Université Claude Bernard Lyon 1, Institut de Science Financière et d'Assurances, LSAF EA2429, F-69366 Lyon, France*

Mathieu Ribatet

*IMAG 34095 Montpellier cedex 05*

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## Abstract

It is known that the historical observed returns used to estimate the expected return provide poor guides to predict the future returns. Consequently, the optimal portfolio weights are extremely sensitive to the return assumptions used. Getting information about the future evolution of different asset returns, could help the investors to obtain more efficient portfolio. The solution will be reached by estimating the portfolio risk by conditional variance or conditional semivariance. This strategy allows us to take advantage of returns prediction which will be obtained by nonparametric univariate methods. Prediction step uses kernel estimation of conditional mean. Application on the Chinese and the American markets are presented and discussed.

*Keywords:* Conditional Semivariance, Conditional Variance, DownSide Risk, Kernel Method, Nonparametric Mean prediction.

## 1. Introduction

Investment strategies and their profitability have always been a hot topic for people with an interest in financial assets. The modern asset allocation theory was originated from the Mean-Variance portfolio model introduced by Markowitz (1952), see also Markowitz (1959) and Markowitz (1990). The original Markowitz model simply dealt with the use of historical returns. Variance is commonly used as a risk measure in portfolio optimisation to find the trade-off between the risk and return. In practice, the expected returns and variances are calculated using historical data observed before the portfolio optimization date and are used as proxies for future returns. This practice is not purely guesswork, there are well-developed nonparametric approaches to obtain good forecasts.

Markowitz's portfolio optimization requires the knowledge of both the expected return and the covariance matrix of the assets. It is well known that the optimum portfolio weights are very sensitive to return expectations which are very difficult to determine. For instance, historical returns are bad predictors of the future returns if we use the classical arithmetic mean (Michaud, 1989; Black and Litterman, 1992; Siegel and Woodgate, 2007). Estimating covariance matrices is a delicate statistical challenge that requires sophisticated methods (see Ledoit and Wolf (2004)). It is fair to state that, due to the large statistical errors of the input of Markowitz's portfolio optimization, its result is not reliable and should be considered very cautiously. This led Levy and Roll (2010) to turn the usual approach up-side-down and found that minor adjustments of the input parameter are much needed, well within the statistical uncertainties.

There are various models to used in forecasting future financial time series. For example, Delatola and Griffin (2011) developed a prediction method based on Bayesian nonparametric modelling of the return distribution with stochastic volatility and Kresta (2015) proposed to use of a GARCH-copula model in Portfolio Optimization .

In this paper, we propose a radical different methods by including informations about the possible future returns in the estimation step obtained by

31 kernel nonparametric prediction. Then, we can improve the quality of portfolio  
 32 optimisation.

33 Firstly, we exhibit the classical Markowitz model developed in 1952.

34 Let us say that there are  $m$  assets to constitute a portfolio  $P$  and denote  
 35 by  $r_{jt}$  the return of asset  $j$  on date  $t$ ,  $t = 1, \dots, N$ , and  $M$  the estimated  
 36 variance-covariance matrix of the returns  $(r_1, \dots, r_m)$ ,

$$M = \frac{1}{N} \sum_{t=1}^N \begin{bmatrix} (r_{1t} - \bar{r}_1)^2 & (r_{1t} - \bar{r}_1)(r_{2t} - \bar{r}_2) & (r_{1t} - \bar{r}_1)(r_{mt} - \bar{r}_m) \\ (r_{2t} - \bar{r}_2)(r_{1t} - \bar{r}_1) & (r_{2t} - \bar{r}_2)^2 & (r_{2t} - \bar{r}_2)(r_{mt} - \bar{r}_m) \\ \vdots & \vdots & \vdots \\ (r_{mt} - \bar{r}_m)(r_{1t} - \bar{r}_1) & (r_{mt} - \bar{r}_m)(r_{2t} - \bar{r}_2) & (r_{mt} - \bar{r}_m)^2 \end{bmatrix}. \quad (1)$$

37 The optimization program is then the following

$$\min_{\omega} \omega^\top M \omega, \text{ subject to } \omega^\top \mu = E^*, \omega^\top \mathbf{1} = 1, \quad (2)$$

38 where  $\omega^\top = (\omega_1, \dots, \omega_m)$  is the portfolio vector weight,  $\mu^\top = (\bar{r}_1, \dots, \bar{r}_m) =$   
 39  $(\frac{1}{N} \sum_{t=1}^N r_{1t}, \dots, \frac{1}{N} \sum_{t=1}^N r_{mt})$  the empirical mean returns and  $E^*$  is a target  
 40 expected portfolio return.

41 Using Lagrangian method, the explicit solution of solution of (2) is :

$$\omega_* = \frac{\alpha E^* - \lambda}{\alpha \theta - \lambda^2} M^{-1} \mu + \frac{\theta - \lambda E^*}{\alpha \theta - \lambda^2} M^{-1} \mathbf{1}, \quad (3)$$

42 where  $\alpha = \mathbf{1}^\top M^{-1} \mathbf{1}$ ,  $\lambda = \mu^\top M^{-1} \mathbf{1}$  and  $\theta = \mu^\top M^{-1} \mu$ .

43 This model depends strictly on the assumptions that the assets returns follow  
 44 normal distribution and investor has quadratic utility function. However, these  
 45 two conditions are not checked. Many researchers have showed that the assets  
 46 returns distribution are asymmetric and exhibit skewness, see Tobin (1958);  
 47 Arditti (1971); Chunnachinda et al. (1997); Prakash et al. (2003). These authors  
 48 have proposed a DownSide Risk (DSR) measures such as Semivariance (SV)  
 49 and conditional value at risk (CVaR). These DSR measures are consistent with  
 50 investor's perception towards risk as they focus on return dispersions below

51 any benchmark return  $B$  chosen by the investor. Below, we focus only on the  
 52 Semivariance risk measure which is often considered as a more plausible risk  
 53 one than the variance. The associated optimization program is the following:

$$\min_{\omega} \omega^{\top} M_{SR} \omega \text{ subject to } \omega^{\top} \mu = E^*, \omega^{\top} \mathbf{1} = 1, \quad (4)$$

54 where  $M_{SR}$  is the matrix with coefficients

$$\Sigma_{ijB} = \frac{1}{N} \sum_{t=1}^V (r_{it} - B)(r_{jt} - B)$$

55 such that  $V$  is the period in which the portfolio underperforms the target return  
 56  $B$ .

57 The major obstacle to get the solution of this problem is that the semico-  
 58 variance matrix is endogenous (see Estrada (2004, 2008)), that is, the change in  
 59 weights affects the periods in which the portfolio underperforms the target rate  
 60 of return, which in turn affects the elements of the Semivariance matrix.

61 Many authors propose different methods to estimate the elements of  $M_{SR}$  in  
 62 order to resolve problem defined in equation (4). Among them, Hogan and War-  
 63 ren (1974) propose to use the Frank-Wolf algorithm but the main disadvantage  
 64 of this algorithm is its slow convergence rate. Moreover, during early iteration,  
 65 this algorithm tends to decrease the objective function. Ang (1975) proposes to  
 66 linearise the Semivariance so that the optimization problem can be solved using  
 67 linear programming. However, this method ignores the inter-correlations be-  
 68 tween securities. Harlow (1991) also considers problem (4) and generates Mean-  
 69 Semivariance efficient frontier, which he compares to the Mean-Variance effi-  
 70 cient frontier. Mamoghli and Daboussi (2008) improve Harlow approach. Their  
 71 model permits to surmount the problem of inequality of the Semicovariance  
 72 measures which occurs in the Mean-Semivariance model of Harlow. Markowitz  
 73 et al. (1993) transform the Mean-Semivariance problem into a quadratic problem  
 74 by adding fictitious securities. Estrada (2008) proposes a simple and accurate  
 75 heuristic approach that yields a symmetric and exogenous Semicovariance ma-  
 76 trix, which enables the determination of Mean-Semivariance optimal portfolios

77 by using the well-known closed-form solutions of Mean-Variance problems. In  
78 Athayde (2001) and (Athayde, 2003), there is an iterative algorithm which is  
79 used to construct a Mean-DownSide Risk portfolio frontier. This algorithm is  
80 improved by Ben Salah et al. (2016a) and Ben Salah et al. (2016b) by intro-  
81 ducing nonparametric estimation of the returns in order to get a better smooth  
82 efficient frontier.

83 A knowledgeable investor should have an overview on market development.  
84 For a fixed amount to invest now, he has to predict, day by day (or month by  
85 month, . . .), the optimal return that his investment could bring to him.

86 In this paper, we will develop a rule of decision to optimize the portfolio  
87 selection through reducing the risk calculated using the classical variance. A  
88 new approach, called conditional Markowitz optimization method, will be used  
89 to determine an optimal portfolio. This portfolio is obtained by minimizing  
90 the so-called *conditional risk*. This risk may in turn be broken down into two  
91 versions: Mean-Variance and Mean-Semivariance approaches. The main idea is  
92 to anticipate the values of the assets returns on the date  $N + 1$  (knowing the  
93 past) and incorporate this information in the optimization model.

94 The rest of the article is organized as follows. Section 2 introduces the non-  
95 parametric conditional risk. We start by giving an overview on nonparametric  
96 regression and prediction, as well as conditional variance and covariance defini-  
97 tions. The conditional Mean-Variance and the conditional Mean-Semivariance  
98 models are exhibited in this Section. Numerical studies based on the Chinese  
99 and the American Market dataset are presented in Section 3. The last Section  
100 is devoted to conclusion and further development.

## 101 **2. Nonparametric Conditional Risk**

102 Here, we start by giving some general concepts and results concerning non-  
103 parametric regression and prediction. Then, we define the Conditional Mean-  
104 Variance and Mean-Semivariance models and we exhibit the corresponding al-  
105 gorithms to get the optimal portfolio.

106 *2.1. Nonparametric Regression Model*

107 In the following subsection, we outline the mechanics of kernel regression  
 108 estimation.

Let  $(X, Y)$  be a pair of explanatory and response random variables with joint density  $f(x, y)$ , and  $(Y_i, X_i)_{i=1, T}$   $T$ -copies of  $(X, Y)$ . We suppose that  $X$  is  $m$ -dimensional. Using a quadratic criterion, the best prediction of  $Y$  based on  $X = x$  is the conditional expectation  $E(Y | X = x) = g(x)$ .

Using  $(Y_i, X_i)_{i=1, T}$ , we will focus on the estimation of the unknown mean response  $g(x)$ .

The regression model is

$$Y_t = g(x_t) + \epsilon_t, \quad (5)$$

where  $g(\cdot)$  is unknown. The errors  $\epsilon_t$  satisfy

$$E(\epsilon_t) = 0, \quad V(\epsilon_t) = \sigma_\epsilon^2, \quad Cov(\epsilon_i, \epsilon_j) = 0 \text{ for } i \neq j. \quad (6)$$

To derive the estimator note that we can express  $g(x)$  in terms of the joint probability density function  $f(x, y)$  as follows:

$$g(x) = E(Y | X = x) = \int y f(y | x) dy = \frac{\int y f(x, y) dy}{\int f(x, y) dy} \quad (7)$$

Using kernel density estimation of the joint distribution and the marginal one (see (Bosq and Lecoutre, 1987)), the Nadaraya-Watson estimator for  $g(x)$  is given by :

$$\hat{g}(x) = \frac{\sum_{t=1}^T Y_t \mathcal{K}_H(X_t - x)}{\sum_{t=1}^T \mathcal{K}_H(X_t - x)} \quad (8)$$

where  $\mathcal{K}_H(x) = |H|^{-1/2} \mathcal{K}(|H|^{-1/2} x)$  with  $H$  is the  $m \times m$  matrix of smoothing parameters which is symmetric and positive definite and  $|H|$  its determinant.

The function  $\mathcal{K} : \mathbb{R}^m \rightarrow [0, \infty)$  is a probability density.

By the way, the conditional variance is given by

$$\sigma^2(x) = E((Y - g(x))^2 | X = x), \quad (9)$$

and its kernel estimator is given by

$$\hat{\sigma}^2(x) = \frac{\sum_{t=1}^T (Y_t - g(\hat{x}))^2 \mathcal{K}_H(X_t - x)}{\sum_{t=1}^T \mathcal{K}_H(X_t - x)}. \quad (10)$$

If  $Y = (Y_1, \dots, Y_m)$  is a random vector, the conditional covariance between  $Y_1$  and  $Y_2$  given  $X = x$  is defined as

$$\sigma_{12}(x) = E(Y_1 - g_1(x))(Y_2 - g_2(x)) \mid X = x), \quad (11)$$

and its kernel estimator is given by

$$\hat{\sigma}_{12}(x) = \frac{\sum_{t=1}^T (Y_{1t} - \hat{g}_1(x))(Y_{2t} - \hat{g}_2(x)) \mathcal{K}_H(X_t - x)}{\sum_{t=1}^T \mathcal{K}_H(X_t - x)}, \quad (12)$$

109 where  $g_1(x)$  and  $\hat{g}_1(x)$  (respectively  $g_2(x)$  and  $\hat{g}_2(x)$ ) are the  $E(Y_1 \mid X = x)$  and  
 110 its kernel estimator (respectively the  $E(Y_1 \mid X = x)$  and its kernel estimator)  
 111 defined above.

112

113 The choice of the bandwidth matrix  $H$  is the most important factor affecting  
 114 the accuracy of the estimator (8), since it controls the orientation and amount  
 115 of smoothing induced.

116 The simplest choice for this matrix is to take  $H = h\mathbb{I}_m$ , where  $h$  is a unidi-  
 117 mensional smoothing parameter and  $\mathbb{I}_m$  is the  $m \times m$  identity matrix. Then we  
 118 have the same amount of smoothing applied in all coordinate directions and the  
 119 kernel estimator (8) has the form

$$\hat{g}(x) = \frac{\sum_{t=1}^T Y_t \mathcal{K}\left(\frac{X_t - x}{h}\right)}{\sum_{t=1}^T \mathcal{K}\left(\frac{X_t - x}{h}\right)} = \frac{\sum_{t=1}^T Y_t \mathcal{K}_h(X_t - x)}{\sum_{t=1}^T \mathcal{K}_h(X_t - x)}, \quad (13)$$

120 where  $\mathcal{K}_h(x) = h^{-m} \mathcal{K}\left(\frac{x}{h}\right)$ . The estimators (10) and (12) are derived similarly.

121 Another choice, relatively easy-to-manage, is to take the bandwidth matrix  
 122 equal to a diagonal matrix, which allows for different amounts of smoothing in



123 each of the coordinates.

124 The following kernel functions are used most often in multivariate nonparametric  
125 estimation:

126 • product kernel:  $\mathcal{K}(x) = K(x_1) \times \dots \times K(x_m) = \prod_{j=1}^m k(x_j)$  where  $k(\cdot)$  is  
127 an univariate density function.

128 • symmetric kernel:  $C_{k,m}k(\|x\|_2^{1/2})$ .

129 The multivariate Gaussian density  $K(x) = (2\pi)^{-m/2} \exp(-\frac{x^\top x}{2})$  is a product and  
130 symmetric kernel.

131 In the rest of this paper, we will consider only the product kernel and the  
132 bandwidth matrix  $H = \text{diag}(h_1, \dots, h_m)$ . Then, for  $x = (x_1, \dots, x_m)$ , the  
133 estimator (8) should be written as follows:

$$\hat{g}(x) = \frac{\sum_{t=1}^T Y_t k\left(\frac{X_{1t}-x_1}{h_1}\right) \times \dots \times k\left(\frac{X_{mt}-x_m}{h_m}\right)}{\sum_{t=1}^T k\left(\frac{X_{1t}-x_1}{h_1}\right) \times \dots \times k\left(\frac{X_{mt}-x_m}{h_m}\right)}, \quad (14)$$

134 with  $h_j = T^{-1/m+4} \hat{\sigma}_j$  where  $\hat{\sigma}_j$  is an estimator of the standard deviation of the  
135 random variable  $X_j$ . These bandwidths are optimal using the minimisation of  
136 the Asymmetrical Mean Integrated Error criterion (see for example (Wasserman,  
137 2006)).

138 The estimators (10) and (12) are derived similarly.

## 139 2.2. Nonparametric Prediction

Nonparametric smoothing techniques can be applied beyond the estimation  
of the autoregression function. Consider a  $m$ -multivariate stationary time series  
 $\{(r_{1t}, \dots, r_{mt}), t = 1, \dots, N\}$ . We consider the processes  $(X_t, Y_t)$  defined as  
follows

$$X_t = (r_{1t}, \dots, r_{mt}) \quad Y_t = r_{j(t+1)}, \quad (15)$$

and we are interested in predicting the return of a given asset  $j$  on time  $N + 1$ .  
This problem is equivalent to the estimation of the regression  $g(\cdot)$  function

presented above. Then

$$g(r_{1N}, \dots, r_{mN}) = \mathbb{E}(Y_N | X_N = x_N) = \mathbb{E}(r_{j(N+1)} | x_N = (r_{1N}, \dots, r_{mN})). \quad (16)$$

140 Let  $T = N - 1$ . Using the kernel method, we get easily the following estimators  
141 for the conditional expectation, variance and covariance:

• **Conditional expectation:**

$$\bar{r}_{c,j} = \hat{g}(r_{1N}, \dots, r_{mN}) = \frac{\sum_{t=1}^T r_{j(t+1)} K\left(\frac{r_{1t}-r_{1N}}{h}\right) \times \dots \times K\left(\frac{r_{mt}-r_{mN}}{h}\right)}{\sum_{t=1}^T K\left(\frac{r_{1t}-r_{1N}}{h}\right) \times \dots \times K\left(\frac{r_{mt}-r_{mN}}{h}\right)}, \quad (17)$$

• **Conditional variance :**

$$\hat{\sigma}_{c,j}^2(r_{1N}, \dots, r_{mN}) = \frac{\sum_{t=1}^T (r_{j(t+1)} - \bar{r}_{c,j})^2 K\left(\frac{r_{1t}-r_{1N}}{h}\right) \times \dots \times K\left(\frac{r_{mt}-r_{mN}}{h}\right)}{\sum_{t=1}^T K\left(\frac{r_{1t}-r_{1N}}{h}\right) \times \dots \times K\left(\frac{r_{mt}-r_{mN}}{h}\right)}, \quad (18)$$

• **Conditional covariance :**

$$\hat{\sigma}_{c,ij}(r_{1N}, \dots, r_{mN}) = \frac{\sum_{t=1}^T (r_{i(t+1)} - \bar{r}_{c,i})(r_{j(t+1)} - \bar{r}_{c,j}) K\left(\frac{r_{1t}-r_{1N}}{h}\right) \times \dots \times K\left(\frac{r_{mt}-r_{mN}}{h}\right)}{\sum_{t=1}^T K\left(\frac{r_{1t}-r_{1N}}{h}\right) \times \dots \times K\left(\frac{r_{mt}-r_{mN}}{h}\right)}, \quad (19)$$

142 **Comments**

- 143 1. To estimate conditional expectation, variance and covariance, we supposed  
144 that the future depends on the immediate past. This hypothesis, consid-  
145 ered as 1-Markovian condition, is natural because the future returns are  
146 strongly correlated to the recent past.
- 147 2. In Statistics literature, many other multivariate kernels are proposed like  
148 the Spherical/radial-symmetric kernel or the multivariate Epanechnikov  
149 (spherical) one.

150 3. Nonparametric methods are typically indexed by a bandwidth or tuning  
 151 parameter which controls the degree of complexity. The choice of band-  
 152 width is often critical to implementation: under- or over-smoothing can  
 153 substantially reduce precision. The standard approach to the bandwidth  
 154 problem is to choose a bandwidth that minimizes some measures of global  
 155 risk for the entire regression function, usually Mean Integrated Squared  
 156 Error (MISE), i.e. the expected squared error integrated over the entire  
 157 curve. The optimal bandwidth is then estimated either using plug-in es-  
 158 timators of the minimizer of the asymptotic approximation to MISE or  
 159 using an unbiased data-based estimator of the MISE . This is the cross-  
 160 validation method. This method was analyzed in Sarda (1993). It is  
 161 recommended by (Altman and Leger, 1995) when large samples are avail-  
 162 able. As to the estimation of the conditional covariance matrix, one may  
 163 use different bandwidths for different elements of this matrix. However,  
 164 the resulting estimation with different bandwidths cannot be guaranteed  
 165 to be positive definite (Li et al. (2007) ). In practice, the positive definite-  
 166 ness is a desirable property. Thus, we suggest using the same bandwidth  
 167 for all elements.

### 168 2.3. Conditional Mean-Variance Model

169 Our goal here is to constitute an optimal portfolio using conditional criterion.  
 170 In our opinion, it is natural to use conditional information to provide a timely  
 171 and effective solution.

172 We suppose that they are, as in the previous section,  $m$  assets to be used  
 173 for constructing a well diversified portfolio. Optimizing asset allocation is sim-  
 174 ply defined as the process of mixing asset weights of a portfolio within the  
 175 constraints of an investor's capital resources to yield the most favourable risk-  
 176 return trade-off. The risk here is defined by the conditional variance of ,the  
 177 portfolio return.

178 Let  $\omega = (\omega_1, \dots, \omega_m)^\top$  be the portfolio weight vector and  $r_{pt} = \omega_1 r_{1t} +$   
 179  $\omega_2 r_{2t} + \dots + \omega_m r_{mt}, t = 1 \dots, N, N$ -realisations of the the portfolio return  $R_p$ .

180 Using conditional mean-variance criterion, the optimization model is

$$\min_{\omega} \omega^{\top} M_c \omega, \text{ subject to } \omega^{\top} \mu_c = E^*, \omega^{\top} \mathbf{1} = 1, \quad (20)$$

181 where  $\mu_c = (\bar{r}_{c,1}, \dots, \bar{r}_{c,m})^{\top}$ ,  $E^*$  is a given target return and  $M_c$  is the condi-  
182 tional covariance matrix whose elements are  $\hat{\sigma}_{c,ij}(r_{1N}, \dots, r_{mN})$ .

183 Using Lagrangian method, an explicit solution of (20) is given by:

$$\omega_{c,*} = \frac{\alpha_c E^* - \lambda_c}{\alpha_c \theta_c - \lambda_c^2} M_c^{-1} \mu_c + \frac{\theta_c - \lambda_c E^*}{\alpha_c \theta_c - \lambda_c^2} M_c^{-1} \mathbf{1}, \quad (21)$$

184 where  $\alpha_c = \mathbf{1}^{\top} M_c^{-1} \mathbf{1}$ ,  $\lambda_c = \mu_c^{\top} M_c^{-1} \mathbf{1}$  and  $\theta_c = \mu_c^{\top} M_c^{-1} \mu_c$ .

185 Then the conditional risk,  $CR$ , is given by

$$CR = (\omega_{c,*}^{\top} M_c \omega_{c,*}). \quad (22)$$

#### 186 2.4. Conditional Mean-Semivariance Model

187 Let us recall that the main criticism to variance, used by Markowitz (1952)  
188 as measure of risk is, in essence, that it gives the same importance and the  
189 same weight to gains and losses, also the use of variance suppose that returns  
190 are normally distributed . That is why Markowitz (1959) argues for another  
191 more plausible measure of risk that he calls the Semivariance which takes into  
192 consideration the asymmetry and the risk perception of investors. For moti-  
193 vations, details and theoretical result on this method, see for example Vasant  
194 et al. (2014) .

195 Let  $B$  the benchmark return(a threshold which captured the risk perspec-  
196 tives from investors to investors). It is a target return which can be equal to 0.  
197 The conditional mean-Semivariance model is the following:

$$\min_{\omega} \omega^{\top} M_{\{c,SR\}} \omega \text{ subject to } \omega^{\top} \mu_c = E^*, \omega^{\top} \mathbf{1} = 1, \quad (23)$$

198 where  $M_{\{c,SR\}}$  is the matrix with coefficients

$$\hat{\sigma}_{ijB} = \frac{\sum_{t=1}^V (r_{it} - B)(r_{jt} - B)K\left(\frac{r_{1t}-r_{1N}}{h}\right) \times \dots \times K\left(\frac{r_{mt}-r_{mN}}{h}\right)}{\sum_{t=1}^V K\left(\frac{r_{1t}-r_{1N}}{h}\right) \times \dots \times K\left(\frac{r_{mt}-r_{mN}}{h}\right)}, \quad (24)$$

199 such that  $V$  is the period in which the portfolio underperforms the target return  
200  $B$ .

201 **Remark.** The coefficients  $\hat{\sigma}_{ijB}$  are computed differently of those from (19).  
202 To do it in the same way, we should re-index the observations (such that portfolio  
203 under performs the target return  $B$ ) in order to get a new time series process  
204 and then apply (19). This modification is not very helpful given the abundance  
205 of data.

206 Resolving this problem is not easy because the matrix  $M_{\{c,SR\}}$  is endoge-  
207 nous. Based on Athayde algorithm (see (Athayde, 2001)), we develop, in the  
208 following, an iterative algorithm that could resolve the optimization problem  
209 without enormous difficulties. The principle is the use of Lagrangian method at  
210 each step.

211 **• Step 0:**

- 212 – Start with  $\omega_{c,0} = (\omega_{c,0}^1, \dots, \omega_{c,0}^m)$ ,
- 213 – compute  $r_{pt}^0 = \omega_{c,0}^1 r_{1t} + \dots + \omega_{c,0}^m r_{mt}$ ,  $t = 1, \dots, N$ ,
- 214 – select the set  $S_0$  of time indices portfolio return observations in which  
215 this portfolio  $\omega_{c,0}$  had negative deviations i.e.  $r_{pt}^0 - B \leq 0$ .

216

– Construct the following positive ( $m \times m$ ) semi-definite matrix  $M_{\{c,SR,0\}}$ :

$$M_{\{c,SR,0\}} = \frac{1}{\sum_{t \in S'_0} K\left(\frac{r_{1(t-1)} - r_{1N}}{h}\right) \times \dots \times K\left(\frac{r_{m(t-1)} - r_{mN}}{h}\right)} \times \sum_{t \in S'_0} \begin{bmatrix} \dots & & \dots & & \dots \\ \vdots & & \vdots & & \vdots \\ \dots & (r_{it} - B)(r_{jt} - B) K\left(\frac{r_{1(t-1)} - r_{1N}}{h}\right) \times \dots \times K\left(\frac{r_{m(t-1)} - r_{mN}}{h}\right) & \dots & & \dots \\ \vdots & & \vdots & & \vdots \\ \dots & & \dots & & \dots \end{bmatrix}, \quad (25)$$

217

where  $S'_0 = S_0$  or  $S_0 = S_0 \setminus \{1\}$  (if  $1 \in S_0$ ).

218

• **Step 1:** find the portfolio  $\omega_{c,1}$  that solves the following problem:

$$\min_{\omega} \omega^\top M_{\{c,SR,0\}} \omega \text{ subject to } \omega^\top \mu_c = E^*, \omega^\top \mathbf{1} = 1. \quad (26)$$

219

Using *Lagrangian Method*, the solution to the problem (26) will be given

220

by:

$$\omega_{c,1} = \frac{\alpha_{c,1} E^* - \lambda_{c,1}}{\alpha_{c,1} \theta_{c,1} - \lambda_{c,1}^2} (M_{\{c,SR,0\}})^{-1} \mu_c + \frac{\theta_{c,1} - \lambda_{c,1} E^*}{\alpha_{c,1} \theta_{c,1} - \lambda_{c,1}^2} (M_{\{c,SR,0\}})^{-1} \mathbf{1}, \quad (27)$$

221

where  $\alpha_{c,1} = \mathbf{1}^\top (M_{\{c,SR,0\}})^{-1} \mathbf{1}$ ,  $\lambda_{c,1} = \mu_{c,1}^\top (M_{\{c,SR,0\}})^{-1} \mathbf{1}$  and  $\theta_{c,1} =$

222

$\mu_c^\top (M_{\{c,SR,0\}})^{-1} \mu_c$ .

223

224

• **Step 2:**

225

– compute  $r_{pt}^1 = \omega_{c,1}^1 r_{1t} + \dots + \omega_{c,1}^m r_{mt}$ ,  $t = 1, \dots, N$ ,

226

– select the set  $S_1$  of index observations in which this portfolio  $\omega_{c,1}$  had

227

negative deviations i.e.  $r_{pt}^1 - B \leq 0$ ,

228

– construct the following positive ( $m \times m$ ) semi-definite matrix  $M_{\{c,SR,1\}}$ :

$$M_{\{c,SR,1\}} = \frac{1}{\sum_{t \in S'_1} K(\frac{r_1(t-1)-r_{1N}}{h}) \times \dots \times K(\frac{r_m(t-1)-r_{mN}}{h})} \times \sum_{t \in S_1} \begin{bmatrix} \dots & \dots & \dots \\ \vdots & \vdots & \vdots \\ \dots & (r_{it} - B)(r_{jt} - B)K(\frac{r_1(t-1)-r_{1N}}{h}) \times \dots \times K(\frac{r_m(t-1)-r_{mN}}{h}) & \dots \\ \vdots & \vdots & \vdots \\ \dots & \dots & \dots \end{bmatrix}, \quad (28)$$

229

where  $S'_1 = S_1$  or  $S'_1 = S_1 \setminus \{1\}$  (if  $1 \in S_1$ ).

230

– find the portfolio  $\omega_{c,2}$  that solves the following problem:

$$\min_{\omega} \omega^\top M_{\{c,SR,1\}} \omega \text{ subject to } \omega^\top \mu_c = E^*, \omega^\top \mathbf{1} = 1. \quad (29)$$

231

Using *Lagrangian Method*, the solution to the problem (29) will be

232

given by:

$$\omega_{c,2} = \frac{\alpha_{c,2} E^* - \lambda_{c,2}}{\alpha_{c,2} \theta_{c,2} - \lambda_{c,2}^2} (M_{\{c,SR,1\}})^{-1} \mu_c + \frac{\theta_{c,2} - \lambda_{c,2} E^*}{\alpha_{c,2} \theta_{c,2} - \lambda_{c,2}^2} (M_{\{c,SR,1\}})^{-1} \mathbf{1}. \quad (30)$$

233

• **Step 3:** iterate the previous process to construct a sequence of matrices

234

$M_{\{c,SR,l\}}$  until getting the first matrix  $M_{\{c,SR,F\}}$  satisfying the criterion

235

$M_{\{c,SR,F\}} = M_{\{c,SR,F+1\}}$ . The Optimal portfolio will be given by:

$$\omega_{c,F+1} = \frac{\alpha_{c,F} E^* - \lambda_{c,F}}{\alpha_{c,F} \theta_{c,F} - \lambda_{c,F}^2} (M_{\{c,SR,F\}})^{-1} \mu_c + \frac{\theta_{c,F} - \lambda_{c,F} E^*}{\alpha_{c,F} \theta_{c,F} - \lambda_{c,F}^2} (M_{\{c,SR,F\}})^{-1} \mathbf{1}. \quad (31)$$

236

and the Conditional Semi Risk value,  $CSR$ , will be

$$CSR = \omega_{c,F+1}^\top M_{\{c,SR,F\}} \omega_{c,F+1} \quad (32)$$

237 **Remarks.**

- 238 1. There is a finite number of iterations to get the optimal solution.
- 239 2. In the prediction step (to get the an unobservable values of the returns),  
240 we treated separately the evolution of each asset. It is possible to make  
241 multivariate (or vectorial) prediction and get jointly the unobservable val-  
242 ues for all the assets.
- 243 3. Short selling is allowed in this model, i.e. the optimal portfolio can have  
244 a negative weight for some assets. To forbid short selling, the additional  
245 constraint  $\omega_j \geq 0$  for  $j = 1, \dots, m$  is necessary.

246 **3. Empirical Analysis**

247 In this section, the performance of the conditional mean-variance and the  
248 conditional mean-semivariance optimisation methods are investigated. They are  
249 compared to Classical Markowitz and DownSide methods. It is supposed that  
250 there is no transaction costs, no taxes and that the benchmark  $B = 0$ .

251 *3.1. Data*

252 A dataset, drawn from Reuters, was used for this analysis. The original  
253 Data are a daily stock returns belonged to two markets:

- 254 • The Chinese market (emerging market) with 16 assets. They are 897  
255 daily observations (returns) for each asset from November the 14th, 2011,  
256 to July 8th, 2015,
- 257 • The American market (developed market) with 19 assets. They are 788  
258 daily observations (returns) for each asset from June, 18th, 2012 to July,  
259 8th, 2015.

260 To compare the efficiency and the performance of the proposed methods, we use  
261 the daily values of



- 262 • The Hang Seng Index-HSI that aims to capture the leadership of the  
263 Hong Kong exchange, and covers approximately 65% of its total market  
264 capitalization,
- 265 • The S&P 500 Index that tracks 500 large U.S. companies across a wide  
266 span of industries and sectors. The stocks in the S&P 500 represent  
267 roughly 70 % of all the stocks that are publicly traded.

268 The assets returns are calculated from stock prices observed on Thomson Reuters  
269 Platform as follows:

$$r_t = \frac{p_t - p_{t-1}}{p_{t-1}}, \quad (33)$$

270 with

- 271 •  $p_t$ : Stock price at date  $t$ ,
- 272 •  $p_{t-1}$ : Stock price at date  $t - 1$

273 *The prices  $p_t$ ,  $t = 1, \dots, N$ ; are adjusted for dividends.*

274 The historical statistics of the asset markets are summarised below.

### 275 *3.2. Historical statistics*

276 In this subsection, historical statistics are proposed. The goal is to check  
277 the normality or not of the returns distribution in order to decide which risk  
278 measure is more appropriate to determine the optimal portfolio.

#### 279 *3.2.1. The Chinese Market*

280 Let us start by the Chinese Market. Over the past two decades, the Chinese  
281 economy and financial markets have undergone a remarkable transformation  
282 and have seen significant growth. More specifically, the Chinese equity market  
283 has grown from a once very rudimentary and closed market to one of the largest  
284 equity markets in the world. Although most of the Chinese equity market still  
285 remains in the hands of controlling parties and domestic investors, authorities  
286 have made significant progress in opening the market to foreign capital and

Table 1: Chinese historical analysis

	Abbreviation	Min	Mean	Sd	Skewness	Kurtosis	Max
HangSeng	HSI	-5.84	0.03	1.01	-0.22	1.86	3.80
Agricultural.Bk	A.Bk	-9.90	0.06	1.61	0.70	9.93	10.12
Bank.of.China	B.Ch	-10.98	0.09	1.78	0.69	11.31	10.14
Ind.And.Com.Bank	IACB	-9.90	0.04	1.48	0.06	9.18	9.04
Petrochina	Pet	-9.21	0.04	1.68	0.87	12.09	10.04
China.life.insurance	Ch.L.I.	-10.01	0.10	2.40	0.71	4.02	10.04
China.petroleum	Ch.P.	-10.04	0.04	1.88	0.28	6.85	10.04
Bank.of.Com.	Bk.C.	-10.00	0.08	1.97	0.74	8.88	10.10
Citic.securit	Ci.Se	-7.14	0.00	1.81	1.14	6.81	12.95
China.telecom	Ch.T.	-4.85	0.01	1.81	0.48	1.49	9.28
China.pacific	Ch.P.	-9.98	0.08	2.38	0.55	3.51	13.50
Chinarailway	Ch.R.	-10.14	0.19	2.92	2.65	20.68	26.59
Huaneng	Hua	-10.35	0.11	2.35	0.21	5.90	14.83
Greatwall	Gre	-10.00	0.18	2.63	0.19	1.38	10.63
Dong.feng	Do.F.	-6.96	-0.01	2.19	0.26	0.55	8.13
China.nat.buil	Ch.N.B	-8.96	-0.03	2.16	0.45	2.39	11.12
Tsingtao	Tsi	-10.04	0.01	1.79	0.27	6.86	12.79

287 increasing the tradable float outstanding-meaning the Chinese equity market has  
288 the potential to become a top dominant force within global portfolios. Taking  
289 all assets together, we observe that

290 •  $-10.98 \leq Min \leq -4.85$

291 •  $3.80 \leq Max \leq 26.59$

292 •  $-0.03 \leq Mean \leq 0.19$

293 •  $1.01 \leq SD \leq 2.92$

294 •  $0.55 \leq Kurtosis \leq 20.68$

Table 2: China Correlation Matrix

	A.Bk	B.Ch	IACB	Pet	C.L.I	Ch.P.	Bk.C.	Ci.Se	Ch.T.	Ch.P.	Ch.R.	Hua	Gre	Do.F.	C.N.B	Tsi
A.Bk	1.00	0.82	0.80	0.57	0.49	0.59	0.76	0.04	0.04	0.25	0.18	0.01	-0.03	0.01	0.03	0.09
B.Ch	0.82	1.00	0.76	0.55	0.49	0.57	0.74	0.03	0.00	0.23	0.23	-0.00	-0.04	-0.01	-0.02	0.05
IACB	0.80	0.76	1.00	0.60	0.45	0.60	0.72	0.02	0.05	0.20	0.16	-0.01	-0.03	0.01	0.05	0.09
Pet	0.57	0.55	0.60	1.00	0.54	0.74	0.52	0.02	0.06	0.23	0.18	0.00	0.00	0.02	0.06	0.09
C.L.I	0.49	0.49	0.45	0.54	1.00	0.45	0.49	-0.01	0.04	0.23	0.23	0.03	-0.02	0.01	0.03	0.09
Ch.P.	0.59	0.57	0.60	0.74	0.45	1.00	0.57	0.09	0.04	0.19	0.18	0.02	-0.00	0.06	0.11	0.06
Bk.C.	0.76	0.74	0.72	0.52	0.49	0.57	1.00	0.03	0.01	0.26	0.19	-0.01	-0.03	0.00	-0.00	0.11
Ci.Se	0.04	0.03	0.02	0.02	-0.01	0.09	0.03	1.00	0.10	0.05	0.03	-0.01	-0.01	0.12	0.13	0.03
Ch.T.	0.04	0.00	0.05	0.06	0.04	0.04	0.01	0.10	1.00	0.01	0.05	-0.01	-0.02	0.12	0.40	-0.03
Ch.P.	0.25	0.23	0.20	0.23	0.23	0.19	0.26	0.05	0.01	1.00	0.15	-0.03	0.01	0.00	0.01	0.13
Ch.R.	0.18	0.23	0.16	0.18	0.23	0.18	0.19	0.03	0.05	0.15	1.00	0.05	0.00	0.01	0.01	0.16
Hua	0.01	-0.00	-0.01	0.00	0.03	0.02	-0.01	-0.01	-0.01	-0.03	0.05	1.00	0.00	0.01	0.06	0.08
Gre	-0.03	-0.04	-0.03	0.00	-0.02	-0.00	-0.03	-0.01	-0.02	0.01	0.00	0.00	1.00	0.02	0.03	0.03
Do.F.	0.01	-0.01	0.01	0.02	0.01	0.06	0.00	0.12	0.12	0.00	0.01	0.01	0.02	1.00	0.12	0.00
C.N.B	0.03	-0.02	0.05	0.06	0.03	0.11	-0.00	0.13	0.40	0.01	0.01	0.06	0.03	0.12	1.00	-0.01
Tsi	0.09	0.05	0.09	0.09	0.09	0.06	0.11	0.03	-0.03	0.13	0.16	0.08	0.03	0.00	-0.01	1.00

295 •  $-0.22 \leq Skwenes \leq 2.65$

296 It means that the behaviour of assets return is different from an asset to an-  
 297 other. However, the returns distribution are more peaked than a Gaussian  
 298 distribution (the Skewness  $\neq 0$ ). This statement should favour the use of the  
 299 DSR methods to optimize a portfolio. As a first time, we process the data as  
 300 they are normally distributed. Our first goal is to compare the efficiency of  
 301 classical Markowitz Mean-Variance method versus the proposed Conditional-  
 302 Mean-Variance method. The second one is to compare the Mean-DownSide  
 303 Risk method to the Conditional one.

304 To analyse the possible correlation between the different assets, It is helpful  
 305 to know the correlation matrix. This matrix is presented in Table (2) It appears

306 that the returns are differently correlated between each other. For example,  
307 Agricultural.Bk is strongly correlated to Bank.of.China and Ind.And.Com.Bank  
308 assets, and very weakly correlated to Dong.feng and China.nat.buil assets. More  
309 generally, the correlation between two stocks is larger when they are from bank-  
310 ing sector than when they belong to different industries. The use of Principal  
311 Component Analysis (PCA) could reduce the number of assets to constitute the  
312 optimal portfolio. This step is omitted in this paper.

313 **Remark.**

314 In this matrix, we have excluded, the correlation between the HangSeng  
315 Index and the other assets because we will use it as a benchmark to compare  
316 its return to the optimal portfolio's.

317 *3.2.2. The American Market*

318 As for the Chinese Market, similar historical statistics are done for the Amer-  
319 ican one. In the beginning, we display and comment the descriptive statistics,  
320 then we exhibit and comment the correlation matrix.

321 Taking all assets together, we observe that

322 •  $-14.34 \leq Min \leq -0.90$

323 •  $2.54 \leq Max \leq 29.56$

324 •  $0.0 \leq Mean \leq 0.21$

325 •  $0.73 \leq SD \leq 2.73$

326 •  $0.85 \leq Kurtosis \leq 21.88$

327 •  $-0.54 \leq Skwenes \leq 2.18$

328 The behaviour of assets return is different from an asset to another. However,  
329 most of distributions are more peaked than a Gaussian distribution (the Skew-  
330 ness  $\neq 0$ ). The SP 500 index could being considered as normally distributed  
331 (Skewness=-0.15 , Kurtosis=0.99).

332 Table (4) is devoted to the Correlation Matrix:

Table 3: USA Historical Statistics

	Abbreviation	Min	Mean	SD	Skewness	Kurtosis	Max
SP500	S&P	-2.50	0.06	0.73	-0.15	0.99	2.54
Apple	App	-12.37	0.07	1.62	-0.54	6.81	8.20
Google	Goo	-8.09	0.09	1.34	1.28	16.27	13.86
Microsoft	Mft	-11.40	0.06	1.44	-0.23	11.57	10.40
Exxon	Exx	-4.18	0.00	0.94	-0.11	1.57	3.22
Berkshire	Ber	-0.90	0.03	0.42	-1.3	14.29	9.34
Wellsfargo	Wel	-4.98	0.08	1.04	0.07	1.92	4.37
Johnson	Joh	-2.88	0.05	0.84	-0.22	0.85	2.57
General.Electric	Gel	-5.35	0.04	1.07	0.33	4.74	7.34
JP.Morgan	JPM	-5.49	0.08	1.24	-0.07	1.49	5.48
Facebook	Fbk	-12.12	0.17	2.73	2.18	21.88	29.56
Wallmart	Wal	-4.36	0.01	0.89	-0.23	2.81	4.72
Procter.and.Gramble	PaG	-6.44	0.04	0.88	-0.20	5.68	4.01
Pfizer	Pfi	-4.41	0.06	0.97	0.00	1.58	4.23
Amazon	Ama	-11.10	0.10	1.87	0.54	11.98	14.11
Walt.Disney	WDi	-8.21	0.12	1.22	0.42	10.48	9.89
Coca.Cola	CCO	-5.96	0.01	0.92	-0.08	4.65	5.44
Gilead.sciences	GiS	-14.34	0.21	1.89	-0.06	8.52	13.72
Visa	Vis	-7.14	0.11	1.27	0.59	7.33	10.29
Citi.group	CGr	-6.22	0.10	1.48	0.16	1.50	6.42

Table 4: USA Correlation Matrix

	App	Goo	Mft	Exx	Ber	Wel	Joh	Gel	JPM	Fbk	Wall	PaG	Pfi	Ama	WDi	CCo	Gis	Vis	CGr
App	1.00	0.23	0.21	0.19	0.01	0.27	0.16	0.01	0.22	0.16	0.20	0.16	0.02	0.16	0.21	0.16	0.16	0.04	0.20
Goo	0.23	1.00	0.35	0.32	-0.03	0.40	0.33	0.08	0.36	0.24	0.25	0.26	0.10	0.49	0.36	0.25	0.28	0.09	0.39
Mft	0.21	0.35	1.00	0.34	0.00	0.37	0.29	0.06	0.36	0.12	0.27	0.27	0.05	0.29	0.30	0.27	0.18	0.12	0.33
Exx	0.19	0.32	0.34	1.00	0.06	0.56	0.48	-0.02	0.51	0.11	0.29	0.40	0.02	0.30	0.40	0.38	0.29	0.08	0.48
Ber	0.01	-0.03	0.00	0.06	1.00	0.04	-0.02	-0.04	-0.01	0.01	0.00	-0.03	0.01	-0.01	0.03	-0.00	0.09	0.05	0.02
Wel	0.27	0.40	0.37	0.56	0.04	1.00	0.53	0.00	0.69	0.16	0.36	0.34	0.02	0.35	0.54	0.38	0.34	0.07	0.66
Joh	0.16	0.33	0.29	0.48	-0.02	0.53	1.00	-0.00	0.44	0.15	0.41	0.50	0.01	0.29	0.43	0.44	0.37	-0.01	0.43
Gel	0.01	0.08	0.06	-0.02	-0.04	0.00	-0.00	1.00	0.01	-0.04	-0.01	-0.04	0.41	0.03	-0.03	-0.04	-0.02	0.36	0.01
JPM	0.22	0.36	0.36	0.51	-0.01	0.69	0.44	0.01	1.00	0.19	0.31	0.32	0.02	0.32	0.44	0.34	0.29	0.06	0.77
Fbk	0.16	0.24	0.12	0.11	0.01	0.16	0.15	-0.04	0.19	1.00	0.05	0.06	-0.04	0.22	0.21	0.07	0.24	-0.02	0.17
Wal	0.20	0.25	0.27	0.29	0.00	0.36	0.41	-0.01	0.31	0.05	1.00	0.43	0.02	0.20	0.41	0.38	0.20	0.02	0.31
PaGr	0.16	0.26	0.27	0.40	-0.03	0.34	0.50	-0.04	0.32	0.06	0.43	1.00	-0.05	0.17	0.34	0.49	0.26	-0.04	0.32
Pfi	0.02	0.10	0.05	0.02	0.01	0.02	0.01	0.41	0.02	-0.04	0.02	-0.05	1.00	0.04	0.01	-0.03	0.06	0.39	0.01
Ama	0.16	0.49	0.29	0.30	-0.01	0.35	0.29	0.03	0.32	0.22	0.20	0.17	0.04	1.00	0.36	0.20	0.27	0.09	0.33
WDi	0.21	0.36	0.30	0.40	0.03	0.54	0.43	-0.03	0.44	0.21	0.41	0.34	0.01	0.36	1.00	0.39	0.26	0.07	0.42
CCo	0.16	0.25	0.27	0.38	-0.00	0.38	0.44	-0.04	0.34	0.07	0.38	0.49	-0.03	0.20	0.39	1.00	0.19	0.01	0.34
Gis	0.16	0.28	0.18	0.29	0.09	0.34	0.37	-0.02	0.29	0.24	0.20	0.26	0.06	0.27	0.26	0.19	1.00	0.07	0.32
Vis	0.04	0.09	0.12	0.08	0.05	0.07	-0.01	0.36	0.06	-0.02	0.02	-0.04	0.39	0.09	0.07	0.01	0.07	1.00	0.06
CGr	0.20	0.39	0.33	0.48	0.02	0.66	0.43	0.01	0.77	0.17	0.31	0.32	0.01	0.33	0.42	0.34	0.32	0.06	1.00

333 In the American market, the correlation between two stocks is larger when  
334 they are from similar sectors, for example Facebook, Google, Amazon and JP  
335 Morgan, City Group and Johnson are positively correlated. There is no signifi-  
336 cant negative correlation between the assets return. All the correlations are low  
337 or medium.

### 338 *3.3. Portfolio Optimisation*

339 Using the data of the two markets, we will use the Conditional Mean-  
340 Variance and the Conditional Mean-Semivariance models to get optimal portfo-  
341 lios that we can invest in each market. The idea is to anticipate the future but  
342 putting into consideration the past. In the classical methods, Mean, Variance  
343 and Semivariance does not take account of the forthcoming data.

344 Our methodology consists in dividing the data into two samples: one for  
345 making the optimization (optimization sample) and the other for testing the  
346 efficiency of the methods (test sample). The optimization sample is used to  
347 determine the optimal weights for each method (classical Mean-Variance, Con-  
348 ditional Mean-Variance, Classical Mean-Semivariance and Conditional Mean-  
349 Semivariance). These weights are used for computing the optimal portfolio  
350 returns for each method.

351 In order to measure the performance of the proposed methods we use the  
352 sample test. The optimal portfolio returns are compared to the naive one and  
353 they are also used to assess performance against the HangSeng index and S&P  
354 500 one. The following parameters and considerations will be used throughout  
355 this section:

- 356 • Optimal portfolio is determined for one period,
- 357 • For the target return  $E^*$ , many values are tested. We have decided to  
358 present only results with  $E^* = 0.075\%$ . Results with other target returns  
359 are similar.
- 360 • The benchmark is chosen equal to 0 ( $B = 0$ ). This choice makes easy  
361 certain calculations.

- The kernel  $\mathcal{K}$  is the multivariate Gaussian density,

$$\mathcal{K}(x) = \frac{1}{(2\pi)^{n/2}} \exp\left(-\frac{x_1^2 + \dots + x_m^2}{2}\right),$$

- 362 • The bandwidths  $h$  are chosen by cross validation method and depend  
363 on each asset return observations (see previous sections). This choice is  
364 motivated by its popularity in nonparametric literature (see Arlot and  
365 Celisse (2010) ).
- 366 • There is no transaction cost,
- 367 • Short selling is allowed.

368 From now on, the following abbreviation will be used:

- 369 • M-V= Classical Mean-Varinace Model
- 370 • C.M-V= Conditional Mean-Varinace Model
- 371 • M-DSR= Classical Mean-Semivarinace Model
- 372 • C.M-DSR= Conditional Mean-Semivarinace Model

373 To test our methods, we used the following procedure:

- 374 • to determine the optimal portfolio, we use all the data collected until May,  
375 31th, 2015,
- 376 • returns collected from June, the 1st, 2015 until July, the 7th, 2015 are used  
377 to compare naive portfolio return (all the weights are equal to  $\omega_i = \frac{1}{m}$ ) to  
378 those obtained by the other methods.

379 For the Chinese Market, results are summarized in the Figure (1). It is clear  
380 that, in term of returns, all the portfolios obtained from the optimization meth-  
381 ods are dominating the naive one. The conditional methods perform better than  
382 the non conditional ones.

383



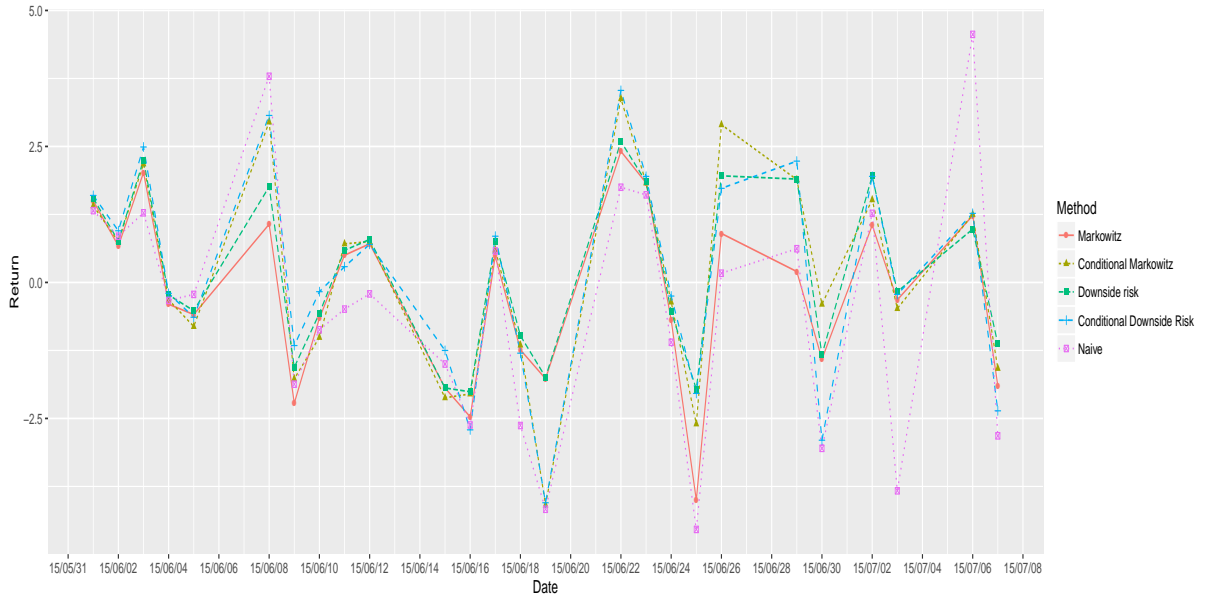


Figure 1: Chinese Portfolio return (%)

384 From Figure (1), we can extract the following comparative Table (5) which  
 385 could inform us about the efficiency of each method compared to effective return  
 386 obtained by the naive portfolio ( $\omega_j = 1/m$ ), computed day after day starting  
 387 from June, the 1st, 2015.

Table 5: Naive versus the other portfolio optimization methods in the Chinese Market

	Naive	M-V.	C.M-V	M-DSR	C.M-DSR
Naive	26/26	16/26	20/26	21/26	<b>22/26</b>

388 In the above table, the numerator of the fraction is the number of days on which  
 389 methods perform better than the naive one. Here we see the supremacy of all  
 390 the methods compared to the naive one. The conditional mean-semivariance  
 391 method seems perform better than all the other methods.

392

393 In the same spirit and under the same conditions, the portfolio optimization  
 394 processes, in the American Market, are summarized in Figure (2).

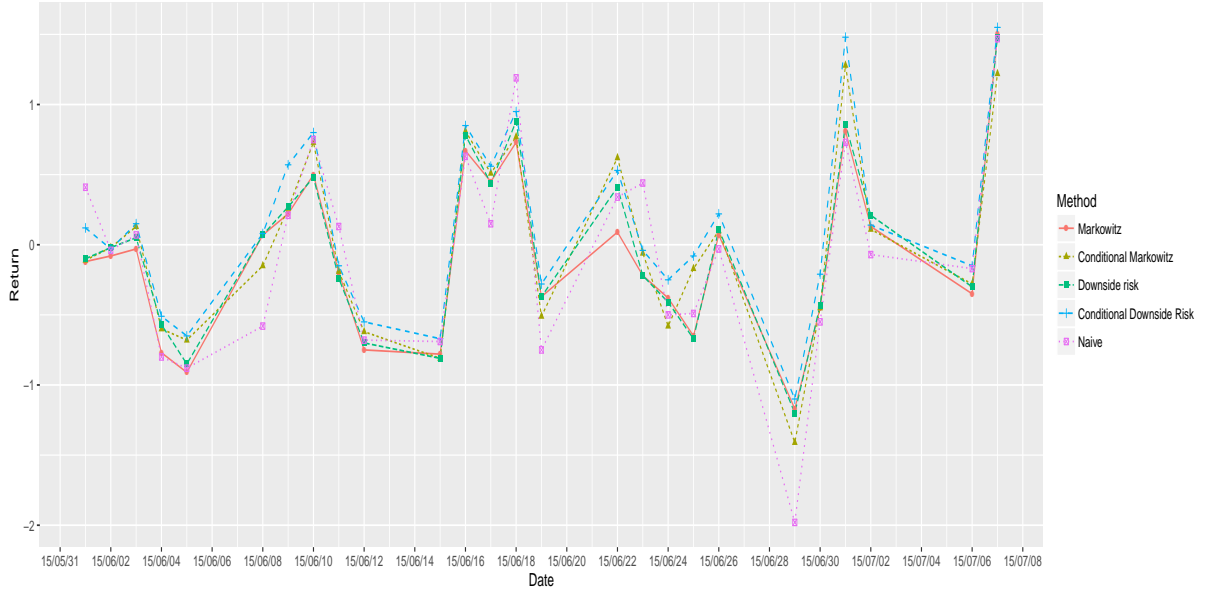


Figure 2: American Portfolio return (%)

395

396 On this market, we observe that the same finding continues to occur as in the  
 397 Chinese Market. All conclusions, concerning the performance of the optimisa-  
 398 tion methods versus the naive one, are summarized in the following comparative  
 399 table (Table 6). The conditionam methods perform better than the classical one.

Table 6: Naive versus the other portfolio optimization methods in the American Market

	Naive	M-V.	C.M-V.	M-DSR	C.M-DSR
Naive	26/26	13/26	17/26	16/26	<b>22/26</b>

400

#### 401 3.4. Another Tool to Test The Efficiency of the Proposed Methods

402 The HangSeng and the S&P 500 indices will contribute to test the efficiency  
 403 of the classical and the new portfolio optimization methods. We will compare  
 404 the daily portfolio return obtained by the weights which are solutions of different

405 optimization programs using all the data until May the 31th , 2015. From June  
406 the 1st, 2015 until July the 7th, 2015, we compute the portfolio returns and we  
407 compare them against the daily HangSeng and the S&P 500 indices. Results  
408 are exhibited in figure 4.1 (the Chinese Market) and figure 4.2 (the American  
409 Market).

- 410 • Red color signifies a negative difference between portfolio return and HangSeng  
411 Index (respectively S&P 500 index).
- 412 • Blue color signifies a positive difference between portfolio return and HangSeng  
413 Index (respectively S&P 500 index).
- 414 • Overall performance is the sum of all the differences during the test period.

415 We note that the comparison is in favour of the Conditional-Mean-Variance and  
416 the Conditional-Mean-Semivariance optimization methods. The overall perfor-  
417 mance varies depending on the method of portfolio optimization and markets:

- 418 • From 0.94% (Classical Markowitz method) to 12.64% (Conditional Mean-  
419 Semivariance method) for the the Chinese Market,
- 420 • From 0.09% (Classical Markowitz method) to 1.42% (Conditional Mean-  
421 Semivariance method) for the American Market.

422 These mouthwatering results deserve more practice and more tests over a long  
423 period of time and on different markets using different financial products.

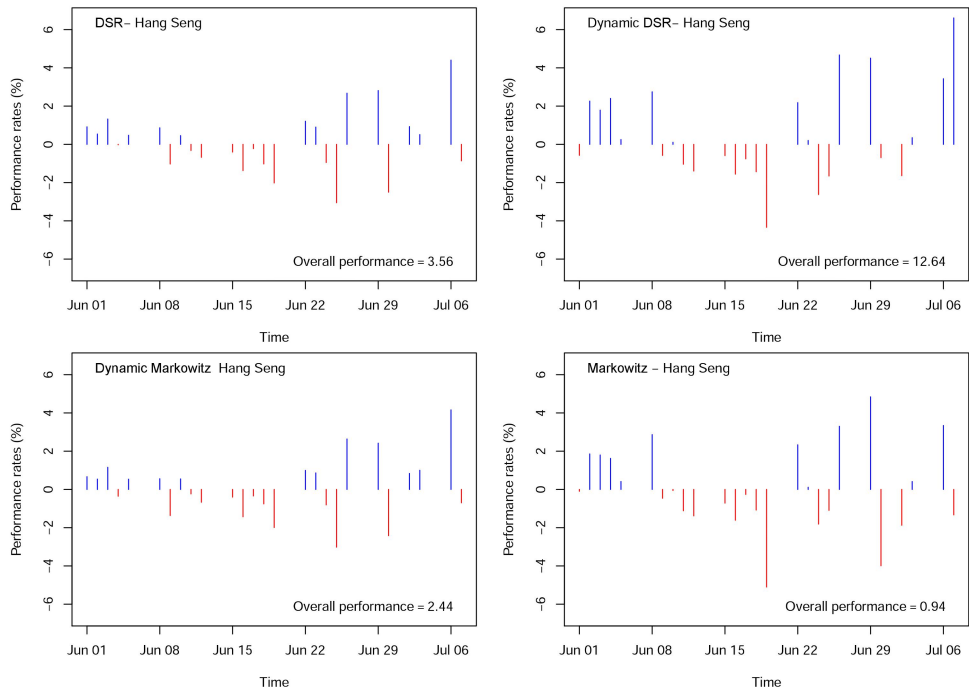


Figure 3: **HangSeng** index versus return of optimal portfolio

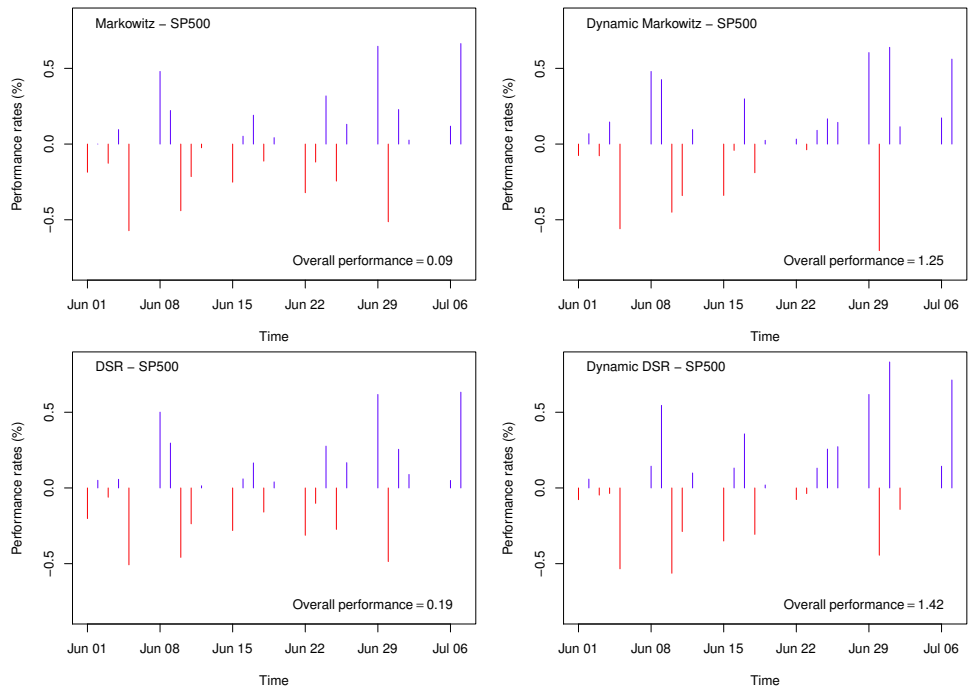


Figure 4: **S&P500** index versus return of optimal portfolio

#### 425 4. Conclusion

426 In this paper, we developed two new approaches in order to get an optimal  
427 portfolio minimizing two conditional risks.

- 428 • The first risk is based on conditional variance. In this case, the optimiza-  
429 tion method is an extension of the classical Mean-Variance model. What  
430 we proposed is to replacing Mean and Variance by Conditional Mean and  
431 Conditional Variance estimators.
- 432 • The second one is based on Conditional Semivariance. In this case, the  
433 optimization method is an extension of the classical Mean-Semivariance  
434 model. What we proposed is to replacing Mean and Semivariance by  
435 Conditional Mean and Conditional semivariance estimators.

436 This novelty, using conditional risk, gives a new approach and a more efficient  
437 alternative to get an optimal portfolio. In fact, all the other methods do not  
438 anticipate the future and just extrapolate to the future what we observed in the  
439 past.

440 In both cases, the optimization algorithm involved using the Lagrangian  
441 method. Even, our results seem interesting, the efficiency of our methods should  
442 be confirmed on other markets and with other various assets. Kernel methods,  
443 belonging to nonparametric methods, are used to estimate Conditional Mean,  
444 Variance and Semivariance. Product Gaussian densities are used as kernel.  
445 It will be very helpful to choose typical multivariate kernels. Similarly, we  
446 should develop a global method to choose the bandwidth which is crucial in  
447 nonparametric estimation.

448 Back to results of this paper: the Conditional Mean-Semivariance seem most  
449 appropriate to get an optimal portfolio using the data of the Chinese and the  
450 American markets. By the way, Conditional Mean-Variance is more efficient  
451 than Mean-Variance method. Similarly, the Conditional Mean-Semivariance is  
452 better than the Mean-Semivariance method.

453 Thanks to its robustness, it is also reasonable to substitute the conditional  
454 median to the conditional mean and to propose an optimization model based  
455 on Conditional Median and conditional variance or conditional Median and  
456 Conditional Semivariance. This topic will be treated as a matter for future  
457 research.

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