A FAMILY OF LINEAR SINGLY DIAGONAL RUNGE-KUTTA METHODS AND HIGH ORDER PADE'S SCHEMES FOR ODE

HÉLÈNE BARUCQ*, **MARC DURUFLÉ**† **AND MAMADOU N'DIAYE***

*Department of Mathematics, University of Pau and INRIA-Bordeaux Sud-Ouest, Team Magique-3D, Pau, France.
email: helene.barucq@inria.fr, mamadou.ndiaye@inria.fr

† Department of Mathematics, University of Bordeaux and INRIA-Bordeaux Sud-Ouest, Team Magique-3D, Bordeaux, France.
email: marc.durufle@inria.fr

The solution of wave equations with locally time-stepping is of particular interest in case of realistic applications for which local high-order space approximations or locally refined mesh are relevant. The numerical simulation of waves inside the Earth is a challenging example. The general form of the corresponding ODE (Ordinary Differential Equations) reads:

$$M_h \frac{dU}{dt} + K_h U \;=\; F(t),$$

where M_h is the mass matrix and K_h is the stiffness matrix. The problem is stiff because the eigenvalues of $M_h^{-1} K_h$ may be large. As a consequence, the stability condition of explicit time schemes might become too restrictive. As a preliminary work for the design of high order locally implicit discretizations of wave problems, we provide a performance assessment of different explicit and implicit one-step schemes. The comparison criteria are based on the amplitude and phase errors.

Following Burrage's work ([1]) we propose a family of Linear Singly Diagonal Implicit Runge-Kutta schemes (Linear SDIRK) and we analyse the construction of high order schemes by optimizing the constant error. Then we present a new high order non-dissipative time scheme derived from the Padé approximation (see [2]) of an exponential function.

Numerical experiments are conducted in 1-D and 2-D for Maxwell's equations. The numerical results are performed with high order finite elements by using the code Montjoie (see [3]).

REFERENCES

[1] Kevin Burrage, "A special family of Runge-Kutta methods for solving stiff differential equations", *BIT*, 1977

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