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# Electromagnetic Optimization of Microwave Filters using Adjoint Sensitivities

Matthias Caenepeel, Fabien Seyfert, Martine Olivi, and Yves Rolain, *Fellow, IEEE*

**Abstract**—This paper introduces a novel computer-aided tuning (CAT) method for coupled-resonator microwave bandpass filters. The method is based on the estimation of the Jacobian of the function that relates the physical filter design parameters to the extracted coupling parameters. Lately commercial full-wave electromagnetic (EM) simulators provide the adjoint sensitivities of the  $S$ -parameters with respect to the geometrical parameters. This information leads to an efficient estimation of the Jacobian since it no longer requires finite difference based evaluation. The tuning method first extracts the physically implemented coupling matrix and estimates the corresponding Jacobian. Next it compares the extracted coupling matrix to the target coupling matrix (golden goal). Using the difference between the coupling matrices and the pseudo-inverse of the estimated Jacobian, a correction that brings the design parameters closer to the golden goal is obtained. This process is repeated iteratively until the correction becomes sufficiently small with respect to a user specified goal.

In the case of coupling structures with multiple solutions, the Jacobian is calculated for each admissible solution. This paper presents a criterion to identify the physical solution among the different solutions.

The CAT method is applied to the design of a cascaded triplet (CT) filter implemented in a microstrip technology. This filter is a well-known examples of a non-canonical coupling structure.

**Index Terms**—Coupling Matrix, Filter Tuning, Parameter Extraction, Jacobian Estimation, Adjoint Sensitivity

## I. INTRODUCTION

THE design of coupled-resonator microwave bandpass filters is widely based on coupling matrix theory [1]. The design starts from a rational scattering ( $S$ -) matrix that fulfills the filter specifications. The next step synthesizes an equivalent low-pass circuit (Fig. 1) that realizes these  $S$ -parameters [2]. The low-pass circuit is also represented by a coupling matrix. The coupling parameters (circuitual elements) are implemented physically by correctly dimensioning an adequate physical layout. The initial dimensioning of the filter uses empirical design curves relating the coupling parameters to the physical parameters [3]. These empirical curves neglect the more complex parasitic interactions between resonators such as loading effects of adjacent resonators. Resulting initial coupling values must hence be optimized or tuned in order to meet for the filter its specifications.

Over the last years several computer-aided tuning (CAT) techniques have been developed to optimize the physical design parameters in various ways [4]–[8]. These tuning procedures extract a coupling matrix starting from simulated or measured scattering parameters of the initial design and compare it to the target coupling matrix (golden goal). The

corrected physical parameters are then used to tune the filter. As this extracted coupling matrix is an approximation, extraction errors will inevitably be present. Even when they are small, their presence can lead to incoherent corrections, possibly destroying the tuning process. This problem becomes central when the coupling topology admits several solutions: in this case the extracted coupling matrix and the implemented coupling matrix may differ totally [9]. Ensuring that the extraction process yields the physically implemented coupling matrix when starting from the simulated  $S$ -parameters is therefore crucial.

The extraction techniques can roughly be divided in two categories. The first one optimizes the electrical parameters of the low-pass equivalent circuit to fit the measured response [4], [10]. These methods yield only the solution closest to the initial guess by construction. Unfortunately, this solution does not necessarily correspond to the physically implemented one. (The solution strongly depends on the initial values for the coupling parameters specified by the user.) The second set [5]–[8] first identifies the characteristic polynomials or a rational form representing the scattering parameters. A coupling matrix is then obtained for the identified model. Both classes of methods yield results with a low residual error, but suffer from the same problem: they only handle circuits with a canonical coupling topology. The coupling topology represents the way that the resonators are coupled to each other. Certain filters such as cascaded triplet (CT), cascaded quartet (CQ) and extended-box filters, have a non-canonical coupling topology [1]. For such filters several circuits that share the same coupling structure, but have different values for the coupling parameters, realize the same response. The extraction method in this paper systematically derives all possible circuitual realizations corresponding to the implemented coupling topology. The main difference with the extraction method used in [9], [11] is that here a single simulation suffices to identify the physically implemented coupling matrix.

The method estimates the Jacobian matrix of the functional model that relates the circuit parameters to the physical parameters. It thereto combines the adjoint sensitivities of the  $S$ -parameters with respect to the circuit parameters on the one hand and the adjoint sensitivities of the  $S$ -parameters with respect to the geometrical parameters on the other hand. Formal expressions exist to calculate the adjoint sensitivities of the  $S$ -parameters with respect to the circuit parameters using only the values of the extracted coupling parameters. Lately commercial EM-simulators, such as CST Microwave Studio [12], provide the adjoint sensitivities of the  $S$ -parameters with respect to the geometrical parameters. One simulation

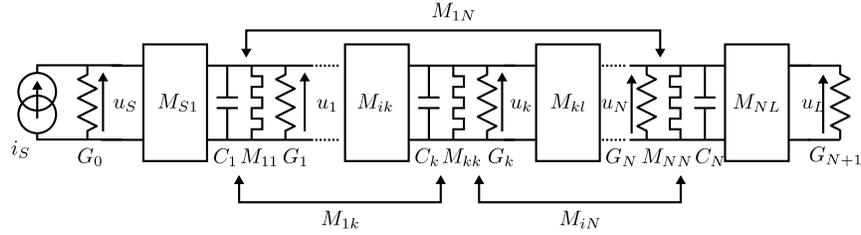


Fig. 1: The equivalent network models a filter consisting of  $N$  multi-coupled resonators.

hence suffices to estimate the Jacobian matrix. This drastically reduces the number of required EM-simulations, by avoidance of finite difference approximations. It also allows to identify the physically implemented coupling matrix in the case of non-canonical topologies. One simulation suffices to estimate the Jacobian for each admissible solution. To use a Jacobian to tune the physical parameters in the case of a coupling topology with multiple solutions, it is necessary to identify the coupling matrix corresponding to the physically implemented couplings. The use of variational information based on adjoint sensitivities computations is here of great help. The idea is that in a physical design, a coupling parameter is mainly affected by a specific set of design parameters. Thus the partial derivatives of the coupling parameter with respect to these parameters are larger than the partial derivatives with respect to the other design parameters having a second order effect at most. This assumption allows to predict which elements of the Jacobian are dominant in the case of a physical relation. Selecting the correct coupling matrix and its associated Jacobian boils down to finding the Jacobian for which those elements are dominant.

The main novelty of this paper is to provide a method to estimate the Jacobian based on adjoint sensitivities and to determine a criterion to select the physically implemented solution for topologies with multiple solutions. Section II describes the equivalent low-pass network to model the filter behavior and derives formal expressions for the adjoint sensitivity of the  $S$ -parameters with respect to the circuit parameters. Section III details the coupling matrix extraction method. Section IV combines the adjoint sensitivities to estimate the Jacobian matrix. Section V presents the optimization method as is used to tune the filter response. Finally section VI applies the method to a cascaded trisection (CT) filter implemented in a microstrip technology. This filter is a typical example of a non-canonical coupling structures.

## II. EQUIVALENT LOW-PASS CIRCUIT

The equivalent network used to model the filters behavior consists of  $N$  coupled low-pass resonators (Fig. 1). The low-pass resonator contains a shunt capacitor  $C_i$ , a frequency invariant reactance (FIR)  $M_{ii}$  [1] and an admittance  $G_i$ . The FIR  $M_{ii}$  models a frequency shift of the individual resonators with respect to the center frequency  $f_c$  of the filter and the admittance  $G_i$  models the loss. The capacitors as well as the input and output loads are all normalized to unity. The resonators are coupled through frequency invariant symmetric couplings  $M_{ik} = M_{ki}$ . The resonators are also coupled to the source (input) and the load (output) by a frequency invariant

coupling labeled  $M_{S_i}$  and  $M_{L_i}$  respectively. Note that the same network is used in [1], [13]. Here the losses are also taken into account and are modeled by  $G_i$ . Solving the nodal equations of the network yields following solution:

$$[\mathbf{G} + j\mathbf{M} + j\omega\tilde{\mathbf{I}}][\mathbf{u}] = [\mathbf{A}][\mathbf{u}] = [\mathbf{i}] \quad (1)$$

where

- $\mathbf{G} \in \mathbb{R}^{(N+2) \times (N+2)}$  is a diagonal matrix with  $[\mathbf{G}]_{ii} = G_i$  and  $[\mathbf{G}]_{11} = [\mathbf{G}]_{N+2, N+2} = 1$ .
- $\tilde{\mathbf{I}} \in \mathbb{R}^{(N+2) \times (N+2)}$  is an identity matrix with elements  $[\tilde{\mathbf{I}}]_{11} = [\tilde{\mathbf{I}}]_{N+2, N+2} = 0$ .
- $\mathbf{M} \in \mathbb{R}^{(N+2) \times (N+2)}$  is the coupling matrix containing the couplings and FIRs.
- $\omega$  is the normalized low-pass frequency.
- $\mathbf{i} = [i_S, \dots, 0]^t$  is the excitation vector of currents of size  $(N+2) \times 1$
- $\mathbf{u} = [u_S, u_1, \dots, u_N, u_L]^t$  is the voltage vector of size  $(N+2) \times 1$
- $\mathbf{A} = [\mathbf{G} + j\mathbf{M} + j\omega\tilde{\mathbf{I}}] \in \mathbb{C}^{(N+2) \times (N+2)}$
- $j^2 = -1$

where  $[\mathbf{A}]_{kl}$  denotes the element at position  $(k, l)$  of the matrix  $\mathbf{A}$ . Writing the  $S$ -parameters as a function of  $\mathbf{A}$  results in [13]:

$$\begin{aligned} S_{11} &= 2[\mathbf{A}^{-1}]_{11} - 1 \\ S_{21} &= 2[\mathbf{A}^{-1}]_{N+2, 1} \\ S_{22} &= 2[\mathbf{A}^{-1}]_{N+2, N+2} - 1 \end{aligned} \quad (2)$$

We now derive a formal expression for  $\frac{\partial S_{ij}}{\partial M_{k,l}}(\omega)$  which is the sensitivity of a scattering parameter  $S_{ij}$  with respect to a coupling parameter  $M_{k,l}$  at a certain frequency  $\omega$ . In section IV these sensitivities are used to estimate the Jacobian of the functional relation between the physical filter parameters and the circuit parameters. The sensitivities can be expressed as a function of  $\mathbf{A}^{-1}$ . Remember that the partial derivative of the inverse of a matrix  $\mathbf{A}$  can be written as:

$$\frac{\partial \mathbf{A}^{-1}}{\partial x} = -\mathbf{A}^{-1} \frac{\partial \mathbf{A}}{\partial x} \mathbf{A}^{-1} \quad (3)$$

Applying (3) to the expressions given in (2) yields:

For  $l \neq k$  :

$$\begin{aligned}\frac{\partial S_{11}}{\partial M_{l,k}} &= -4j([\mathbf{A}^{-1}]_{1,l}[\mathbf{A}^{-1}]_{1,k}) \\ \frac{\partial S_{21}}{\partial M_{l,k}} &= -2j([\mathbf{A}^{-1}]_{N+2,l}[\mathbf{A}^{-1}]_{k,1} + [\mathbf{A}^{-1}]_{1,k}[\mathbf{A}^{-1}]_{l,N+2}) \\ \frac{\partial S_{22}}{\partial M_{l,k}} &= -4j([\mathbf{A}^{-1}]_{N+2,l}[\mathbf{A}^{-1}]_{N+2,k})\end{aligned}$$

For  $l = k$  :

$$\begin{aligned}\frac{\partial S_{11}}{\partial M_{l,l}} &= -2j[\mathbf{A}^{-1}]_{1,l}^2 \\ \frac{\partial S_{21}}{\partial M_{l,l}} &= -2j[\mathbf{A}^{-1}]_{1,l}[\mathbf{A}^{-1}]_{l,N+2} \\ \frac{\partial S_{22}}{\partial M_{l,l}} &= -2j[\mathbf{A}^{-1}]_{N+2,l}^2\end{aligned}\quad (4)$$

Remark that these expressions are equivalent to those found in [14] where the dual circuit based on impedance inverters and series inductors is used.

### III. PARAMETER EXTRACTION

The parameter extraction method synthesizes a canonical coupling matrix starting from a rational approximation of the  $S$ -parameters. This coupling matrix is then reconfigured to match the implemented coupling topology. In the case of multiple solutions, all admissible solutions are determined.

#### A. Rational Approximation

The extraction method first estimates a stable common-denominator rational matrix of prescribed McMillan degree  $N$  for the simulated  $S$ -parameters of the filter. The rational approximation of the data is done using the software PRESTO-HF [15]. This software is based on analytic and rational  $H^2$  approximation [16]. The scattering data is transformed to the low-pass domain [1]. The software identifies the following mathematical model for the low-pass domain data:

$$\begin{bmatrix} e^{j\frac{\alpha}{2}\omega} & 0 \\ 0 & e^{j\frac{\beta}{2}\omega} \end{bmatrix} \mathbf{R}(j\omega) \begin{bmatrix} e^{j\frac{\alpha}{2}\omega} & 0 \\ 0 & e^{j\frac{\beta}{2}\omega} \end{bmatrix} \quad (5)$$

where  $\mathbf{R}$  is the rational matrix and  $\alpha$  and  $\beta$  are the delays introduced by the access lines that are assumed to be ideal lines. The McMillan degree of the matrix  $\mathbf{R}$  corresponds to the number of resonators  $N$  that are present in the filter.

#### B. Coupling Matrix Synthesis

A canonical arrow form coupling matrix  $\mathbf{M}$  is synthesized starting from the rational matrix  $\mathbf{R}$ . The coupling structure imposed by the arrow form does not necessarily correspond to the implemented coupling structure. Therefore  $\mathbf{M}$  must be reconfigured to match the actual coupling structure of the filter by performing a series of Givens rotations. In the case of non-canonical circuits several solutions exist for this reduction problem. Using a Groebner basis and homotopy based

approach is possible to determine all admissible coupling matrices. This requires solving a related non-linear system of equations [17]. This technique has been implemented in the Matlab toolbox DEDALE-HF [18]. Whenever multiple solutions exist, all admissible solutions are determined. In the next section we show how the Jacobian is estimated for each of these solutions and give a criterion to determine which one corresponds to the physical solution.

### IV. ESTIMATION OF THE JACOBIAN MATRIX $\mathbf{J}$

The tuning procedure is based on the relation of the geometrical design parameters to the extracted circuit parameters describing the physical implementation of the filter. Assume that there are  $N_g$  geometrical parameters  $\mathbf{g}$  and  $N_c$  circuitual parameters  $\mathbf{m}$ . The coupling matrix extraction process formally relates the geometrical design parameters with the extracted electrical parameters. We will first suppose that this relation is single-valued, that is that we are dealing with a topology admitting a single solution to the synthesis process, and we denote it by:

$$f(\mathbf{g}) = \mathbf{m} \quad (6)$$

where  $\mathbf{g}$  is a vector of size  $N_g \times 1$  containing the geometrical parameters and  $\mathbf{m}$  is a vector of size  $N_c \times 1$  containing the extracted circuitual parameters. The Jacobian matrix  $\mathbf{J}$  of  $f$  is estimated by combining the adjoint sensitivities of the  $S$ -parameters.

Commercial EM-simulators, such as Computer Simulation Technology (CST) [19], provide the adjoint sensitivities of the  $S$ -parameters with respect to the geometrical parameters as a function of the frequency. This information together with the adjoint sensitivities of the  $S$ -parameters with respect to the circuitual parameters (calculated using (4)) is used to estimate the Jacobian of  $f$ .

Using the chain rule of differential calculus, the sensitivity of  $S_{ij}$  with respect to a geometrical parameter  $g_s$  can be written as

$$\frac{\partial S_{ij}}{\partial g_s}(\omega) = \sum_{k=1}^{N_c} \frac{\partial S_{ij}}{\partial m_k}(\omega) \frac{\partial m_k}{\partial g_s} \quad (7)$$

where  $\frac{\partial S_{ij}}{\partial m_k}$  is the adjoint sensitivity of  $S_{ij}$  with respect to the  $k^{th}$  circuitual parameter of  $\mathbf{m}$  and  $\frac{\partial m_k}{\partial g_s}$  is element  $[\mathbf{J}]_{k,s}$  of the Jacobian  $\mathbf{J}$ . From formulas (4) it is possible to calculate the sensitivity of  $S_{ij}$  with respect to each extracted circuit parameter at each simulated frequency. The simulated frequencies which are transformed to the normalized low-pass domain and are ordered in a vector  $\Omega = [\omega_1, \dots, \omega_{N_f}]^t$ , where  $N_f$  is the number of frequencies. The column wise ordering these sensitivities yields a matrix of size  $4N_f \times N_c$  :

$$\mathbf{S}_{dM} = \begin{bmatrix} \frac{\partial S_{11}}{\partial m_1}(\omega_1) & \dots & \frac{\partial S_{11}}{\partial m_k}(\omega_1) & \dots & \frac{\partial S_{11}}{\partial m_{n_c}}(\omega_1) \\ \vdots & & \vdots & & \vdots \\ \frac{\partial S_{11}}{\partial m_1}(\omega_{n_F}) & \dots & \frac{\partial S_{11}}{\partial m_k}(\omega_{n_F}) & \dots & \frac{\partial S_{11}}{\partial m_{n_c}}(\omega_{n_F}) \\ \frac{\partial S_{12}}{\partial m_1}(\omega_1) & \dots & \frac{\partial S_{12}}{\partial m_k}(\omega_1) & \dots & \frac{\partial S_{12}}{\partial m_{n_c}}(\omega_1) \\ \vdots & & \vdots & & \vdots \\ \frac{\partial S_{12}}{\partial m_1}(\omega_{n_F}) & \dots & \frac{\partial S_{12}}{\partial m_k}(\omega_{n_F}) & \dots & \frac{\partial S_{12}}{\partial m_{n_c}}(\omega_{n_F}) \\ \frac{\partial S_{21}}{\partial m_1}(\omega_1) & \dots & \frac{\partial S_{21}}{\partial m_k}(\omega_1) & \dots & \frac{\partial S_{21}}{\partial m_{n_c}}(\omega_1) \\ \vdots & & \vdots & & \vdots \\ \frac{\partial S_{21}}{\partial m_1}(\omega_{n_F}) & \dots & \frac{\partial S_{21}}{\partial m_k}(\omega_{n_F}) & \dots & \frac{\partial S_{21}}{\partial m_{n_c}}(\omega_{n_F}) \\ \frac{\partial S_{22}}{\partial m_1}(\omega_1) & \dots & \frac{\partial S_{22}}{\partial m_k}(\omega_1) & \dots & \frac{\partial S_{22}}{\partial m_{n_c}}(\omega_1) \\ \vdots & & \vdots & & \vdots \\ \frac{\partial S_{22}}{\partial m_1}(\omega_{n_F}) & \dots & \frac{\partial S_{22}}{\partial m_k}(\omega_{n_F}) & \dots & \frac{\partial S_{22}}{\partial m_{n_c}}(\omega_{n_F}) \end{bmatrix} \quad (8)$$

Similarly, the sensitivity of  $S_{ij}$  with respect to the geometrical parameters provided by the simulator are ordered columnwise in a matrix of size  $4N_f \times N_g$

$$\mathbf{S}_{dG} = \begin{bmatrix} \frac{\partial S_{11}}{\partial g_1}(\omega_1) & \dots & \frac{\partial S_{11}}{\partial g_k}(\omega_1) & \dots & \frac{\partial S_{11}}{\partial g_{N_g}}(\omega_1) \\ \vdots & & \vdots & & \vdots \\ \frac{\partial S_{11}}{\partial g_1}(\omega_{n_F}) & \dots & \frac{\partial S_{11}}{\partial g_k}(\omega_{n_F}) & \dots & \frac{\partial S_{11}}{\partial g_{N_g}}(\omega_{n_F}) \\ \frac{\partial S_{12}}{\partial g_1}(\omega_1) & \dots & \frac{\partial S_{12}}{\partial g_k}(\omega_1) & \dots & \frac{\partial S_{12}}{\partial g_{N_g}}(\omega_1) \\ \vdots & & \vdots & & \vdots \\ \frac{\partial S_{12}}{\partial g_1}(\omega_{n_F}) & \dots & \frac{\partial S_{12}}{\partial g_k}(\omega_{n_F}) & \dots & \frac{\partial S_{12}}{\partial g_{N_g}}(\omega_{n_F}) \\ \frac{\partial S_{21}}{\partial g_1}(\omega_1) & \dots & \frac{\partial S_{21}}{\partial g_k}(\omega_1) & \dots & \frac{\partial S_{21}}{\partial g_{N_g}}(\omega_1) \\ \vdots & & \vdots & & \vdots \\ \frac{\partial S_{21}}{\partial g_1}(\omega_{n_F}) & \dots & \frac{\partial S_{21}}{\partial g_k}(\omega_{n_F}) & \dots & \frac{\partial S_{21}}{\partial g_{N_g}}(\omega_{n_F}) \\ \frac{\partial S_{22}}{\partial g_1}(\omega_1) & \dots & \frac{\partial S_{22}}{\partial g_k}(\omega_1) & \dots & \frac{\partial S_{22}}{\partial g_{N_g}}(\omega_1) \\ \vdots & & \vdots & & \vdots \\ \frac{\partial S_{22}}{\partial g_1}(\omega_{n_F}) & \dots & \frac{\partial S_{22}}{\partial g_k}(\omega_{n_F}) & \dots & \frac{\partial S_{22}}{\partial g_{N_g}}(\omega_{n_F}) \end{bmatrix} \quad (9)$$

$\mathbf{S}_{dG}$  can be written as a function of  $\mathbf{S}_{dM}$  using the Jacobian  $\mathbf{J} = [\frac{\partial m_k}{\partial g_s}]$ :

$$\mathbf{S}_{dG} = \mathbf{S}_{dM} \mathbf{J} \quad (10)$$

This overdetermined equation can be solved for the Jacobian  $\mathbf{J}$  in the least squares sense:

$$\mathbf{J} = (\mathbf{S}_{dM}^* \mathbf{S}_{dM})^{-1} \mathbf{S}_{dM}^* \mathbf{S}_{dG} \quad (11)$$

where  $\mathbf{S}_{dM}^*$  is the conjugate transpose of  $\mathbf{S}_{dM}$ . Note that we have implicitly supposed that the coupling topologies we are dealing with are non-redundant [17], meaning that locally no variation of a coupling can be compensated by the re-tuning of others. This property can be shown to mathematically imply the invertibility of the matrix  $\mathbf{S}_{dM}^* \mathbf{S}_{dM}$ , provided the number  $N_f$  of sampled frequencies is big enough.

*Determination of the physical coupling matrix:*

In the case of non-canonical coupling topologies, several equivalent circuits having the same coupling structure but different coupling values exist. Assume that the number of equivalent solutions is  $N_s$ . This means that there are  $N_s$  equivalent functions that relate the design parameters  $\mathbf{g}$  to the coupling parameters  $\mathbf{m}$ . Among them only one function  $f_p$  relates  $\mathbf{g}$  to the physically implemented coupling parameters  $\mathbf{m}_p$ . To identify  $f_p$  among all others, we assume that in a physical design a coupling is mainly determined by a physical parameter and that it is possible to determine which physical parameters influence the couplings the most. This assumption is also implicitly made during the initial dimensioning of the filter since also then second order effects, such as loading of the resonators, are neglected. One can therefore predict in advance which partial derivatives  $\frac{\partial m_k}{\partial g_s}$  are dominant for the physical design. These derivatives correspond to the dominant elements of the Jacobian matrix  $\mathbf{J}_p$  of  $f_p$ . Let  $V$  denote the set of indices corresponding to these elements. For each estimated  $\mathbf{J}$ , the relative influence of these predicted elements is calculated:

$$c = \frac{\sum_{k,l \in V} [\mathbf{J}]_{l,k}^2}{\sum_{m=1}^{N_c} \sum_{n=1}^{N_g} [\mathbf{J}]_{m,n}^2} \quad (12)$$

In the set of equivalent Jacobian matrices one selects the matrix for which  $c$  is maximal. This matrix corresponds to the solution for which the design parameters have the expected main effects on the coupling parameters. This solution can then safely be assumed to be the physical solution.

## V. OPTIMIZATION METHOD

### A. Correction of the Design Parameters

The optimization method updates the design parameters  $\mathbf{g}$  such that the corresponding circuital parameters  $f(\mathbf{g}) = \mathbf{m}$  coincide maximally with the target coupling parameters  $\tilde{\mathbf{m}}$  of the golden goal. Evaluating the function  $f$  for the initial design parameters  $\mathbf{g}_0$  yields the difference between the circuital parameters of the initial design  $\mathbf{m}_0$  and the target parameters  $\tilde{\mathbf{m}}$  which allows to evaluate the design error:

$$\Delta \mathbf{m}_0 = \tilde{\mathbf{m}} - f(\mathbf{g}_0) \quad (13)$$

The corrections of  $\mathbf{g}_0$  necessary to improve the filter response are therefore readily obtained as:

$$\Delta \mathbf{g}_0 = \mathbf{g}_1 - \mathbf{g}_0 = f^{-1}(\mathbf{m}_0 + \Delta \mathbf{m}_0) - \mathbf{g}_0 \quad (14)$$

where  $\mathbf{g}_1$  are the design parameters of the improved filter. A linear approximation of  $f$  using the estimated Jacobian  $\mathbf{J}_0$  in  $\mathbf{g}_0$  yields:

$$\mathbf{m}_1 = f(\mathbf{g}_1) \approx \mathbf{J}_0 \Delta \mathbf{g}_0 + \mathbf{g}_0 \quad (15)$$

Expression (15) approximates the necessary correction of the design parameters using the pseudo-inverse of the Jacobian  $\mathbf{J}_0^{-1}$ :

$$\Delta \mathbf{g}_0 \approx \mathbf{J}_0^{-1} \Delta \mathbf{m}_0 \quad (16)$$

A simulation of the updated filters response yields the coupling parameters  $\mathbf{m}_1$  and the Jacobian  $\mathbf{J}_1$  in the same way as above. Repeating the process above then allows to refine the correction until the necessary corrections become sufficiently small:

$$\|\Delta \mathbf{g}_k\|_\infty = \max_l [\Delta \mathbf{g}_k]_l < \delta \quad (17)$$

where  $\delta$  is a user-defined value. Since it has no use to model the filter for geometrical parameters more precisely than the fabrication tolerance, a valid choice for  $\delta$  is readily found to be the accuracy of the fabrication process. For all of the considered examples, the optimization method converged after a number of iterations smaller than the number of design parameters to be tuned.

### B. Optimization Algorithm

- 1) Generation of the ideal rational scattering matrix that fulfills the specifications.
- 2) Synthesis of the target coupling matrix  $\tilde{\mathbf{M}}$  (which are ordered in a vector  $\tilde{\mathbf{m}}$ ). In the case of multiple solutions, one solution is chosen as the target matrix.
- 3) Computation of the initial design parameters  $\mathbf{g}_k$  of the filters physical implementation, where  $k = 0$ .
- 4) Full-wave EM-simulation of the filters  $S$ -parameters in the frequency band of interest and the adjoint sensitivities of the  $S$ -parameters with respect to the design parameters  $\mathbf{g}_k$ .
- 5) Extraction of the coupling parameters (Section III):
  - a) Rational approximation of the simulated  $S$ -parameters.
  - b) Coupling matrix synthesis.
  - c) Determination of all the possible solutions corresponding to the implemented coupling topology.
- 6) Estimation of the Jacobian for each solution and determination of the physically implemented coupling matrix using expression (12) (Section IV).
- 7) Computation of the error of the circuit parameters  $\Delta \mathbf{m}_k$ .
- 8) Estimation of the correction for the design parameters  $\Delta \mathbf{g}_k \approx \mathbf{J}_k^{-1} \Delta \mathbf{m}_k$ .
- 9) Termination of the optimization process if  $\|\Delta \mathbf{g}_k\|_\infty < \delta$ .
- 10) Update of the design parameters:  $\mathbf{g}_{k+1} = \mathbf{g}_k + \Delta \mathbf{g}_k$  and return to step 4.

## VI. EXAMPLES

### A. Cascaded Triplet (CT) Filter

The optimization method is applied to the design of a 6<sup>th</sup> order CT filter. The filter is designed to have center frequency  $f_c = 1.2$  GHz, a fractional bandwidth  $FBW = 0.04$ , a minimum return loss  $RL$  of 20 dB and 2 finite transmission zeros (TZs) at  $\omega = 1.05$  and  $\omega = 2.1$ . The target coupling matrix is synthesized using Dededale-HF [18]. There are 2 solutions to the reduction problem of which one is chosen (arbitrarily). In the chosen solution, the first triplet realizes the TZ at  $\omega_1 = 2.1$  and the second one the TZ at  $\omega_2 = 1.05$ . The filter is implemented on a RO4360 substrate with a thickness of 1.016 mm and  $\epsilon_r = 6.15$ . To implement the  $\frac{\lambda}{2}$ -resonators we use a variation of the square open-loop (SOLR) resonator,

which is not square but rectangular. Figure 2 shows the top-view of the layout of the CT filter. Table I contains the initial dimensions of the filter. There are 15 design parameters which are the spacing  $d_{kl}$  between the resonators  $k$  and  $l$  and the distance between the ends  $g_i$  of the resonator  $i$ . Note that the positions of the feeding lines  $t_i$  are also design parameters. They are however not included in the optimization method, since CST is not capable to calculate the adjoint sensitivity with respect to a parameter that influences the position of a port. Therefore these parameters have to be tuned manually.

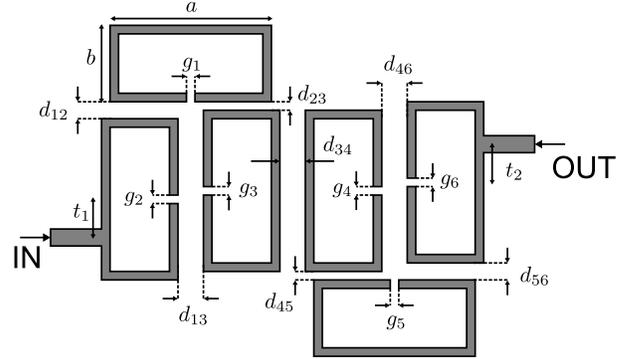


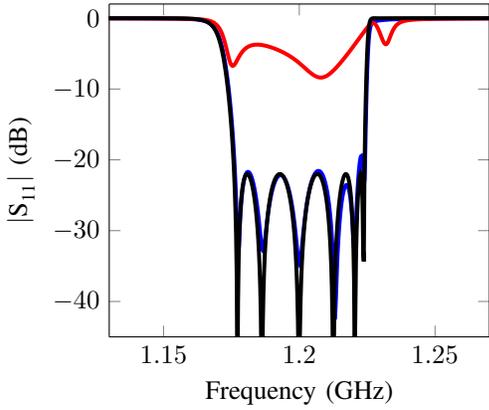
Fig. 2: Top-view of the layout of a sixth order cascaded triplet filter.

Physical parameter	Initial Value (mm)	Final Value (mm)
$a$	16.7	16.7
$b$	16.5	16.5
$w$	1	1
$t_1$	4.6	4.6
$t_2$	4.6	4.6
$d_{12}$	0.6	0.74
$d_{23}$	1.4	1.16
$d_{13}$	2.1	1.9
$d_{34}$	2.2	2.74
$d_{45}$	1.5	1.71
$d_{56}$	1.4	1.05
$d_{46}$	1.3	1.37
$g_1$	0.75	0.61
$g_2$	1.1	0.8
$g_3$	0.65	0.47
$g_4$	0.6	0.48
$g_5$	1.3	1.22
$g_6$	0.75	0.55

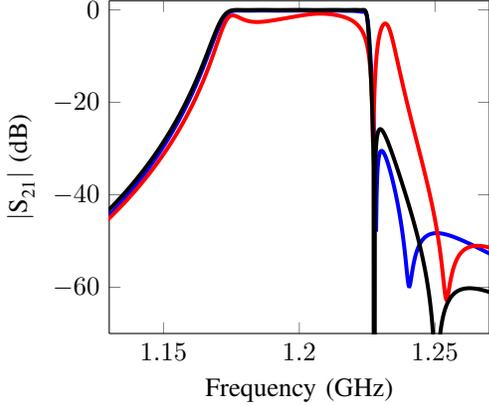
TABLE I: Initial and final values for the physical design parameters of the CT filter.

Figure 3 shows the  $S$ -parameters of the initial design. Note that the initial design is far away from the target design. We did not use design curves to obtain initial values for the mixed couplings  $M_{12}$ ,  $M_{23}$ ,  $M_{45}$  and  $M_{56}$  but we have chosen values within the range of values found for the other spacings using visual inspection.

The coupling matrices are extracted using the method described in Section III. There are 2 solutions to the reconfiguration problem and thus there are 2 possible Jacobian matrices too. The parameters are ordered such that dominant elements should be positioned on the main diagonal of the matrix. When we calculate the relative influence of these dominant elements over the others for the first iteration, we find the following values for  $c_p$  as defined in (12):



(a)  $|S_{11}|$  for the initial design (—), the final loss-less design (—) and the ideal response (—)



(b)  $|S_{21}|$  for the initial design (—), the final loss-less design (—) and the ideal response (—)

Fig. 3:  $|S_{11}|$  and  $|S_{21}|$  for the initial design, the final loss-less design and the ideal response of the CT filter.

$$c_p = [0.9473 \quad 0.2216] \quad (18)$$

This indicates that the first solution corresponds to the physically implemented coupling matrix. Table II contains the coupling parameters associated to the initial design parameters.

The value of  $\delta_{corr}$  was set to 0.025 mm and after 4 iterations (5 EM-simulations) we found a  $\|\Delta g_k\|_\infty = 0.024$  mm. On average, an EM-simulation takes 1 hour 5 min (with adjoint sensitivities included). Note that the adaptive meshing is included in this time and this already takes 40 minutes. An EM-simulation without adjoint sensitivities takes 45 minutes on average, indicating that the extra time that is needed to calculate the adjoint sensitivities is acceptable. Figure 4 shows the evolution of the inter-resonator couplings and of the self-coupling during the tuning process. The figure shows that after 2 iterations, the coupling parameters are already very close to the target values. The last 2 iterations can be seen as fine tuning. Note that for clarity of the figure, we have not taken into account the sign of the inter-resonator couplings. Figure 5a shows the evolution of the 2-norm and the  $\infty$ -norm of the design error  $\Delta m$ . Note that  $\infty$ -norm is the smallest after iteration 2 (at simulation 3), meaning that the

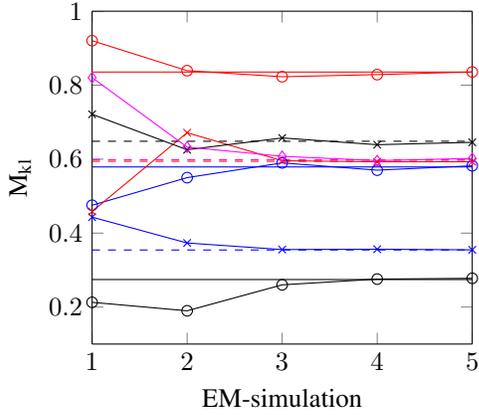
Parameter	Initial	Final	Target
$M_{S1}$	1.0603	1.0495	1.0422
$M_{L8}$	1.0480	1.0404	1.0422
$M_{12}$	0.9203	0.8357	0.8355
$M_{23}$	0.4754	0.5820	0.5789
$M_{13}$	0.2128	0.2777	0.2745
$M_{34}$	-0.8204	-0.6019	-0.5986
$M_{45}$	0.4432	0.3547	0.3544
$M_{56}$	0.4585	0.5930	0.5942
$M_{46}$	0.7213	0.6455	0.6483
$M_{11}$	-0.0835	0.0230	0.0394
$M_{22}$	-0.7540	-0.3511	-0.3377
$M_{33}$	-0.1244	0.1052	0.1138
$M_{44}$	-0.0356	0.1193	0.1349
$M_{55}$	-0.8585	-0.8270	-0.8252
$M_{66}$	-0.3083	0.0326	0.0394
$M_{SL}$	0	-0.0002	0
$M_{S6}$	0	-0.0001	0
$M_{L1}$	0	-0.0001	0
$M_{16}$	-0.0012	-0.0005	0
$M_{26}$	0.0158	0.0140	0
$M_{36}$	0.0037	0.0047	0

TABLE II: Extracted and target coupling parameters of the SOLR CT filter.

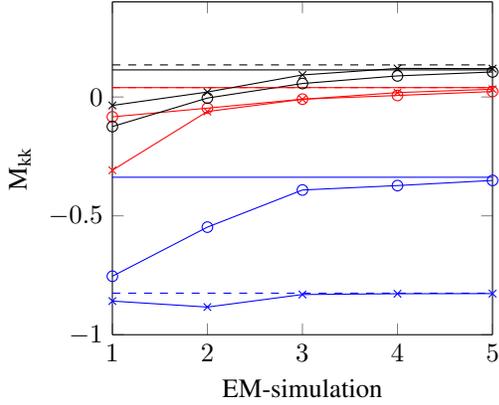
maximum difference with respect to the target value is the lowest at that iteration. However the 2-norm of the vector  $\Delta m$  is clearly minimal after the 4<sup>th</sup> iteration. Figure 5b shows the evolution of the 2-norm and the  $\infty$ -norm of the correction vector  $\Delta g$  during the tuning process. The maximum correction proposed for the second iteration is larger than for the first, for the following iteration it clearly decreases. Note that these corrections are in mm. Finally remark that the TZs do not perfectly coincide. This is due to the presence of the unwanted parasitic couplings  $M_{16}$ ,  $M_{26}$  and  $M_{36}$  (shown in Table II). We have not re-distributed the parasitic couplings, so note that they are not necessarily physically located at those positions. Figure 6 shows the response that is obtained from the ideal coupling matrix when the parasitic couplings are added. It is clear that then the TZs now coincide. Note however that the parasitic couplings are small, we therefore choose not to re-optimize the target matrix.

## VII. CONCLUSION

This paper introduces a novel CAT method which optimizes the coupling parameters. The method estimates the Jacobian of the function relating the physical design parameters of the filter to the implemented coupling parameters of the filter. The estimation combines the adjoint sensitivities of the  $S$ -parameters with respect to the coupling parameters and adjoint sensitivities of the  $S$ -parameters with respect to the design parameters. Formal expressions to calculate the adjoint sensitivities of the  $S$ -parameters with respect to the coupling parameters are given in this paper. Commercial EM-simulators are able to calculate adjoint sensitivities of the  $S$ -parameters with respect to the design parameters without drastically increasing simulation time. This makes that Jacobian is numerically cheap to estimate and only requires 1 EM simulation. Another major advantage is that the Jacobian provides a criterion to determine the implemented solution in the case of non-canonical topologies. The novel method is applied to a design having multiple solutions: a CT filter



(a) Evolution of the inter-resonator couplings during the tuning process.



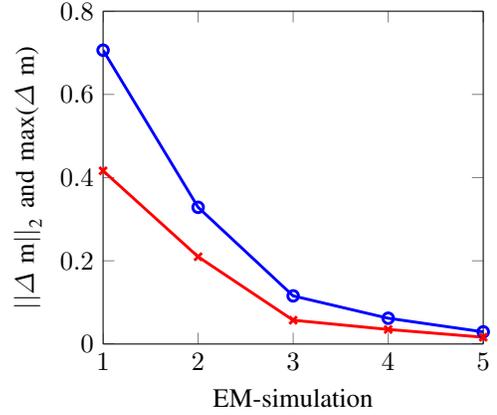
(b) Evolution of the self-couplings during the tuning process.

Fig. 4: Evolution of the coupling parameters during the tuning process. The full lines mark the target values of the first triplet and the dashed lines mark the target values of the second triplet.

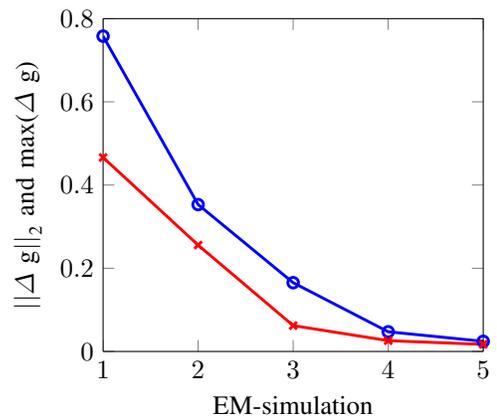
implemented in a microstrip technology. The example proves that the number of required EM-simulations is very low (less than half of the number of the design parameters).

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(a) The 2-norm of the error vector  $\|\Delta \mathbf{m}\|_2$  (-o) and the maximal error  $\|\Delta \mathbf{m}\|_\infty$  (-x).



(b) The 2-norm of the correction vector  $\|\Delta \mathbf{g}\|_2$  (-o) and the maximal correction  $\|\Delta \mathbf{g}\|_\infty$  (-x) (mm).

Fig. 5: Evolution of the 2-norm and  $\infty$ -norm of the error vector  $\Delta \mathbf{m}$  and the correction vector  $\Delta \mathbf{g}$ .

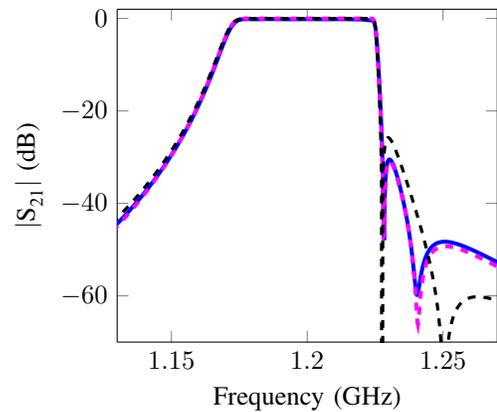


Fig. 6:  $|S_{21}|$  (dB) of the final (—) design and response obtained from the ideal coupling matrix (- - -) when the parasitic couplings are added versus the ideal response (- - -).

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