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Abstract

The Planck constant is the minimal area of one bit.

1 Introduction

The constant c is not just a constant occurring in some formulae, it is also the magnitude of something: of the speed of light. So it is easy to answer the question: what is the constant c the magnitude of?

Answering the same question for the Planck constant seems to be more difficult. Two signs point at this difficulty. The constant c does not have a name. It is neither called "the Rømer constant", nor "the Einstein constant". To name it, we just say what it is the magnitude of: "the speed of light". But we do not seem to know what the Planck constant is the magnitude of, so we call it "the Planck constant". Moreover, there is only one constant c, while at least two versions of the Planck constant are in competition: $\hbar$ and $h = 2\pi\hbar$, each being more or less convenient, depending on the formula we consider.

We suggest here that Bekenstein's postulate, according to which the amount of information stored in a region of space is bounded, would allow us to give an answer to this question: the Planck constant is the minimal area of one bit.

2 Units

The Planck constant is expressed in kg m^2 s^{-1}. But since Special Relativity, we have been knowing that seconds and kilograms are just other units for distance, like inches and ångströms.

Indeed, a duration t can just as well be expressed in meters, as the distance ct travelled by light during the duration t. In this respect, $c = 2.998\ 10^8$ m s^{-1} is no more a fundamental constant than the constant 0.0254 m in^{-1} which transforms inches in meters.

When distances and durations are expressed in meters, a gravitational field E no longer needs to be expressed in m s^{-2}, it may be substituted by $e = E/c^2$ which is expressed in m^{-1}. Then the gravitational field created by a point mass M at a distance d is

$$e = \frac{E}{c^2} = \frac{\mathcal{G}M/c^2}{d^2} = \frac{m}{d^2}$$

where $m = \mathcal{G}M/c^2$ is the mass expressed in meters. Again, notice that here $\mathcal{G}$, or more precisely $\mathcal{G}/c^2 = 7.426\ 10^{-28}$ m kg^{-1}, is no more a fundamental constant than the constant that permits to transform inches in meters.

The Planck constant is expressed in m^2 kg s^{-1}. But as seconds and kilograms are just other units for distance, it can as well be expressed in m^2. Its valuea in m^2 is

$$a_P = \hbar\frac{\mathcal{G}}{c^2}\frac{1}{c} = 2.612\ 10^{-70}\ \mathrm{m}^2$$

more often called "the Planck area".

3 Bekenstein's postulate

Bekenstein's postulate states that the amount of information that can be stored in a sphere of radius R is bounded, and that the bound is

$$I_{max} = \frac{1}{4\ln(2)}\frac{c^3}{\hbar\mathcal{G}}4\pi R^2 = \frac{1}{4\ln(2)a_P}4\pi R^2$$

This postulate, and in particular the fact that this amount of information grows like R^2, and not R^3, is still controversial, but if we assume it, then we can deduce that the minimal area to store one bit is $4\ln(2)a_P$. Thus, the Planck constant expressed in m^2, a_P, modulo the factor $4\ln(2)$, is just the minimal area of one bit. This answers the question of what the Planck constant is the magnitude of.

So, the meaningful version of the Planck constant does not seem to be $\hbar$, h, or a_P, but rather $b = 4\ln(2)a_P = 7.243\ 10^{-70}\ \mathrm{m}^2$, which is the minimal area of one bit. In the usual units, the meaningful value does not seem to be $\hbar$ or h, but $4\ln(2)\hbar = 2.924\ 10^{-34}\ \mathrm{m}^2\ \mathrm{kg}\ \mathrm{s}^{-1}$.

Defining the constant b as the minimal area of one bit makes Bekenstein's formula

$$I_{max} = \frac{4\pi R^2}{b}$$

very natural: the maximal amount of information that can be stored in a sphere is its area divided by the minimal area of one bit.

Just like the speed of light stops to be a fundamental constant when we express durations in meters, the constant b stops to be a fundamental constant if we use it to define a natural distance unit. This unit is not the Planck length $l_p = \sqrt{a_p} = 1.616\ 10^{-35}$ m, but $l = \sqrt{b} = 2\sqrt{ln(2)}l_p = 2.691\ 10^{-35}$ m. Using this unit to measure the radius of a sphere, the maximal information that can be stored in a sphere is just its area.

References

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