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ABSTRACT

In this paper, we investigate the problem of simultaneous state and parameter estimation for a class of nonlinear systems which can be transformed into an output depending normal form. A new and simple adaptive observer for such class of systems is presented. Sufficient condition for the existence of the proposed observer is derived. A concrete application is given in order to highlight the effectiveness of the proposed result.

I. Introduction

Observer design problem has been widely studied in the literature since the last four decades. Many methods have been developed, and one famous approach is based on differential geometric method, which enables us to easily reuse the existing observers. The literature is vast about nonlinear observer normal form approach (see for example [1, 2, 3, 4, 5, 6, 7]).

For those proposed normal forms, many different types of observers are stated in the literature to estimate the states of the studied system [8, 9]. In [10], a Kalman like adaptive observer was presented for state-affine systems with linear time-varying matrix, and a high gain observer was proposed in [11] for the nonlinear system with triangular form. After that, other adaptive observers are studied to estimate of the states for more general normal forms, including the output depending normal form [12, 13].

On the other hand, there exist as well many works to simultaneously estimate the state and the parameters by using adaptive observers. In [14], the authors proposed adaptive observers for a class of uniformly observable nonlinear systems with general linear/nonlinear parameterizations which is of triangular form with linear constant part. [15] treated the simultaneous of states and parameters for linear systems with nonlinear parameterized perturbations. Another adaptive observer was introduced in [16] in order to treat the nonlinear parameterization case. Except adaptive method, other approaches can be used as well to simultaneously estimate the state and the parameters, such as sliding mode technique [17].

However, the mentioned methods are almost for nonlinear systems which are either of the output injection normal form, or of the triangular normal form, and no results are reported for the output depending normal form. Since those proposed observers cannot be applied directly to such a form, thus this paper is motivated to propose a new and simple adaptive observer for the simultaneous estimations of the state and the parameters of this normal form. As an extension of [18], the novelties of this paper are the following two aspects. Firstly necessary and sufficient conditions are deduced which guarantee the existence of a parameter-independent diffeomorphism transforming a class of nonlinear systems into a more general output depending nonlinear observer form with output injection. Secondly we show that if some sufficient conditions are satisfied, then a very simple adaptive observer can be designed to simultaneously estimate the state and parameter of the proposed normal form.

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II. Problem statement

Consider the following nonlinear system:

$$\left\{\begin{array}{l} \dot{x}=F(x)+G(x)\theta+Q(x)u \\ y=H(x) \end{array}\right. \tag{1}$$

where $x \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}^q$ is the input, $\theta \in \mathbb{R}^m$ is the constant parameter, $y \in \mathbb{R}$ is the output, $F : \mathbb{R}^n \to \mathbb{R}^n$, $G = [G_1, \cdots, G_m]$ with $G_i : \mathbb{R}^n \to \mathbb{R}^n$, $Q = [Q_1, \cdots, Q_q]$ with $Q_i : \mathbb{R}^n \to \mathbb{R}^n$, and $H : \mathbb{R}^n \to \mathbb{R}$ are smooth. It is supposed in this paper that $F(0) = 0$, $G(0) = 0$, $Q(0) = 0$ and $H(0) = 0$, and the pair (H, F) satisfies the observability rank condition, i.e. $\mathrm{rank}\{dL_F^{i-1}H, 1 \le i \le n\} = n$, where $L_F^k H$ is the k^{th} Lie derivative of H along F and $dL_F^{i-1}H$ stands for the associated differentiation for $1 \le i \le n$ with $dL_F^0 H = dH$. Then, one can construct the well-known Krener & Isidori [1] frame: $\tau = [\tau_1, \cdots, \tau_n]$ with the first vector field τ_n given by the following algebraic equations:

$$\left\{\begin{array}{l} dL_F^{i-1}H(\tau_n)=0 \text{ for } 1 \le i \le n-1 \\ dL_F^{n-1}H(\tau_n)=1 \end{array}\right.$$

whereas the rest of vector fields are obtained by iterating on i: $\tau_i = [\tau_{i+1}, F]$ for $1 \le i \le n-1$ where $[,]$ denotes the conventional Lie bracket.

In [19], the commutativity of Lie bracket, i.e. $[\tau_i, \tau_j] = 0$ for $1 \le i \le n$ and $1 \le j \le n$, is the necessary and sufficient condition to transform system (1) with $G(x) = Q(x) = 0$ into the following well-known nonlinear observer form with output injection:

$$\left\{\begin{array}{l} \dot{z}=Az+\beta(y) \\ y=Cz \end{array}\right.$$

where $\beta(y) : \mathbb{R} \to \mathbb{R}^n$, and the matrices A and C are of the Brunovsky form.

This result was extended to study system (1) with input and parameter in [20], and the necessary and sufficient conditions are given which guarantee the existence of a diffeomorphism such that system (1) can be transformed as well into the above similar output injection form and an adaptive observer was proposed to estimate the state and parameter of the studied system. In that paper, the commutativity of Lie bracket condition was still imposed, i.e. $[\tau_i, \tau_j] = 0$ for $1 \le i \le n$ and $1 \le j \le n$.

The problem now is raised when this commutativity condition cannot be satisfied, then one cannot apply the method proposed in [20]. The main contributions of this paper are: 1) to propose a new output injection form; 2) to deduce the necessary and sufficient conditions to transform the studied system into the proposed form; 3) to design a very simple adaptive observer which simultaneously estimate the state and the parameters.

III. Assumptions, notations and preliminary results

Suppose that the the commutativity of Lie bracket condition is not satisfied for the family of vector fields $\tau = [\tau_1, \cdots, \tau_n]$ proposed by [19], this implies that the studied system cannot be transformed into the simple output injection form with constant matrix A of the famous Brunovsky form, then according to [4] we can construct another frame $\bar{\tau} = [\bar{\tau}_1, \cdots, \bar{\tau}_n]$ from τ as follows:

$$\left\{\begin{array}{l} \bar{\tau}_n=\pi(y)\tau_n \\ \bar{\tau}_i=\frac{1}{a_i(y)}[\bar{\tau}_{i+1}, F], \text{ for } 1 \le i \le n-1 \end{array}\right. \tag{2}$$

where $\pi(y) = \prod_{i=1}^{n-1} a_i(y)$, and $a_i(y)$ for $1 \le i \le n-1$ being non vanishing functions of the output to be determined. Based on this new frame, one can state the following theorem, which is in fact an extension of our previous result in [4].

Theorem 1 *There exists a parameter-independent diffeomorphism $z = \phi(x)$ which transforms system (1) into the following output depending nonlinear observer form with output injection:*

$$\left\{\begin{array}{l} \dot{z}=A(y)z+\beta(y)+g(y)\theta+q(y)u \\ y=Cz \end{array}\right. \tag{3}$$

where $g(y)$ and $q(y)$ are of the appropriate dimensions, $A(y)$ is of the output depending Brunovsky form:

$$A(y)=\begin{bmatrix} 0 & a_1(y) & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & a_{n-2}(y) & 0 \\ 0 & 0 & \cdots & 0 & a_{n-1}(y) \\ 0 & 0 & \cdots & 0 & 0 \end{bmatrix} \tag{4}$$

and $C = [1, 0, \cdots, 0]$, if and only if the following conditions are satisfied:

1. $[\bar{\tau}_i, \bar{\tau}_j] = 0$ *for* $1 \le i \le n$ *and* $1 \le j \le n$;
2. $[\bar{\tau}_i, Q_j] = 0$ *for* $1 \le i \le n-1$ *and* $1 \le j \le q$;
3. $[\bar{\tau}_i, G_j] = 0$ *for* $1 \le i \le n-1$ *and* $1 \le j \le m$.

Proof 1 *It has been proven in [4] (see Theorem* 3.1 *in [4]) that there exists a diffeomorphism* $z=\phi(x)$ *such that system (1) with* $G(x)=0$ *and* $Q(x)=0$ *can be transformed into the form (3) with* $q(y)=0$ *and* $g(y)=0$ *if and only if the first condition of Theorem 1 is satisfied. Following the same procedure, it is easy to prove that the second and the third conditions of Theorem 1 are necessary and sufficient to guarantee that* $G(x)\theta$ *and* $Q(x)u$ *can be transformed into* $g(y)\theta$ *and* $q(y)u$ *by the deduced diffeomorphism* $z=\phi(x)$.

Remark 1 *If* $a_i=1$ *in the matrix* $A(y)$ *defined in (4), system (3) can be seen as an extension of the output injection normal form proposed in [19], for which an adaptive observer was proposed in [20] to simultaneously estimate the state and the parameters. Besides, in [14], the authors proposed the adaptive observers for a class of uniformly observable nonlinear systems with linear/nonlinear parameterizations. However, the proposed observer is complicated since it is based on the adaptive observer proposed by [21] whose gain depends on the solutions of two differential equations. Moreover, the proposed observer in [14] cannot be applied for (3) with the output depending matrix* $A(y)$ *since it works only for a constant matrix* A.

Due to the physical constraint, the control and the state values of the practical systems are always bounded. Therefore, in what follows, we make the following standard (see [11, 14], for example) assumption in the estimation theory on the boundedness of the state and the input for system (1).

Assumption 1 *For the studied system (1), it is assumed that the state* $x(t)$ *is bounded for the given bounded parameter* θ *and the input* u, *i.e.* $x(t)\in\mathscr{X}\subset\mathbb{R}^n$ *under the given* $u(t)\in\mathscr{U}\subset\mathbb{R}^q$ *for any* $t\geq 0$ *and* $\theta\in\Theta\subset\mathbb{R}^m$ *where* $\mathscr{X}$, $\mathscr{U}$ *and* Θ *are the compact sets.*

Since it is assumed that system (1) can be transformed via a smooth diffeomorphism $z=\phi(x):\mathscr{X}\to\mathscr{Z}$ into the output depending normal form (3), therefore the state z and the output y of (3) are both bounded as well, i.e. $z(t)\in\mathscr{Z}\subset\mathbb{R}^n$ and $y(t)\in\mathscr{Y}\subset\mathbb{R}$ for any $t\geq 0$ where $\mathscr{Z}$ and $\mathscr{Y}$ are two corresponding compact sets. Therefore, for any $a_i(y)\neq 0$ of system (3), there always exist positive constants $\underline{a}_i>0$, $\overline{a}_i>0$ and $\sigma_i>0$ for $1\leq i\leq n-1$ such that

$$0<\underline{a}_i<|\, a_i(y)\,|<\overline{a}_i<\infty,\ \forall y\in\mathscr{Y} \tag{5}$$

and

$$|\,\frac{da_i(y(t))}{dt}\,|<\sigma_i<\infty,\ \forall y\in\mathscr{Y} \tag{6}$$

For the sake of simplicity and without loss of generality, let us define $\underline{a}=\min_{1\leq i\leq n-1}\{\underline{a}_i\}$ and $\overline{a}=\max_{1\leq i\leq n-1}\{\overline{a}_i\}$.

Assumption 2 *It is assumed that the vector-function* $g(y)$ *in (3) is persistently exciting for all* $y\in\mathscr{Y}$, *i.e. there exist positive constants* $b>0$ *and* $T>0$ *such that*

$$\int_t^{t+T} g^T(y(s))g(y(s))ds\geq b\mathbf{I}_{\mathbf{m}\times\mathbf{m}}$$

For the studied normal form, we define the following variables which will be used in the sequel. Denote

$$A_0=\begin{pmatrix} 0 & 1 & \cdots & 0 & 0\\ \vdots & \vdots & \ddots & \vdots & \vdots\\ 0 & 0 & \cdots & 0 & 1\\ 0 & 0 & \cdots & 0 & 0\end{pmatrix} \tag{7}$$

and

$$\Gamma(y,\rho)=diag\{\lambda_1,\cdots,\lambda_n\} \tag{8}$$

where $\lambda_1=1$, and $\lambda_i=\frac{\prod_{j=1}^{i-1}a_j(y)}{\rho^{i-1}}$ for $2\leq i\leq n$ with positive constant ρ being freely chosen, which determines the gain of the proposed observer. Since $0<\underline{a}_i<|\,a_i(y)\,|<\overline{a}_i<\infty$, then $\Gamma(y,\rho)$ is always invertible and bounded. Define

$$\Lambda(y(t),\dot{y}(t),\rho)=\dot{\Gamma}(y(t),\dot{y}(t),\rho)\Gamma^{-1}(y(t),\rho) \tag{9}$$

where $\dot{\Gamma}(y(t),\dot{y}(t),\rho)=\frac{d\Gamma(y(t),\rho)}{dt}=\dot{y}(t)\frac{\partial\Gamma(y(t),\rho)}{\partial y}$.

If Assumption 1 is satisfied, then $\Lambda(y(t),\dot{y}(t),\rho)$ defined in (9) is bounded for all $y(t)\in\mathscr{Y}$ and $t\geq 0$, and there always exists a positive constant $\bar{\lambda}$ such that each element of $\Lambda(y(t),\dot{y}(t),\rho)$ satisfies $|\Lambda_{i,j}(y(t),\dot{y}(t),\rho)|<\bar{\lambda}<\infty$, $\forall y\in\mathscr{Y}$ for $1\leq i\leq n$ and $1\leq j\leq n$. Let S_ρ be the solution of the following equation:

$$0=\rho S_\rho+A_0^TS_\rho+S_\rho A_0-C^TC \tag{10}$$

with A_0 defined in (7), where S_ρ is symmetric positive definite [10].

IV. Main result

Consider system (3) with $\Gamma(y,\rho)$ and S_ρ defined in the last section, let us denote

$$\mathscr{G}=\left[\mathscr{G}_1^T,\cdots,\mathscr{G}_m^T\right]^T\triangleq g^T(y)\Gamma(y,\rho)S_\rho \tag{11}$$

with the ith row in $\mathscr{G}$ noted as $\mathscr{G}_i:\mathbb{R}^n\to\mathbb{R}^n$ for $1\leq i\leq m$. Moreover, let us denote $\mathscr{Y}$ as the field of the output function, based on which we can define the module spanned by C over $\mathscr{Y}$ as follows:

$$\Omega = \mathrm{span}_{\mathscr{Y}}\{dy\} \tag{12}$$

then the following assumption is imposed.

Assumption 3 *With $\Gamma(y,\rho)$ and S_ρ respectively defined in (8) and (10), it is assumed that $\mathscr{G}_i \in \Omega$ for $1 \leq i \leq m$ where $\mathscr{G}_i$ and Ω are defined in (11) and (12), respectively.*

Lemma 1 *If Assumption 3 is satisfied, then there exists an output depending matrix*

$$K(y) = g^T(y)\Gamma(y,\rho)S_\rho C^T \tag{13}$$

such that $g^T(y)\Gamma(y,\rho)S_\rho = K(y)C$.

Proof 2 *According to the definition of Ω in (12), if Assumption 3 is satisfied, then for each $\mathscr{G}_i$, there exists $K_i(y)$ such that $\mathscr{G}_i = K_i(y)C$ for $1 \leq i \leq m$. Hence we have*

$$g^T(y)\Gamma(y,\rho)S_\rho = \begin{bmatrix} \mathscr{G}_1 \\ \vdots \\ \mathscr{G}_m \end{bmatrix} = \begin{bmatrix} K_1(y) \\ \vdots \\ K_m(y) \end{bmatrix} C \stackrel{\triangle}{=} K(y)C$$

where $C = [1, 0, \cdots, 0]$. From the above equation we can prove that, if Assumption 3 is satisfied, then by choosing $K(y) = g^T(y)\Gamma(y,\rho)S_\rho C^T$ one has always the equality $g^T(y)\Gamma(y,\rho)S_\rho = K(y)C$.

Remark 2 *For a class of nonlinear systems which can be transformed into the following output injection form:*

$$\begin{cases} \dot{z} = Az + \bar{\beta}(y,u) + b\bar{g}(y,u)\theta \\ y = Cz \end{cases} \tag{14}$$

one can design a very simple adaptive observer, proposed in [20], if there exists a positive definite symmetric matrix P such that $b^T P = C$. It is clear this sufficient condition is a special form of (13) since the form (14) is a special case of the form (3).

Theorem 2 *If Assumption 1, 2 and 3 are satisfied, then there exist three positive constants $\gamma > 0$, $\rho > 0$ and $\eta > 0$ such that the following dynamics*

$$\begin{cases} \dot{\hat{z}} = & A(y)\hat{z} + \beta(y) + g(y)\hat{\theta} + q(y)u \\ & +\gamma\Gamma^{-1}(y,\rho)S_\rho^{-1}C^T(y - C\hat{z}) \\ \dot{\hat{\theta}} = & \eta K(y)(y - C\hat{z}) \end{cases} \tag{15}$$

where $K(y)$ is defined in (13), is an adaptive observer to simultaneously estimate the state z and the parameter θ of system (3).

Proof 3 *Denote $e_z = z - \hat{z}$ and $e_\theta = \theta - \hat{\theta}$, thus the observation error dynamics is governed by the following system:*

$$\dot{e}_z = \left(A(y) - \gamma\Gamma^{-1}(y,\rho)S_\rho^{-1}C^T C\right)e_z + g(y)e_\theta \tag{16}$$

Since $a_i(y) \neq 0$ for all $y \in \mathscr{Y}$, then $\Gamma(y,\rho)$ is always invertible. Therefore we can make the following change of coordinates $\varepsilon_z = \Gamma(y,\rho)e_z$ and $\varepsilon_\theta = e_\theta$. According to the definition of $\Gamma(y,\rho)$, we have

$$\begin{aligned} \dot{\varepsilon}_z &= \left[\Gamma(y,\rho)A(y) - \gamma S_\rho^{-1}C^T C\right]\Gamma^{-1}(y,\rho)\varepsilon_z \\ &\quad +\Gamma(y,\rho)g(y)e_\theta + \dot{\Gamma}(y,\rho)\Gamma^{-1}(y,\rho)\varepsilon_z \end{aligned}$$

Since $\Gamma(y,\rho)A(y)\Gamma^{-1}(y,\rho) = \rho A_0$ and $C\Gamma^{-1}(y,\rho) = C$, then one has

$$\begin{aligned} \dot{\varepsilon}_z &= \left[\rho A_0 - \gamma S_\rho^{-1}C^T C\right]\varepsilon_z + \Lambda(y,\dot{y},\rho)\varepsilon_z \\ &\quad +\Gamma(y,\rho)g(y)\varepsilon_\theta \end{aligned} \tag{17}$$

which is due to the fact that $\Lambda(y,\dot{y},\rho) = \dot{\Gamma}(y,\rho)\Gamma^{-1}(y,\rho)$.

Since $\varepsilon_\theta = e_\theta$ and $\varepsilon_z = \Gamma(y(t),\rho)e_z$ with $\Gamma(y(t),\rho)$ being always bounded and invertible for all $y \in \mathscr{Y}$, then the convergence of e_z and e_θ to zero are equivalent to the convergence of ε_z and ε_θ to zero. In what follows, we will prove the convergence of ε_z and ε_θ to zero.

For this, let us consider the following Lyapunov function $V(\varepsilon_z,\varepsilon_\theta) = \varepsilon_z^T S_\rho \varepsilon_z + \frac{\varepsilon_\theta^T \varepsilon_\theta}{\eta}$, then one has

$$\begin{aligned} \dot{V}(\varepsilon_z,\varepsilon_\theta) &= \varepsilon_z^T\left(-2\gamma C^T C + \rho C^T C\right)\varepsilon_z + 2\frac{\varepsilon_\theta^T\dot{\varepsilon}_\theta}{\eta} \\ &\quad +\varepsilon_z^T\left(-\rho^2 S_\rho + 2\Lambda S_\rho\right)\varepsilon_z + 2\varepsilon_z^T S_\rho \Gamma g(y)\varepsilon_\theta \end{aligned}$$

Since Assumption 3 is supposed to be satisfied, then according to Lemma 1 we can find an output depending vector $K(y)$ such that $g^T(y)\Gamma S_\rho = K(y)C$. Due to the definition of $\dot{\hat{\theta}}$ in (15), we have

$$\dot{\varepsilon}_\theta = \dot{\theta} - \dot{\hat{\theta}} = -\eta K(y)(y - C\hat{z}) = -\eta g^T(y)\Gamma S_\rho \varepsilon_z$$

where θ is the constant parameters to be identified. Then we have $2\varepsilon_z^T S_\rho \Gamma g(y)\varepsilon_\theta + 2\frac{\varepsilon_\theta^T\dot{\varepsilon}_\theta}{\eta} = 0$. Therefore we obtain the following equivalence:

$$\begin{aligned} \dot{V}(\varepsilon_z,\varepsilon_\theta) &= \varepsilon_z^T\left(-2\gamma C^T C + \rho C^T C\right)\varepsilon_z \\ &\quad +\varepsilon_z^T\left(-\rho^2 S_\rho + 2\Lambda S_\rho\right)\varepsilon_z \end{aligned}$$

Since $|\Lambda_{i,j}(y(t),\dot{y}(t),\rho)| < \bar{\lambda} < \infty$, $\forall y \in \mathscr{Y}$, if we choose $\rho > \rho_0 + \sqrt{\bar{\lambda}/2}$ with any positive constant $\rho_0 > 0$ and $\gamma > \gamma_0 + \frac{\rho}{2}$ where γ_0 is any positive constant, *then we can conclude that* $\dot{V}(\varepsilon_z,\varepsilon_\theta) \le -\rho_0 \varepsilon_z^T S_\rho \varepsilon_z$. *Integration of this inequality yields*

$$\begin{aligned} V(\varepsilon_z(t),\varepsilon_\theta(t)) &- V(\varepsilon_z(t_0),\varepsilon_\theta(t_0)) \\ &\le -\rho_0 \int_{t_0}^t \varepsilon_z^T(s) S_\rho \varepsilon_z(s) ds \end{aligned}$$

Since $V(\varepsilon_z(t),\varepsilon_\theta(t)) \ge 0$*, then*

$$\rho_0 \int_{t_0}^t \varepsilon_z^T(s) S_\rho \varepsilon_z(s) ds \le V(\varepsilon_z(t_0),\varepsilon_\theta(t_0))$$

Due to the fact that $V(\varepsilon_z(t_0),\varepsilon_\theta(t_0))$ *is bounded, thus* $\varepsilon_z(t) \in \mathscr{L}_2$*. Moreover, since* $\Lambda(y,\dot{y},\rho)$*,* $\Gamma(y,\rho)$ *and* $g(y)$ *are all bounded for all* $y \in \mathscr{Y}$*, then from* (17) *one can conclude that* $\dot{\varepsilon}_z(t) \in \mathscr{L}_\infty$*. Finally we have* $\varepsilon_z(t) \in \mathscr{L}_2 \cap \mathscr{L}_\infty$ *and* $\dot{\varepsilon}_z(t) \in \mathscr{L}_\infty$*, and according to Barbalat theorem, this implies that* $\lim_{t\to\infty} \varepsilon_z(t) = 0$.

From (17)*, we can see that* $\dot{\varepsilon}_z(t)$ *is bounded and uniformly continuous, thus we have* $\lim_{t\to\infty} \dot{\varepsilon}_z(t) = 0$ *by applying Barbalat theorem. Since both* $\lim_{t\to\infty} \varepsilon_z(t) = \lim_{t\to\infty} \dot{\varepsilon}_z(t) = 0$*,* (17) *implies* $\lim_{t\to\infty} \Gamma(y,\rho) g(y) \varepsilon_\theta(t) = 0$*. Due to the fact that* $\Gamma(y,\rho)$ *is bounded and positive definite symmetric for all* $y \in \mathscr{Y}$*, and* $g(y)$ *is persistently exciting according to Assumption* 2*, then we can prove that* $\lim_{t\to\infty} \varepsilon_\theta(t) = 0$.

V. Application to batch reactor

Let us consider a real batch reactor system adopted from [2], with two chemicals: $A \xrightarrow{k} B$ where k is the reaction rates. According to [2], the dynamics can be described as follows:

$$\begin{cases} \frac{dC_A}{dt} = -ke^{-\frac{E}{RT_e}} C_A \\ \frac{dC_B}{dt} = ke^{-\frac{E}{RT_e}} C_A \\ \frac{dT_e}{dt} = Jke^{-\frac{E}{RT_e}} C_A + \Delta H_V(T_e) \\ y = T_e \end{cases} \tag{18}$$

where C_A and C_B are the concentrations, J, R, E are known constants, $\Delta H_V(T_e)$ is a *known* function of the temperature T_e. From the above equation, it can be seen that $C_A + C_B = const$ which implies that the conservation law is satisfied, therefore, we need only to estimate C_A. Suppose that system (18) is perturbed with an *unknown* constant disturbance θ in the following way:

$$\begin{cases} \frac{dC_A}{dt} = -ke^{-\frac{E}{RT_e}} C_A - \left[e^{-\frac{E}{RT_e}} - e^{-\frac{2E}{RT_e}}\right]\theta \\ \frac{dT_e}{dt} = Jke^{-\frac{E}{RT_e}} C_A + \Delta H_V(T_e) + e^{-\frac{2E}{RT_e}}\theta \\ y = T_e \end{cases} \tag{19}$$

and the objective is to simultaneously estimate both the concentration C_A and the unknown parameter θ. Following the procedure presented in [4], one obtains the following diffeomorphism: $z = \phi(x) = [T_e, JkC_A + kT_e]^T$ which transforms (19) into the following normal form:

$$\begin{aligned} \dot{z}_1 &= e^{-\frac{E}{Ry}} z_2 + \Delta H_V(y) + e^{-\frac{2E}{Ry}}\theta \\ \dot{z}_2 &= k\Delta H_V(y) + \left[Jke^{\frac{-E}{Ry}} - Jke^{\frac{-2E}{Ry}} + ke^{\frac{-2E}{Ry}}\right]\theta \\ y &= z_1 \end{aligned} \tag{20}$$

Since the studied system is a real batch reactor, thus the boundedness assumption of state is satisfied, and one can follow the proposed method to design an observer of the form (15). For the simulation settings, we set $J = 1, k = 100, \theta = -3$, $\rho = 15$, $\eta = 102500$ and $\gamma = 5$. The estimation errors are presented in Fig. 1.

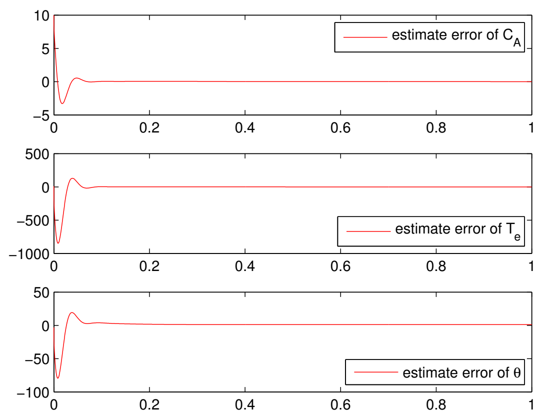

Fig. 1. Estimation errors for C_A, T_e and θ.

VI. Conclusion

This paper dealt with adaptive observer design for a special class of nonlinear systems, which can be transformed into an output depending normal form. Sufficient conditions were presented to guarantee the simultaneously asymptotic estimations of the state and the parameter for the studied system. The result was applied to estimate the simultaneous state and parameter estimations for a real batch reactor system.

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