Stabilizability of Some Neutral Delay Systems with Chains of Poles Clustering on the Imaginary Axis

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Abstract— **When dealing with phenomena such as transport, propagation or communication, it is a crucial issue to take delays into account to avoid bad performances. Neutral type delay systems are the most delicate to analyze as they may have chains of poles clustering the imaginary axis. We investigate here the H_∞-stabilizability of two particular neutral delay systems with a chain of poles clustering the imaginary axis: the first one having a chain of poles in the left-half plane and the second one having a chain of poles in the right-half plane. In both case, we show how to construct a coprime factorization over H_∞ proving then they are both H_∞-stabilizable.**

Index Terms— **Neutral systems, chain of poles, stabilizability; AMS Subject classifications: 93C25, 93D15**

I. INTRODUCTION

Neutral delay systems with commensurate delays possess chains of poles asymptotic to vertical lines in the complex plane. When those asymptotic axes lie in the open left-half plane ie when the systems only possess a finite number of unstable poles their H_∞-stability properties are easy to determine: as for delay systems of retarderd type (and as for finite-dimensional systems) the absence of poles in the closed right halp-plane is a necessary and sufficient condition to ensure H_∞-stability. With this condition in hand, coprime and Bézout factors can be determined following the scheme developed for retarded systems, allowing then the parametrization of all H_∞-controllers. The more delicate case of neutral systems having chains of poles clustering the imaginary axis has been studied in [5], [2] where necessary and sufficient conditions of H_∞-stability have been given. Our aim in this paper is to investigate now the existence of coprime factorization over H_∞-stability for such systems and it is well-known ([6]) that this is equivalent to their stabilizability over H_∞-stability. We restrict here the study to systems with a transfer function having a numerator equal to one and a denominator being a quasi-polynomial with one delay and leading polynomial of degree two. We are interested in both cases of systems having a chain of poles clustering the imaginary axis from left or from right. For sake of simplicity we present here our method on two particular systems belonging to each class.

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II. SYSTEMS WITH A CHAIN OF POLES CLUSTERING ON THE IMAGINARY AXIS FROM RIGHT

In this section we consider a particular neutral delay systems with one delay and a chain of poles asymptotic to the imaginary axis from the right with transfer function given by

$$G(s) = \frac{1}{(s+1)(s+1+(s+2)e^{-sT})} \tag{1}$$

We have $\dfrac{s+1}{s+2} = \alpha - \dfrac{\beta}{s} + \dfrac{\gamma}{s} + o(s^{-2})$ with $\alpha = 1, \beta = -1, \gamma = 2$ satisfying $\dfrac{\gamma}{\alpha} - \dfrac{\beta^2}{2} > 0$ so that from [5] we know that the chain of poles is in the right halp-plane.

Proposition 1 *Let G be the one given by (1). Then $(N(s), D(s))$ defined by*

$$N(s) = \frac{1}{(-\frac{1}{3}s^2 - \frac{2}{3}s + \frac{1}{2}) + (-\frac{1}{3}s^2 - s - 1)e^{-sT}}$$

and,

$$D(s) = \frac{(s+1)(s+1+(s+2)e^{-sT})}{(-\frac{1}{3}s^2 - \frac{2}{3}s + \frac{1}{2}) + (-\frac{1}{3}s^2 - s - 1)e^{-sT}}$$

gives a coprime factorization of G over H_∞

Proof:

Consider the quasi-polynomial $as^2+bs+c+(ds^2+es+f)e^{-sT}$ as a potential candidate for the denominator of $N(s)$ and $D(s)$. Write

$$N(s) = \frac{1}{as^2+bs+c+(ds^2+es+f)e^{-sT}},$$

and

$$D(s) = \frac{(s+1)(s+1+(s+2)e^{-sT})}{as^2+bs+c+(ds^2+es+f)e^{-sT}}.$$

We then have

$$\begin{aligned} D(s) &= \frac{(s+1)(s+1+(s+2)e^{-sT}}{as^2+bs+c+(ds^2+es+f)e^{-sT}} \\ &= \frac{1}{a}\left(1 + \frac{(2a-b)s+a-c}{d(s)}\right) \\ &\quad + \frac{1}{a}\left(\frac{(a-d)s^2+s(3a-e)+2a-f}{d(s)}\right)e^{-sT} \end{aligned}$$

Taking $b = 2a$, $d = a$ and $e = 3a$, we see that $D(s)$ is in H_∞ provided that $as^2+bs+c+(ds^2+es+f)e^{-sT}$ has no zeros in the closed right-half plane.

Using the Walton-Marshall method [3] we obtain some conditions on the coefficients to ensure that $as^2+bs+c+$

$(ds^2+es+f)e^{-sT}$ has no unstable poles in the closed right half-plane: $a=-\frac{1}{3}, c=-\frac{1}{2}, f=-1$.

Now, it is easy to verify that (N,D) satisfies the Corona condition [1] $\inf_{\mathrm{Re}\, s>0}(|N(s)|+|D(s)|)>0$. ■

III. Systems with a chain of poles clustering on the imaginary axis from left

In this section we consider another neutral delay systems with one delay and a chain of poles asymptotic to the imaginary axis from the left with transfer function given by

$$G(s)=\frac{1}{(s+1)(s-1+se^{-sT})}. \tag{2}$$

We have $(s-1)/s=\alpha+\beta/s+\gamma/s+o(s^{-2})$ with $\alpha=1, \beta=-1, \gamma=0$ satisfying $\gamma/\alpha-\beta^2/2>0$, so that from [5] we see that the chain of poles is in the left half-plane. However, G may possess unstable poles of small modulus and there is also a need to determine a coprime factorization for G.

Proposition 2 *Let G be given by (2). The following* $(N(s),D(s))$

$$N(s)=\frac{1}{(s^2-\frac{3}{2})+(s^2+s-\frac{3}{4})e^{-sT}}$$

and,

$$D(s)=\frac{s-1+se^{-sT}}{(s^2-\frac{3}{2})+(s^2+s-\frac{3}{4})e^{-sT}}$$

gives a coprime factorization of G over H_∞.

Proof:

Consider the quasi-polynomial $d(s)=as^2+bs+c+(ds^2+es+f)e^{-sT}$ as a potential candidate for the denominator of $N(s)$ and $D(s)$. Write

$$N(s)=\frac{1}{as^2+bs+c+(ds^2+es+f)e^{-sT}},$$

and

$$D(s)=\frac{(s+1)(s-1+se^{-sT})}{as^2+bs+c+(ds^2+es+f)e^{-sT}}.$$

We then have

$$\begin{aligned} D(s) &= \frac{(s+1)(s-1+se^{-sT})}{as^2+bs+c+(ds^2+es+f)e^{-sT}} \\ &= \frac{1}{a}\left(1+\frac{-bs-c-a}{d(s)}\right) \\ &\quad +\frac{1}{a}\left(\frac{(a-d)s^2+s(a-e)+2a-f}{d(s)}\right)e^{-sT} \end{aligned}$$

The conditions $b=0, d=a, e=a$ will ensure that $D(s)$ is in H_∞ if $d(s)$ has no unstable zeros.

The coefficients $a=1, c=-\frac{3}{2}, f=-\frac{3}{4}$ ensure that $d(s)$ has no zeros in the closed right half-plane. The rest is the same as the previous subsection. ■

IV. Conclusion

We have determined coprime factorizations for special neutral systems having chains of poles asymptotic to the imaginary axis. The method developed here is being generalized to systems of the same class, that is with one delay and a leading polynomial of degree 2. This result proves that these systems are H_∞-stabilizable. Further studies include the determination of Bezout factors.

References

[1] J. B. Garnett, *Bounded Analytic Functions*, Academic Press, 1981.

[2] C. Bonnet, A.R. Fioravanti and J. R. Partington, Stability of neutral systems with commensurate delays and poles asymptotic to the imaginary axis, SIAM J. Control Optimization, Vol. 49, N. 2, pp 498-516.

[3] J. E. Marshall and H. Górecki and K. Walton and A. Korytowski, *Time Delay Systems: Stability and Performance Criteria with Applications*, Ellis Horwood, 1992.

[4] L. H. V. Nguyen and C. Bonnet, Stabilization of some fractional neutral delay systems which possibly possess an infinite number of unstable poles, ADDS series, Springer, 2014.

[5] J. R. Partington and C. Bonnet, H_∞ and BIBO stabilization of delay systems of neutral type, Systems and Control Letters, vol 52, pp 283-288, 2004.

[6] M. C. Smith, On stabilization and the existence of coprime factorizations, IEEE Trans. Automatic Control, Vol. 34, N. 9, pp 1005-1007, 1989.

[7] Y. Yamamoto, pseudo-rational transfer functions - A survey of a class of infinite-dimensional systems, Proc. 46th IEEE CDC, pp 848-853, 2007.

[8] M. Vidyasagar, Control System Synthesis : a factorization approach, MIT Press, Cambridge, MA, 1985.