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# Optimization methods for phase estimation in differential-interference-contrast (DIC) microscopy

Simone Rebegoldi<sup>1</sup>, Lola Bautista<sup>2,3</sup>, Laure Blanc-Féraud<sup>3</sup>, Marco Prato<sup>1</sup>, Luca Zanni<sup>1</sup> and Arturo Plata<sup>2</sup>

<sup>1</sup>Università di Modena e Reggio Emilia, <sup>2</sup>Universidad Industrial de Santander, <sup>3</sup>Université Côte d'Azur,

## The DIC phase estimation problem

Differential Interference Contrast Schematic

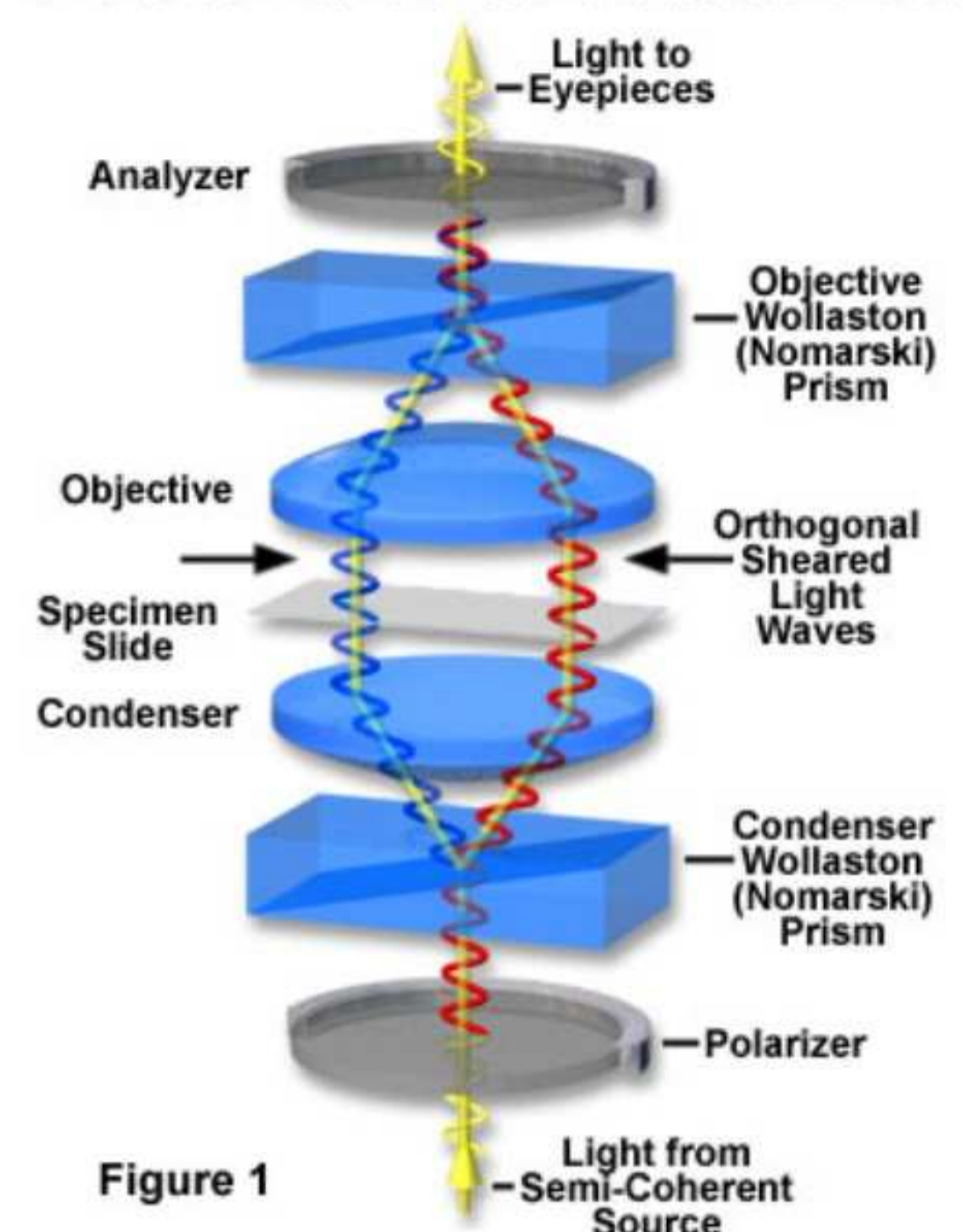
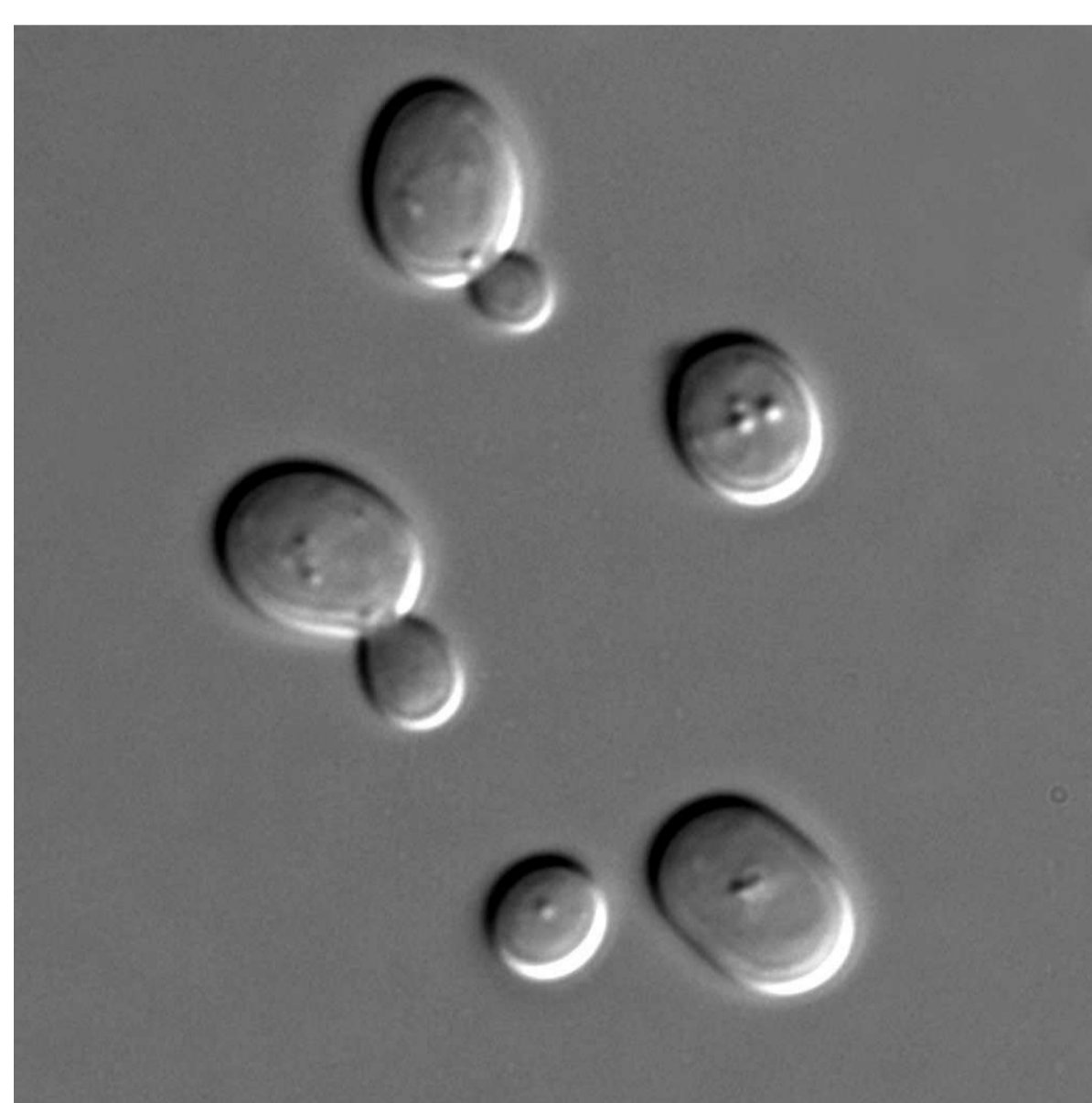


Figure 1



The DIC image is formed by the interference of two orthogonally polarized beams that have a lateral displacement (called *shear*) and are phase shifted relatively one to each other. The resulting image has a 3D high contrast appearance, which can be enhanced by introducing a uniform phase difference between the beams (called *bias*).

**Model:** the DIC image formation is described by the polychromatic rotational-diversity model [1,2]

$$(o_{k,\lambda_\ell})_j = \left| (h_{k,\lambda_\ell} \otimes e^{-i\phi/\lambda_\ell})_j \right|^2 + (\eta_{k,\lambda_\ell})_j, \quad k = 1, \dots, K, \ell = 1, 2, 3, j \in \chi$$

- $k$  is the index of the rotation of the specimen w.r.t. the horizontal axis,  $\ell$  is the index denoting one of the RGB channels and  $j = (j_1, j_2)$  is a 2D-index varying in the set  $\chi = \{1, \dots, M\} \times \{1, \dots, P\}$
- $\lambda_\ell$  is the  $\ell$ -th illumination wavelength
- $o_{k,\lambda_\ell} \in \mathbb{R}^{MP}$  is the  $\ell$ -th color component of the  $k$ -th observed image  $o_k = (o_{k,\lambda_1}, o_{k,\lambda_2}, o_{k,\lambda_3}) \in \mathbb{R}^{MP \times 3}$
- $\phi \in \mathbb{R}^{MP}$  is the unknown phase vector and  $e^{-i\phi/\lambda_\ell} \in \mathbb{C}^{MP}$  is defined by  $(e^{-i\phi/\lambda_\ell})_j = e^{-i\phi_j/\lambda_\ell}$
- $h_{k,\lambda_\ell} \in \mathbb{C}^{MP}$  is the discretization of the continuous DIC Point Spread Function
- $\eta_{k,\lambda_\ell} \in \mathbb{R}^{MP}$  is the noise corrupting the data,  $\eta_{k,\lambda_\ell} \sim \mathcal{N}(\mathbf{0}, \sigma^2 I_{(MP)^2})$ .

**Problem:** given the rotationally diverse images  $o_1, \dots, o_K$ , retrieve the phase vector  $\phi$  by solving

$$\min_{\phi \in \mathbb{R}^{MP}} J(\phi) \equiv J_0(\phi) + J_{TV}(\phi), \quad (\text{P})$$

- $J_0(\phi) = \sum_{\ell=1}^3 \sum_{k=1}^K \sum_{j \in \chi} \left[ (o_{k,\lambda_\ell})_j - |(h_{k,\lambda_\ell} \otimes e^{-i\phi/\lambda_\ell})_j|^2 \right]^2$  is the nonconvex least-squares term
- $J_{TV}(\phi) = \mu \sum_{j \in \chi} \sqrt{((\mathcal{D}\phi)_{j_1})^2 + ((\mathcal{D}\phi)_{j_2})^2 + \delta^2}$  is the total variation (TV) functional, where  $\mu > 0$  is the regularization parameter and  $\delta \geq 0$  is the smoothing parameter ( $\delta = 0 \rightarrow$  standard TV).

## Optimization methods

**Case  $\delta > 0$ :** problem (P) is differentiable  $\rightarrow$  we use a gradient-descent method.

**Algorithm 1** Limited Memory Steepest Descent (LMSD) method [3]

Set  $\rho, \omega \in (0, 1)$ ,  $m > 0$ ,  $\alpha_0^{(0)}, \dots, \alpha_{m-1}^{(0)} > 0$ ,  $\phi^{(0)} \in \mathbb{R}^{MP}$ ,  $G = [ ]$ ,  $\Theta = [ ]$ ,  $n = 0$ .

WHILE True

FOR  $l = 1, \dots, m$

1. Compute the smallest non-negative integer  $i_n$  such that  $\alpha_n = \alpha_n^{(0)} \rho^{i_n}$  satisfies

$$J(\phi^{(n)} - \alpha_n \nabla J(\phi^{(n)})) \leq J(\phi^{(n)}) - \omega \alpha_n \|\nabla J(\phi^{(n)})\|^2.$$

2. Compute the new point as  $\phi^{(n+1)} = \phi^{(n)} - \alpha_n \nabla J(\phi^{(n)})$ .

3. Update  $G = [G \quad \nabla J(\phi^{(n)})]$  and  $\Theta = [\Theta \quad \alpha_n^{-1}]$ .

4. Set  $n = n + 1$ .

END

6. Define the  $(m+1) \times m$  matrix  $\Gamma = \begin{bmatrix} \text{diag}(\Theta) \\ \text{zeros}(1, m) \end{bmatrix} - \begin{bmatrix} \text{zeros}(1, m) \\ \text{diag}(\Theta) \end{bmatrix}$ .

7. Compute the Cholesky factorization  $R^T R$  of the  $m \times m$  matrix  $G^T G$ .

8. Solve the linear system  $R^T r = G^T \nabla J(\phi^{(n)})$ .

9. Define the  $m \times m$  matrix  $\Phi = [R, r] \Gamma R^{-1}$  and its approximation

$$\tilde{\Phi} = \text{diag}(\Phi) + \text{tril}(\Phi, -1) + \text{tril}(\Phi, -1)^T,$$

which is symmetric and tridiagonal.

10. Compute eigenvalues  $\theta_1, \dots, \theta_m$  of  $\tilde{\Phi}$  and define  $\alpha_{n+i-1}^{(0)} = 1/\theta_i$ ,  $i = 1, \dots, m$ .

END

**Case  $\delta = 0$ :** problem (P) is non differentiable  $\rightarrow$  we use a proximal-gradient method.

**Algorithm 2** Inexact Linesearch based Algorithm (ILA) [4]

Set  $\rho, \omega \in (0, 1)$ ,  $0 < \alpha_{\min} \leq \alpha_{\max}$ ,  $\tau > 0$ ,  $\phi^{(0)} \in \mathbb{R}^{MP}$ ,  $n = 0$ .

WHILE True

1. Set  $\alpha_n = \max \left\{ \min \left\{ \alpha_n^{(0)}, \alpha_{\max} \right\}, \alpha_{\min} \right\}$ , where  $\alpha_n^{(0)}$  is chosen as in Algorithm 1.

2. Let  $\psi^{(n)} = \text{prox}_{\alpha_n J_{TV}}(\phi^{(n)} - \alpha_n \nabla J_0(\phi^{(n)})) = \text{argmin}_{\phi \in \mathbb{R}^{MP}} h^{(n)}(\phi)$ .

Compute  $\tilde{\psi}^{(n)}$  such that  $h^{(n)}(\tilde{\psi}^{(n)}) - h^{(n)}(\psi^{(n)}) \leq \epsilon_n$  and  $0 \leq \epsilon_n \leq -\tau h^{(n)}(\tilde{\psi}^{(n)})$ .

3. Set  $d^{(n)} = \tilde{\psi}^{(n)} - \phi^{(n)}$ .

4. Compute the smallest non-negative integer  $i_n$  such that  $\lambda_n = \rho^{i_n}$  satisfies

$$J(\phi^{(n)} + \lambda_n d^{(n)}) \leq J(\phi^{(n)}) + \omega \lambda_n h^{(n)}(\tilde{\psi}^{(n)}).$$

5. Compute the new point as  $\phi^{(n+1)} = \phi^{(n)} + \lambda_n d^{(n)}$ .

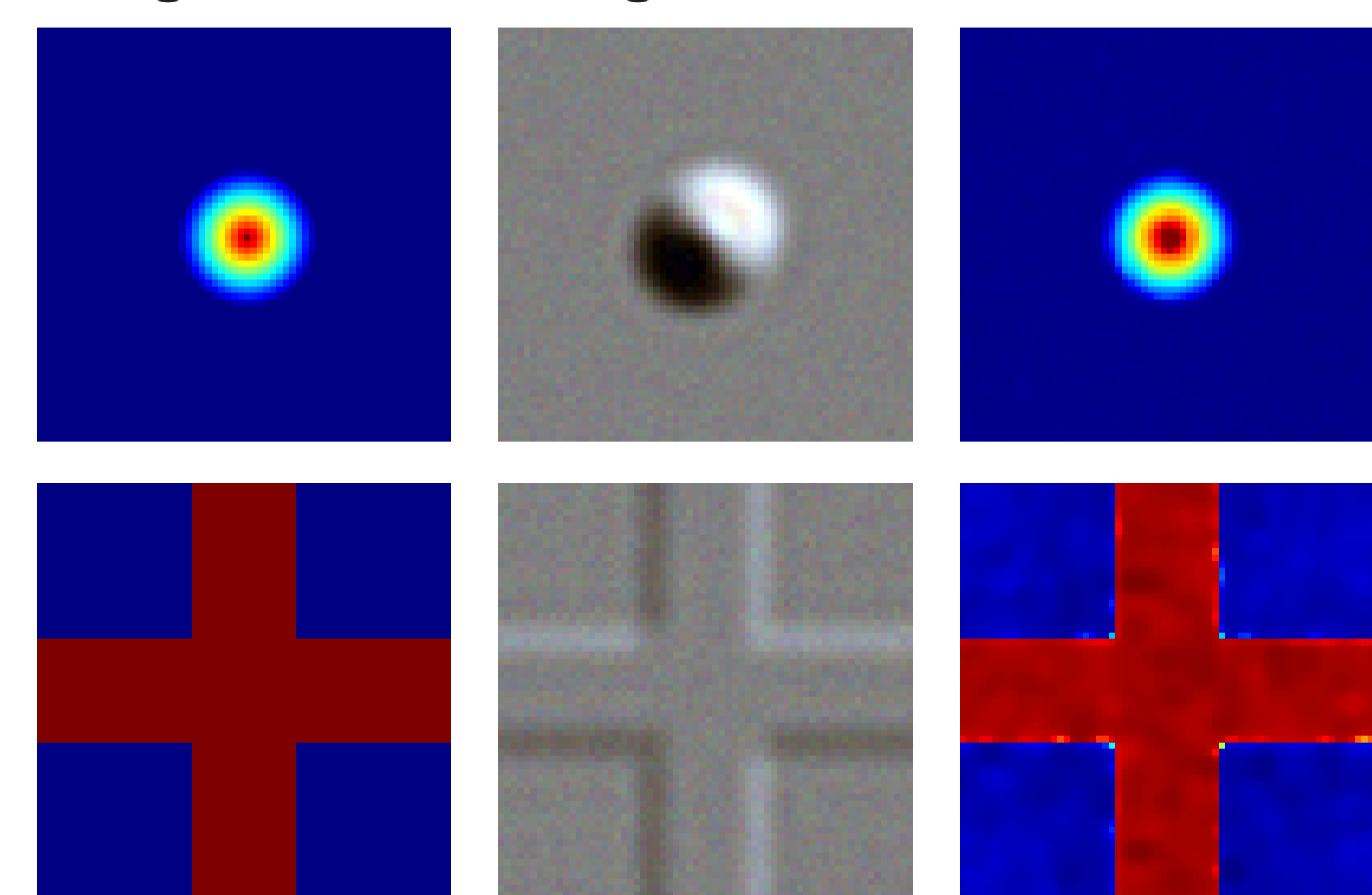
6. Set  $n = n + 1$ .

END

## Convergence and numerical results

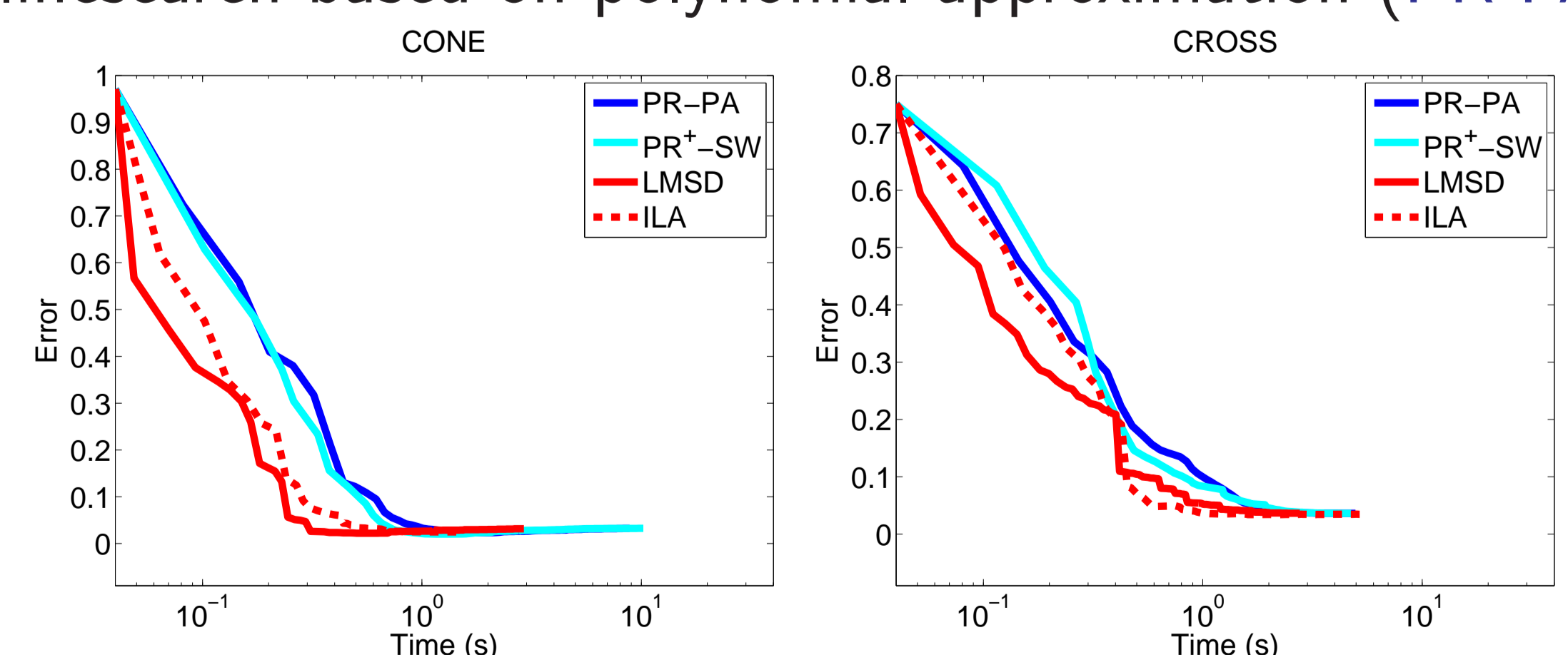
**Convergence:** Any limit point of Algorithm 1 and 2 is stationary for problem (P). Since  $J$  satisfies the Kurdyka-Łojasiewicz property, Algorithm 1 converges to a limit point; the same result can be proved for Algorithm 2 when the proximal point is computed exactly [4].

**Results:** for both objects, cone (top row) and cross (bottom row),  $K = 2$  DIC images have been generated.



True phase Noisy DIC image Rec. phase

The parameters of the methods have been tuned as follows:  $\rho = 0.5$ ,  $\omega = 10^{-4}$ ,  $m = 4$ ,  $\alpha_{\min} = 10^{-5}$ ,  $\alpha_{\max} = 10^2$ ,  $\tau = 10^6 - 1$ ,  $\phi^{(0)} = 0$ . The methods are compared with the Polak-Ribière conjugate gradient method equipped with the strong Wolfe conditions (PR<sup>+</sup>-SW) and a linesearch based on polynomial approximation (PR-PA) [1].



Object	Algorithm	Iterations	# f	# g	Error
Cone	PR-PA	98	997	98	3.63 %
	PR <sup>+</sup> -SW	98	326	326	3.63 %
	LMSD	152	221	152	3.64 %
	ILA	97	179	97	3.46 %

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[4] S. Bonettini et al., ArXiv: 1605.03791, 2016.