A Brief Tour of Theoretical Tile Self-Assembly

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Abstract. The author gives a brief historical tour of theoretical tile self-assembly via chronological sequence of reports on selected topics in the field. The result is to provide context and motivation for the these results and the field more broadly.

Introduction. This tour covers only a subset of the research topics in theoretical tile self-assembly. It is intended for readers who are familiar with the basics of the field and wish to obtain a better understanding of how the multitude of models, problems, and results relate. As such, it is neither a survey nor an introduction; for these, the reader is referred to the excellent works of Doty [18], Patitz [37], Woods [50], and Winfree [47]. Moreover, it does not cover work in experimental DNA tile self-assembly.

The aTAM of Winfree (1990s). It is common for work on theoretical tile self-assembly (hereafter *tile assembly*), to begin "In his Ph.D. thesis, Winfree [47] introduced the *abstract tile assembly model (aTAM)* ...". The ubiquity of this opener matches the importance this work plays in the field: it is the point of conception, and nearly 20 years later, its reading connotes initiation to the area. Moreover, the sustained popularity of tile assembly is due in large part to the elegance and hidden intricacy of this original model. Such intricacy is perhaps most crystallized in a simple yet devious question: is universal computation possible in the aTAM at temperature 1?

As with any research, the concept of the aTAM and its experimental implementations were not in isolation. Several other models of (linear) DNA-based computation also introduced around this time, including the filtering-based models of Adleman [2] and Beaver [8] and the splicing systems of Pâun, Kari, Yokomori, and others [27,39,51].

Benchmark Problems (2000-2004). Beyond defining the aTAM, the Winfree thesis also contains a proof of the computational universality of the aTAM at temperature 2. This result established algorithmic universality, but not the ability to assemble shapes *efficiently*, i.e., using systems of few tile types. Rothemund and Winfree [41] soon established this, achieving $n \times n$ square assembly with $O(\log n)$ tile types. Following this work, the twin capabilities of universal computation and efficient square assembly became the de facto benchmarks for powerful models of tile assembly.[1]

[1] As discussed later, these simple challenges ultimately proved insufficient detailed for distinguishing between some powerful but unequal models.

Followup work by Adleman et al. [4,13] closed the small gaps in optimality left by the construction Rothemund and Winfree and introduced a new metric of efficiency: (expected) assembly time. This metric was ported from simultaneous work on the dynamics of linear assemblies by Adleman et al. [3,6] and considered in other models later [10]. Shortly after, efficient assembly of general (non-square) shapes was proved NP-complete by Adleman et al. [5], while Soloveichik and Winfree [45] established the *geometric universality* [46] of the aTAM: the construction of all shapes efficiently (if scaling is permitted).

Even beyond techniques for information encoding and construction analysis introduced in [4,45], perhaps the most persistent single contribution of work in this era was implicit conjecture in [41] that the aTAM at temperature 1 is not capable of (universal) computation, based on related conjectured lower bound of $2n-1$ tiles to assemble $n \times n$ squares.

Error-Prone Models (2002-2011). Some of the earliest variations on the aTAM were those concerned with the design of systems robust to various errors in the assembly process. Such errors included incorrect tile attachments [11,12,42,44,49], assembly "damage" via partial deletion [44,48], temperature fluctuations [24], and unseeded growth [43]. In an ironic turn, the adversarial "seedless" growth addressed by Schulman et al. [43] was later used to achieve efficient constructions impossible with seeded growth [9]. The collection of error-prone models and results demonstrated that even small changes to aTAM yield rich new ideas. New model variations remain the largest catalyst of new work in tile assembly.

The Temperature 1 Problem (2005-ongoing). The *Temperature-1 Problem* is the most notorious open problem in tile assembly: is the temperature-1 aTAM computationally universal? The widely conjectured answer is a resounding "Obviously not!"[2], but the problem has resisted nearly all progress, permitting only negative results that use either stronger conjectures [33,34], weaker models [32,40], or the assumption of other plausible conjectures [25].

One primary difficulty is even obtaining a precise formal statement of what constitutes "computation" in tile assembly. The second is developing a proof approach that passes the "3D test": the proof must break in 3D, implied by the result of Cook, Fu, and Schweller [14].

Computational Universality via Weak Cooperation (2007-2012). The computationally universality of the aTAM at temperature 2 and the temperature 1 problem beg the question of whether computational universality can be achieved by adding other features to the temperature-1 aTAM. Several variations of the temperature-1 aTAM were considered for which the answer proved to be "Yes". These included models using the third dimension [14], negative-strength glues [21,38], non-square tiles [26,29], and tiles with triggerable "signals" [30,36]. These models commonly exploit either "weak cooperation" in the form of repelling forces or the ability to "jump over walls".

Handedness (2008-2014) In addition to models adding features to the temperature-1 aTAM, other modifications to the aTAM were under consider-

[2] Usually accompanied by an exaggerated shoulder shrug and waving, upturned palms.

ation. In particular, the elimination of the seed was considered in the *hierarchical* or *two-handed (2HAM)* tile assembly models, hinted at in several settings [7,15,31] before reaching the formulation used currently [1,24]. Surprisingly, the removal of the seed causes numerous unexpected effects, including increased power [9], runaway growth [19,20], and no improvement in assembly time [10].

Intrinsic Universality (2010-present). By 2010, the introduction of new tile assembly models was occurring regularly. As previously described, many of these models obtained computational universality, but through alternative "weakly cooperative" means. With a few exceptions where direct simulation was possible (e.g., 2HAM simulation of aTAM [9]), the understanding of the relative power of these models was unsatisfyingly coarse: models are either computationally universal or not.

Adapting definitions of a geometric notion of simulation from cellular automata (see [28,35]), Doty et al. [22,23] established that the temperature-2 aTAM is *intrinsic universal* for all aTAM systems: there exists a single temperature-2 tile set that simulates the behavior of any aTAM system (when provided with a seed assembly encoding the system). Subsequent work used this new comparative metric to prove positive and negative intrinsic universality results for variations of the 2HAM, aTAM, and polygonal tile model [16,17,29,34]. In progress towards the Temperature 1 Problem, the temperature-1 aTAM was proved not intrinsically universal for the higher temperature aTAM [34], failing to match temperature 2.

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