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Optimization methods for phase estimation in differential-interference-contrast (DIC) microscopy

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The DIC phase estimation problem

Differential Interference Contrast Schematic

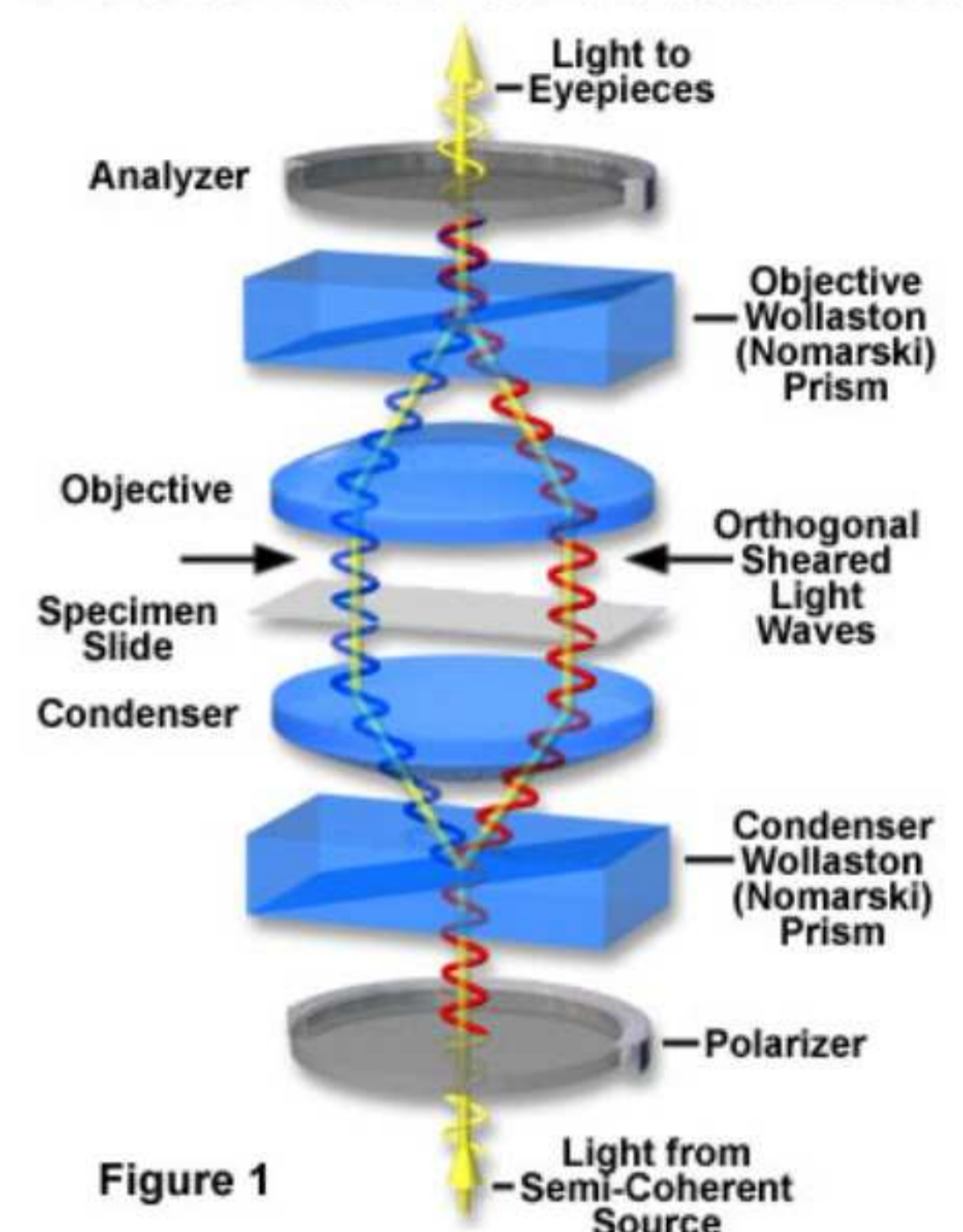
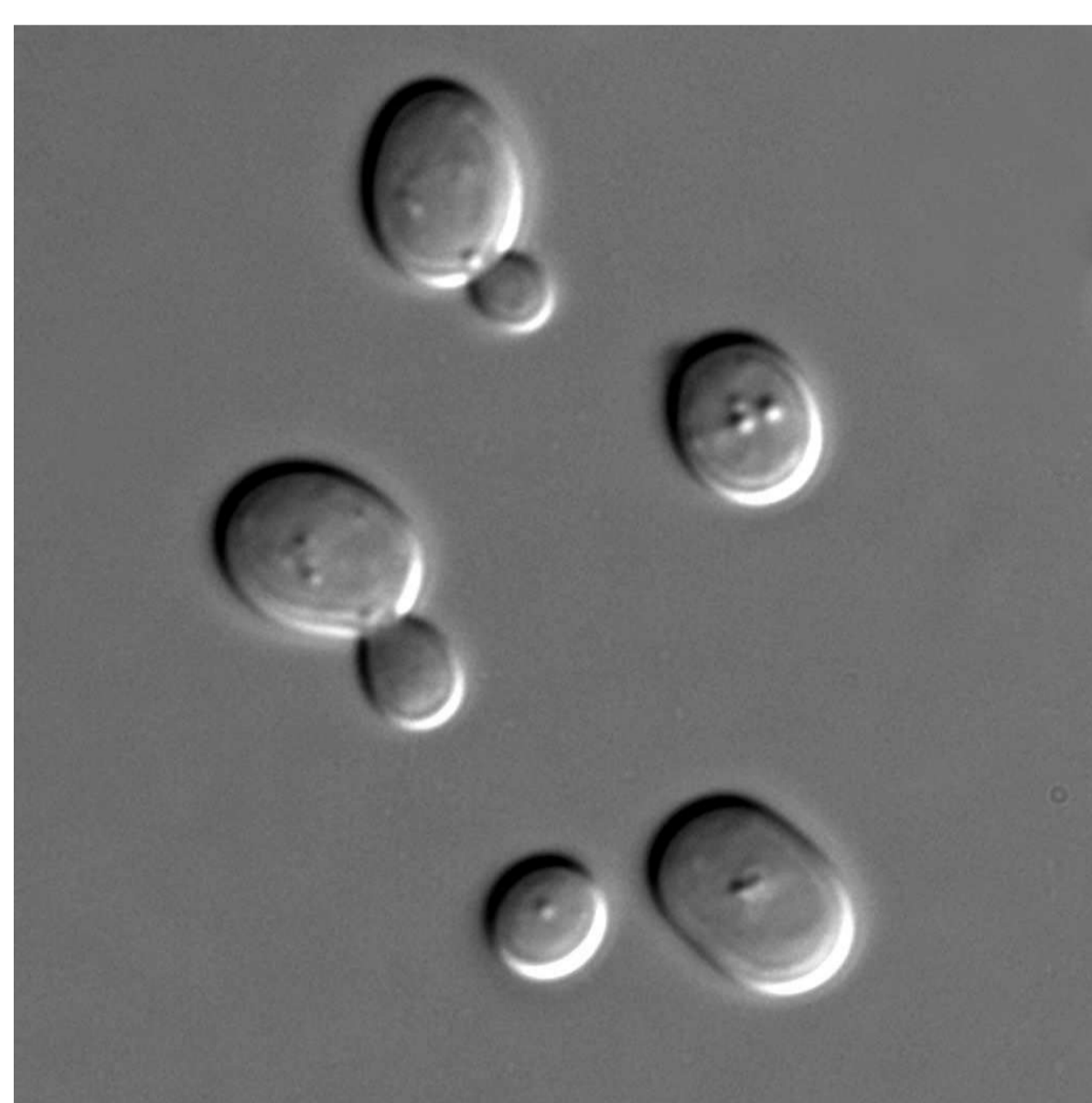


Figure 1



The DIC image is formed by the interference of two orthogonally polarized beams that have a lateral displacement (called *shear*) and are phase shifted relatively one to each other. The resulting image has a 3D high contrast appearance, which can be enhanced by introducing a uniform phase difference between the beams (called *bias*).

Model: the DIC image formation is described by the polychromatic rotational-diversity model [1,2]

$$(o_{k,\lambda_\ell})_j = \left| (h_{k,\lambda_\ell} \otimes e^{-i\phi/\lambda_\ell})_j \right|^2 + (\eta_{k,\lambda_\ell})_j, \quad k = 1, \dots, K, \ell = 1, 2, 3, j \in \chi$$

- k is the index of the rotation of the specimen w.r.t. the horizontal axis, ℓ is the index denoting one of the RGB channels and $j = (j_1, j_2)$ is a 2D-index varying in the set $\chi = \{1, \dots, M\} \times \{1, \dots, P\}$
- λ_ℓ is the ℓ -th illumination wavelength
- $o_{k,\lambda_\ell} \in \mathbb{R}^{MP}$ is the ℓ -th color component of the k -th observed image $o_k = (o_{k,\lambda_1}, o_{k,\lambda_2}, o_{k,\lambda_3}) \in \mathbb{R}^{MP \times 3}$
- $\phi \in \mathbb{R}^{MP}$ is the unknown phase vector and $e^{-i\phi/\lambda_\ell} \in \mathbb{C}^{MP}$ is defined by $(e^{-i\phi/\lambda_\ell})_j = e^{-i\phi_j/\lambda_\ell}$
- $h_{k,\lambda_\ell} \in \mathbb{C}^{MP}$ is the discretization of the continuous DIC Point Spread Function
- $\eta_{k,\lambda_\ell} \in \mathbb{R}^{MP}$ is the noise corrupting the data, $\eta_{k,\lambda_\ell} \sim \mathcal{N}(\mathbf{0}, \sigma^2 I_{(MP)^2})$.

Problem: given the rotationally diverse images o_1, \dots, o_K , retrieve the phase vector ϕ by solving

$$\min_{\phi \in \mathbb{R}^{MP}} J(\phi) \equiv J_0(\phi) + J_{TV}(\phi), \quad (\text{P})$$

- $J_0(\phi) = \sum_{\ell=1}^3 \sum_{k=1}^K \sum_{j \in \chi} \left[(o_{k,\lambda_\ell})_j - |(h_{k,\lambda_\ell} \otimes e^{-i\phi/\lambda_\ell})_j|^2 \right]^2$ is the nonconvex least-squares term
- $J_{TV}(\phi) = \mu \sum_{j \in \chi} \sqrt{((\mathcal{D}\phi)_{j_1})^2 + ((\mathcal{D}\phi)_{j_2})^2 + \delta^2}$ is the total variation (TV) functional, where $\mu > 0$ is the regularization parameter and $\delta \geq 0$ is the smoothing parameter ($\delta = 0 \rightarrow$ standard TV).

Optimization methods

Case $\delta > 0$: problem (P) is differentiable \rightarrow we use a gradient-descent method.

Algorithm 1 Limited Memory Steepest Descent (LMSD) method [3]

Set $\rho, \omega \in (0, 1)$, $m > 0$, $\alpha_0^{(0)}, \dots, \alpha_{m-1}^{(0)} > 0$, $\phi^{(0)} \in \mathbb{R}^{MP}$, $G = []$, $\Theta = []$, $n = 0$.

WHILE True

FOR $l = 1, \dots, m$

1. Compute the smallest non-negative integer i_n such that $\alpha_n = \alpha_n^{(0)} \rho^{i_n}$ satisfies

$$J(\phi^{(n)} - \alpha_n \nabla J(\phi^{(n)})) \leq J(\phi^{(n)}) - \omega \alpha_n \|\nabla J(\phi^{(n)})\|^2.$$

2. Compute the new point as $\phi^{(n+1)} = \phi^{(n)} - \alpha_n \nabla J(\phi^{(n)})$.

3. Update $G = [G \quad \nabla J(\phi^{(n)})]$ and $\Theta = [\Theta \quad \alpha_n^{-1}]$.

4. Set $n = n + 1$.

END

6. Define the $(m+1) \times m$ matrix $\Gamma = \begin{bmatrix} \text{diag}(\Theta) \\ \text{zeros}(1, m) \end{bmatrix} - \begin{bmatrix} \text{zeros}(1, m) \\ \text{diag}(\Theta) \end{bmatrix}$.

7. Compute the Cholesky factorization $R^T R$ of the $m \times m$ matrix $G^T G$.

8. Solve the linear system $R^T r = G^T \nabla J(\phi^{(n)})$.

9. Define the $m \times m$ matrix $\Phi = [R, r] \Gamma R^{-1}$ and its approximation

$$\tilde{\Phi} = \text{diag}(\Phi) + \text{tril}(\Phi, -1) + \text{tril}(\Phi, -1)^T,$$

which is symmetric and tridiagonal.

10. Compute eigenvalues $\theta_1, \dots, \theta_m$ of $\tilde{\Phi}$ and define $\alpha_{n+i-1}^{(0)} = 1/\theta_i$, $i = 1, \dots, m$.

END

Case $\delta = 0$: problem (P) is non differentiable \rightarrow we use a proximal-gradient method.

Algorithm 2 Inexact Linesearch based Algorithm (ILA) [4]

Set $\rho, \omega \in (0, 1)$, $0 < \alpha_{\min} \leq \alpha_{\max}$, $\tau > 0$, $\phi^{(0)} \in \mathbb{R}^{MP}$, $n = 0$.

WHILE True

1. Set $\alpha_n = \max \left\{ \min \left\{ \alpha_n^{(0)}, \alpha_{\max} \right\}, \alpha_{\min} \right\}$, where $\alpha_n^{(0)}$ is chosen as in Algorithm 1.

2. Let $\psi^{(n)} = \text{prox}_{\alpha_n J_{TV}}(\phi^{(n)} - \alpha_n \nabla J_0(\phi^{(n)})) = \text{argmin}_{\phi \in \mathbb{R}^{MP}} h^{(n)}(\phi)$.

Compute $\tilde{\psi}^{(n)}$ such that $h^{(n)}(\tilde{\psi}^{(n)}) - h^{(n)}(\psi^{(n)}) \leq \epsilon_n$ and $0 \leq \epsilon_n \leq -\tau h^{(n)}(\tilde{\psi}^{(n)})$.

3. Set $d^{(n)} = \tilde{\psi}^{(n)} - \phi^{(n)}$.

4. Compute the smallest non-negative integer i_n such that $\lambda_n = \rho^{i_n}$ satisfies

$$J(\phi^{(n)} + \lambda_n d^{(n)}) \leq J(\phi^{(n)}) + \omega \lambda_n h^{(n)}(\tilde{\psi}^{(n)}).$$

5. Compute the new point as $\phi^{(n+1)} = \phi^{(n)} + \lambda_n d^{(n)}$.

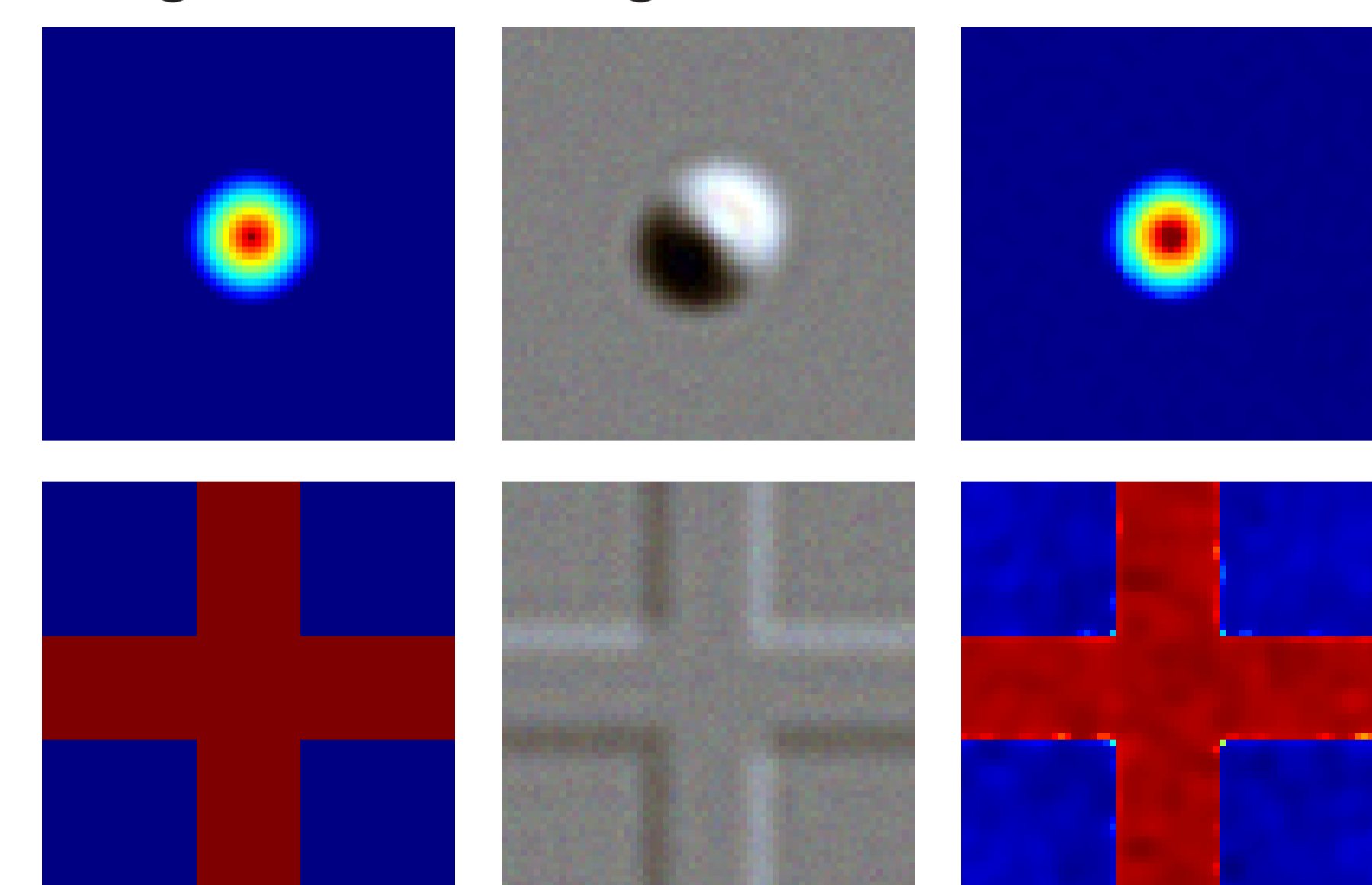
6. Set $n = n + 1$.

END

Convergence and numerical results

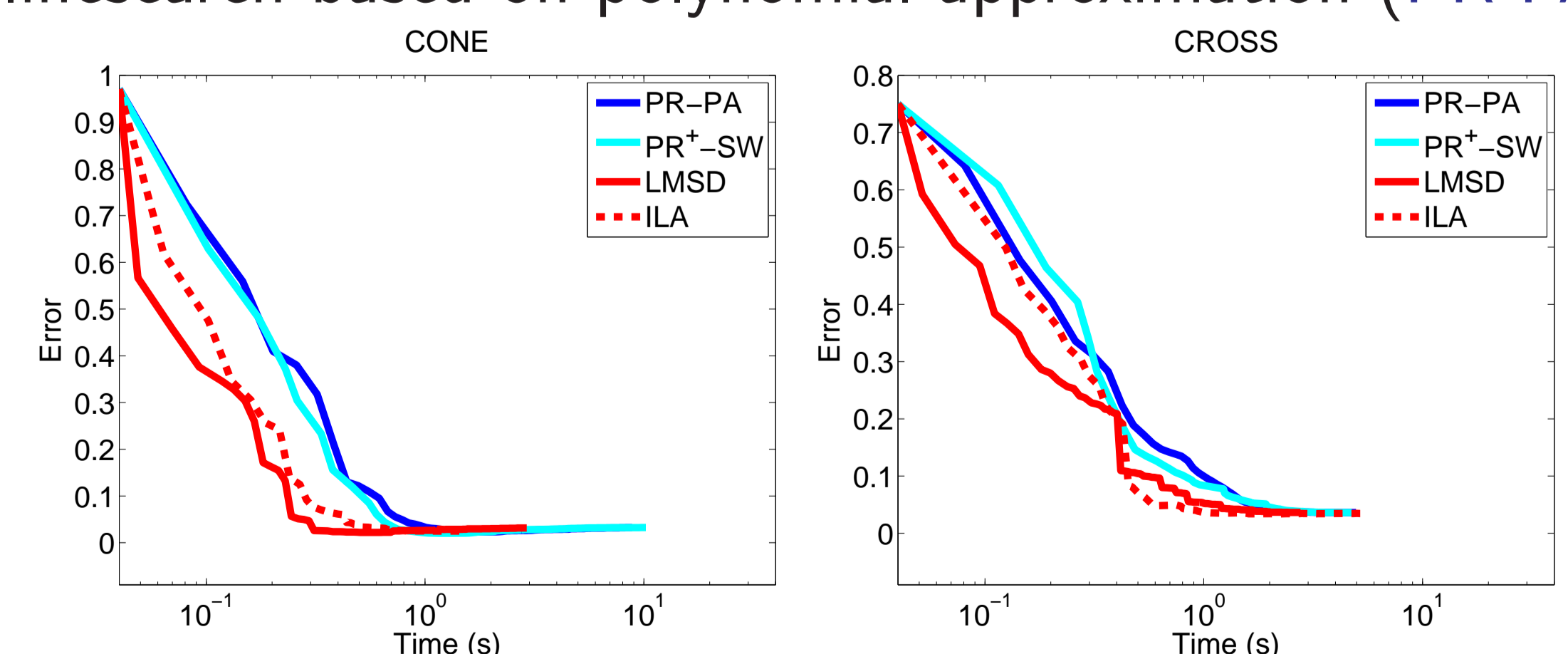
Convergence: Any limit point of Algorithm 1 and 2 is stationary for problem (P). Since J satisfies the Kurdyka-Łojasiewicz property, Algorithm 1 converges to a limit point; the same result can be proved for Algorithm 2 when the proximal point is computed exactly [4].

Results: for both objects, cone (top row) and cross (bottom row), $K = 2$ DIC images have been generated.



True phase Noisy DIC image Rec. phase

The parameters of the methods have been tuned as follows: $\rho = 0.5$, $\omega = 10^{-4}$, $m = 4$, $\alpha_{\min} = 10^{-5}$, $\alpha_{\max} = 10^2$, $\tau = 10^6 - 1$, $\phi^{(0)} = 0$. The methods are compared with the Polak-Ribière conjugate gradient method equipped with the strong Wolfe conditions (PR⁺-SW) and a linesearch based on polynomial approximation (PR-PA) [1].



Object	Algorithm	Iterations	# f	# g	Error
Cone	PR-PA	98	997	98	3.63 %
	PR ⁺ -SW	98	326	326	3.63 %
	LMSD	152	221	152	3.64 %
	ILA	97	179	97	3.46 %

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[4] S. Bonettini et al., ArXiv: 1605.03791, 2016.