The effects of the diffusive inclusions in the bidomain model: theoretical and numerical study. Application to the rat heart.

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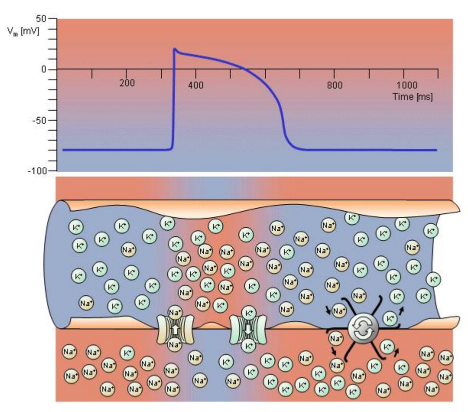
Cardiac Electrophysiology: Multiscale Modelling



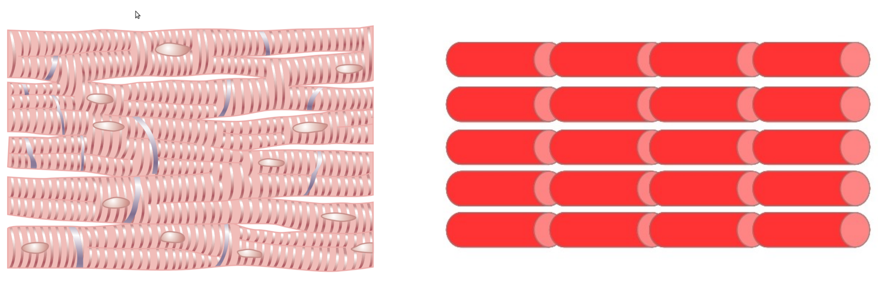
CELL SCALE – Ionic models

$$\frac{dv_m}{dt} = -I_{ion}(v_m, h) - I_{stim}$$

$$\frac{dh}{dt} = -g(v_m, h)$$

$$v_m = u_i - u_e$$

TISSUE SCALE - Bidomain model



$$\begin{aligned} \partial_t h + g(v_m,h) &= 0 \\ \partial_t v_m + I_{ion}(v_m,h) &= \nabla\cdot(\sigma^i \nabla u^i) \\ \partial_t v_m + I_{ion}(v_m,h) &= -\nabla\cdot(\sigma^e \nabla u^e) \end{aligned}$$

Main Factors

- Nonlinear ionic currents
- **MICROSTRUCTURE**
- Gap Junctions

QUESTIONS: scars, fibrosis, etc.

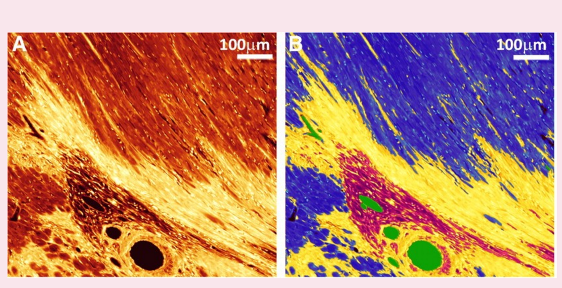


Figure : The infarct border zone.(Rutherford, 2012)

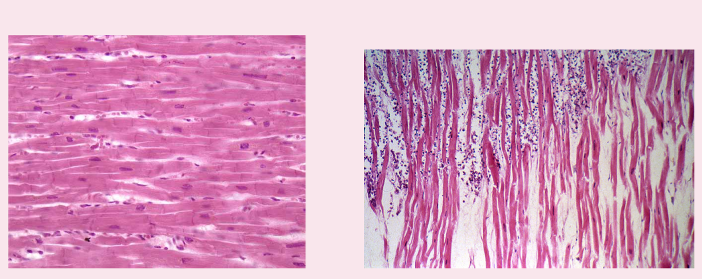
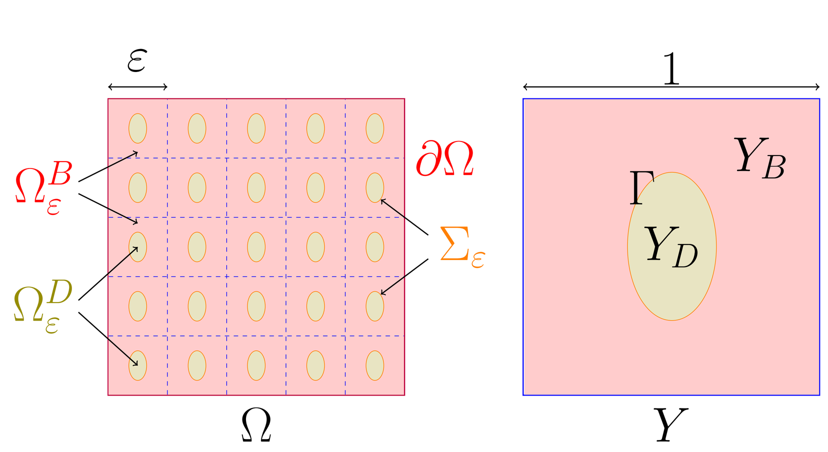


Figure : Normal vs myocardial infarction in human.

METHOD: HOMOGENISATION OF THE MESOSCALE MODEL

$$\begin{aligned} \partial_t h_\varepsilon + g(v_\varepsilon, h_\varepsilon) &= 0, && \text{in } \Omega^B_\varepsilon, \\ \partial_t v_\varepsilon + I_{ion}(v_\varepsilon, h_\varepsilon) &= \nabla\cdot(\sigma^i_\varepsilon \nabla u^i_\varepsilon), && \text{in } \Omega^B_\varepsilon, \\ \partial_t v_\varepsilon + I_{ion}(v_\varepsilon, h_\varepsilon) &= -\nabla\cdot(\sigma^e_\varepsilon \nabla u^e_\varepsilon), && \text{in } \Omega^B_\varepsilon. \end{aligned}$$

$$\nabla\cdot(\sigma^d_\varepsilon \nabla u^d_\varepsilon) = 0, \quad \text{in } \Omega^D_\varepsilon.$$



Assumptions

- **Periodic** inclusions of size ε.
- Isotropic conductivity in the inclusions.
- No cells in the inclusions - **passive conductors**.

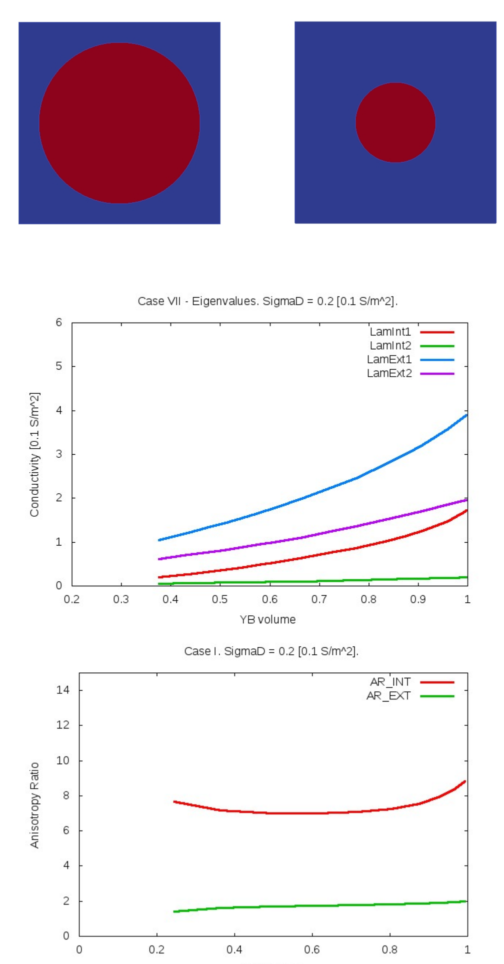
Theoretical Result: MODIFIED BIDOMAIN MODEL

- Limit macroscale model

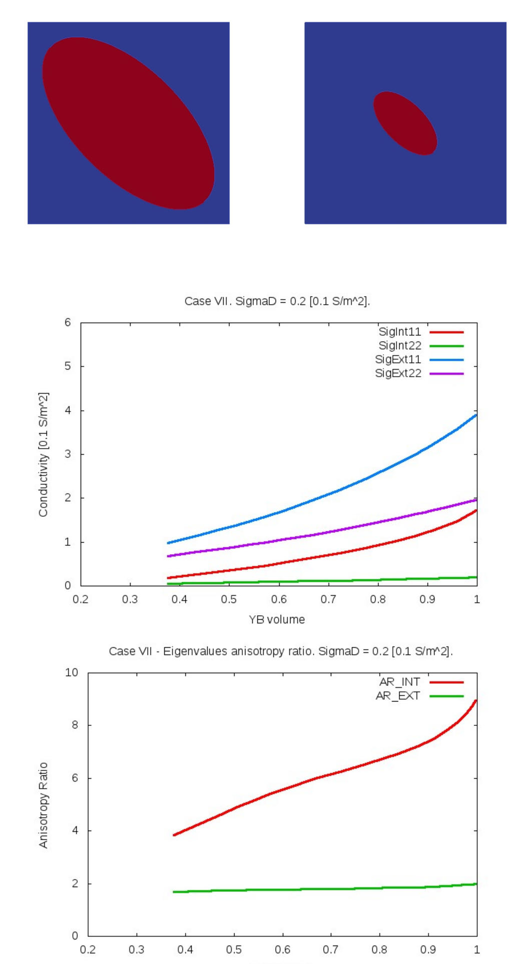
$$\begin{aligned} \partial_t h + g(v_m,h) &= 0, \\ |Y_B|(\partial_t v_m + I_{ion}(v_m,h)) &= \nabla\cdot(\tilde{\sigma}_i \nabla u^i), && \tilde{\sigma}_i = \sigma^i |Y_B| + A^i(\sigma_i, w^i), \\ |Y_B|(\partial_t v_m + I_{ion}(v_m,h)) &= -\nabla\cdot(\tilde{\sigma}_e \nabla u), && \tilde{\sigma}_e = \sigma^e |Y_B| + \sigma^d |Y_D| + A(\sigma_e, \sigma_d, w). \end{aligned}$$

- A_i, A matrices obtained by solving the cell problems on Y for w^i and w.
- **UPDATED CONDUCTIVITIES** - depend on the volume fraction AND geometry.

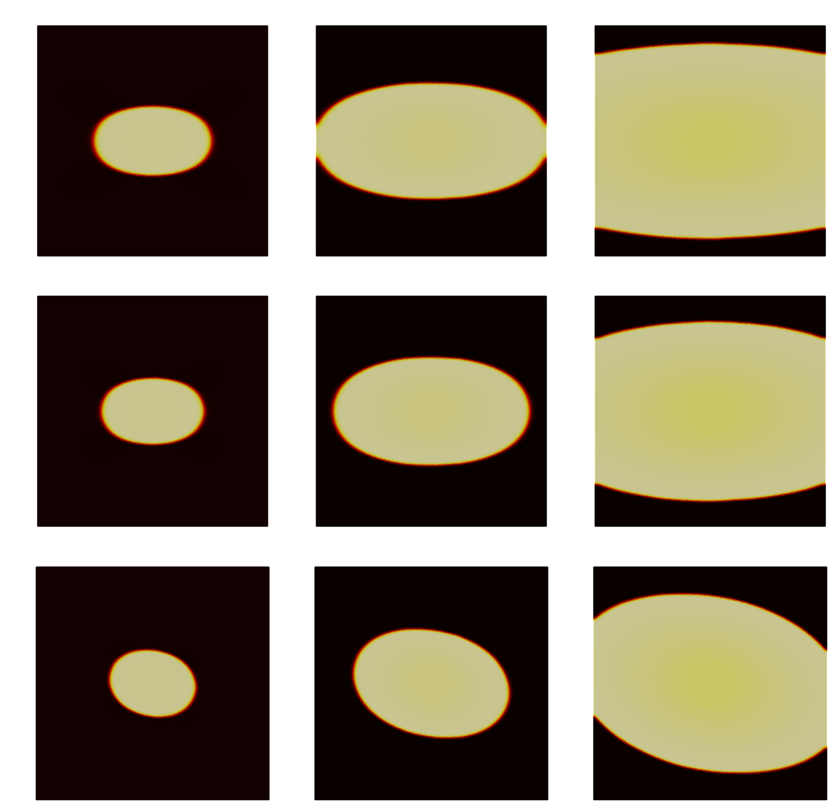
Numerical results: study of the effects in 2D



(a) Shape 1: effects on the values of the conductivity tensors and on the anisotropy ratio.

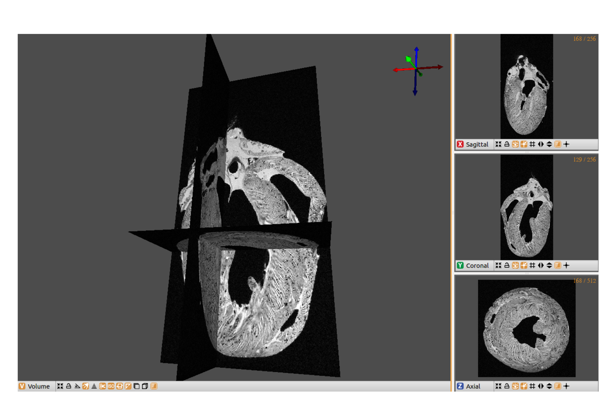


(b) Shape 2: effect on the eigenvalues of the conductivity tensors and on the anisotropy ratios.

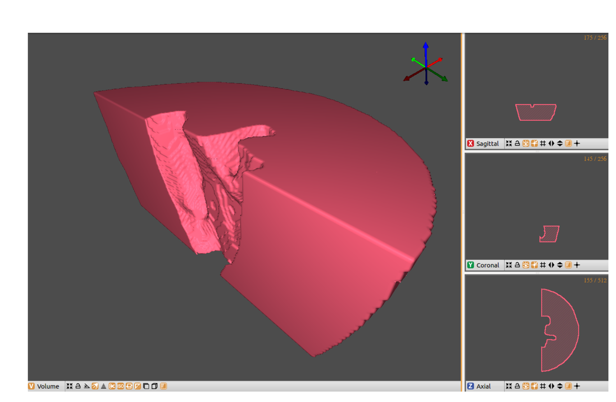


(c) Simulation of the circular wave propagation. Up – standard model, center – modified model with inclusion of the shape 1, down – modified model with inclusion of the shape 2. We observe the difference in speed and alternation in the principle direction of the propagation.

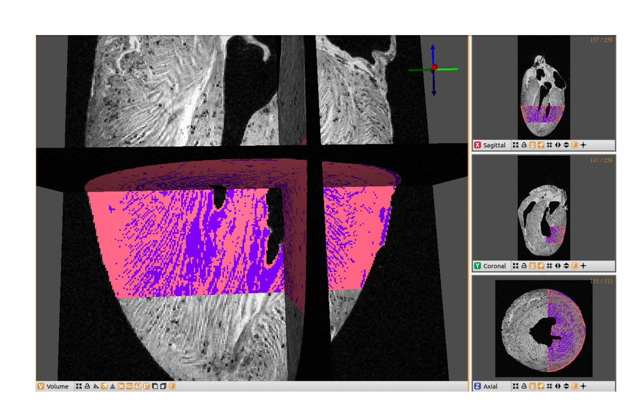
Application of the model to the real HD-MRI of the rat heart.



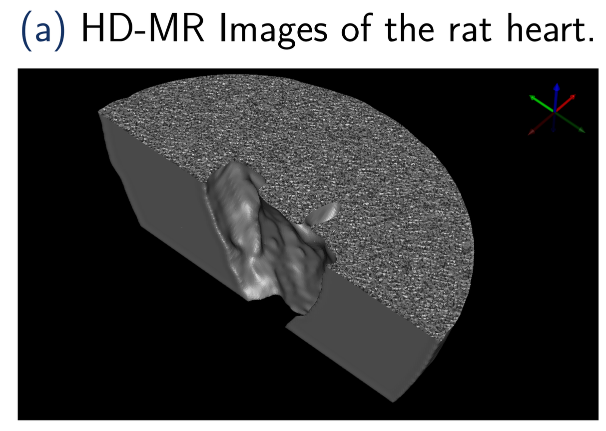
(a) HD-MR Images of the rat heart.



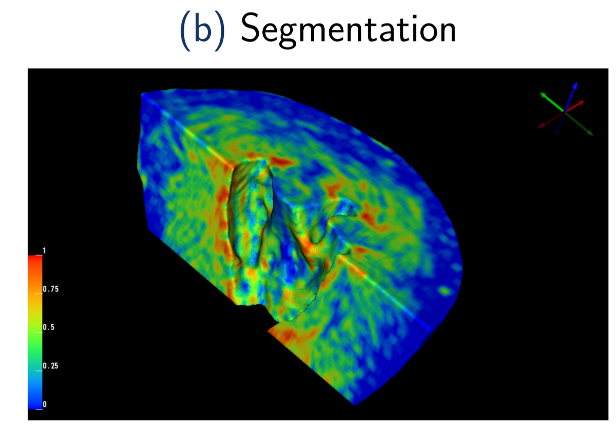
(b) Segmentation



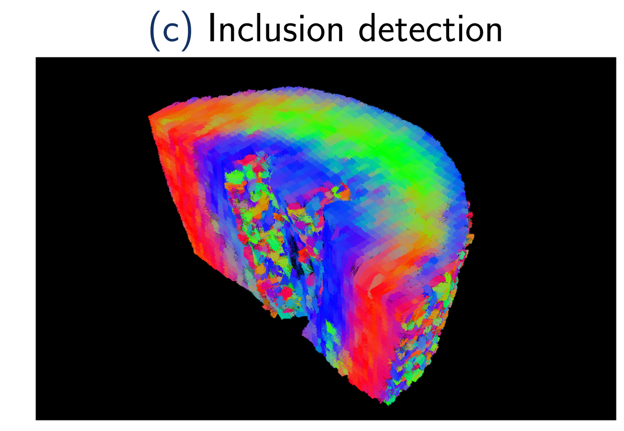
(c) Inclusion detection



(d) Mesh



(e) Volume fraction



(f) Fibers

Figure : Data from Giblert *et al.* (2012), provided by IHU-Lyric, Bordeaux. The size of the image $256 \times 256 \times 512$. Image analysis steps to obtain the required parameters for the simulation. Software used: SEG3D, SCIRun, MATLAB, Perl.

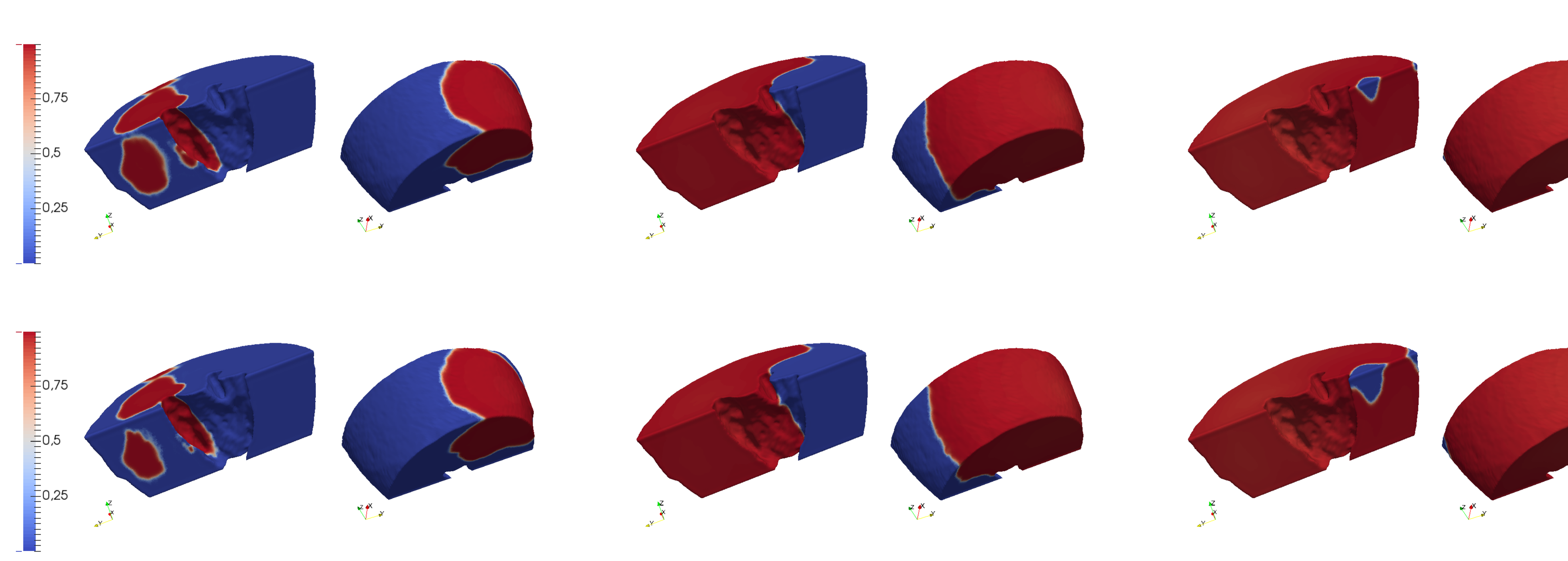


Figure : Simulation on the slab of the rat heart, up – standard model, down – modified model. Times: $20ms, 40ms, 60ms$.

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