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***Abstract*—This paper investigates the subcarrier and power allocation for the downlink of a multicarrier non-orthogonal multiple access (MC-NOMA) system. A three-step algorithm is proposed to deal with the sum rate maximization problem. In Step 1, we assume that each user can use all the subcarriers simultaneously and apply the synchronous iterative waterfilling algorithm (SIWA) to obtain a power vector for each user. In Step 2, subcarriers are assigned to users by a heuristic greedy method based on the achieved power allocation result of Step 1. In Step 3, SIWA is used once again to further improve the system performance with the obtained subcarrier assignment result of Step 2. The convergence of SIWA in Step 3 is proved when the number of multiplexed users is no more than two. Since SIWA is applied twice, we call our three-step method Double Iterative Waterfilling Algorithm (DIWA). Numerical results show that the proposed DIWA achieves comparable performance to an existing near-optimal solution but with much lower time complexity.**

***Index Terms*—Multicarrier non-orthogonal multiple access (MC-NOMA), successive interference cancellation (SIC), iterative waterfilling algorithm (IWA), resource allocation.**

I. Introduction

Orthogonal frequency division multiple access (OFDMA) has been widely used in 3GPP Long Term Evolution (LTE) and LTE-Advanced (LTE-A) cellular systems, in which the whole frequency spectrum is divided into orthogonal subcarriers and each subcarrier is allocated to at most one user during each time slot at each base station (BS). OFDMA can avoid intra-cell interference after performing user transmission scheduling at the BS. Besides, it can be implemented with low-complexity receiver. However, it is known that the spectral resource is in general under-utilized due to the requirement of channel access orthogonality.

Since data traffic for cellular networks is expected to increase by 1000 folds by 2020, improving the spectral efficiency becomes one of the key criteria for meeting the dramatically increasing demand. Non-orthogonal multiple access (NOMA) has recently received significant attention and has been regarded as a promising approach for 5G cellular systems as it allows the multiplexing of multiple users on the same frequency resource, which could provide a higher system spectral efficiency [1]–[3]. Since multiple users are allowed to use the same subcarrier at the same time, successive interference cancellation (SIC) is adopted at the receiver side to mitigate the resultant co-channel interference.

Since SIC is applied, transmit power must be allocated properly among multiplexed users such that interfering signals can be correctly decoded and subtracted from the received signal of some users [4]. Fractional transmit power control (FTPC) is a sub-optimal but common power control strategy for user sum rate maximization, which allocates power according to the individual link condition of each user [5]. In [6] and [7], distributed power allocation algorithms are proposed to minimize total power consumption with data rate requirement of each user taken into account for downlink and uplink multi-cell NOMA, respectively. In addition, there exists some other works that investigate the power control for single-carrier multi-antenna NOMA systems [8]–[10] and network NOMA [11], [12].

For multicarrier NOMA (MC-NOMA) systems, user scheduling (subcarrier assignment) and power allocation are two interacted factors for achieving high spectral efficiency. In practical LTE cellular systems, it is shown in [13], by realistic computer simulation that MC-NOMA has better system level downlink performance in terms of user throughput than that of OFDMA. In [14], a greedy user selection and sub-optimal power allocation scheme based on difference-of-convex (DC) programming is presented to maximize the weighted user sum rate. Note that the optimization of power allocation among different subcarriers and different users are all conducted using the DC programming. It is observable that the scheme has high computational complexity. In [15], various user pairing algorithms for MISO MC-NOMA system are investigated. However, the performance gain of [15] is limited due to the use of naive power control schemes such as fixed power allocation (FPA) and FTPC for multiplexed users. Additionally, a joint power and channel allocation problem for MC-NOMA is formulated in [16], which is proved to be NP-hard and solved by a near-optimal solution based on Lagrangian duality and dynamic programming (LDA).

Motivated by the aforementioned observations, we propose

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the DIWA, which could achieve competitive performance to LDA with much fewer computation operations. First, the synchronous iterative waterfilling algorithm (SIWA) is applied to allocate power for each user with the assumption that each user could use all the subcarriers simultaneously. Second, we use a heuristic greedy method to assign each subcarrier to at most two users. This kind of setting is based on an implementation point of view, i.e., reducing the receiver complexity and error propagation due to SIC [1], [17]. Third, SIWA is applied once again to further improve the system performance with the obtained subcarrier allocation result of Step 2. The convergence of Step 3 is proved. Numerical results show that our proposed DIWA could achieve comparable performance to LDA with much lower computational complexity.

The rest of this paper is organized as follows. In Section II, we present the system model and formulate the problem mathematically. In Section III, the SIWA is introduced for solving the problem. Our proposed subcarrier and power allocation scheme and the convergence of its Step 3 is derived and analyzed in Section IV. In Section V, we evaluate the performance of our proposed resource allocation algorithm by computer simulations. Finally, Section VI contains the conclusion.

II. System Model and Problem Formulation

A. System Model

Consider the downlink of a multi-user cellular system with one base station (BS) serving K users. Denote the set of indices of all users by $\mathcal{K} \triangleq \{1, 2, \ldots, K\}$. The overall bandwidth W is divided into N subcarriers. We denote the index set of these N subcarriers by $\mathcal{N} \triangleq \{1, 2, \ldots, N\}$. For $n \in \mathcal{N}$, let W_n be the bandwidth of subcarrier n, where $\sum_{n\in\mathcal{N}} W_n = W$. Assume there is no interference among different subcarriers because of the orthogonal frequency division.

For $k \in \mathcal{K}$ and $n \in \mathcal{N}$, let g_k^n be the link gain of user k on subcarrier n. We assume a block fading channel. Let $p_k^n \geq 0$ be the allocated transmit power of user k on subcarrier n. User k is said to be multiplexed on subcarrier n if $p_k^n > 0$. There is a sum-power constraint for each user k, such that $\sum_{n=1}^{N} p_k^n \leq \bar{p}_k$, where $\bar{p}_k > 0$. Let η_k^n be the receiver noise power of user k on subcarrier n. For notation simplicity, we normalize the noise power as $\tilde{\eta}_k^n \triangleq \eta_k^n / g_k^n$.

We assume that the BS allocates subcarriers to users and multiplexes users on a given subcarrier using superposition coding. Let $\mathcal{U}_n$ be the set of users to whom subcarrier n is assigned. Each subcarrier can be modeled as a multi-user Gaussian broadcast channel and SIC is applied at the receiver side when it is possible to eliminate the intra-band interference.

As SIC is applied, we need to consider the decoding order of users on the same subcarrier. For $n \in \mathcal{N}$, let Π_n be the set of all possible permutations of $\mathcal{U}_n$. For example, if users u and v are multiplexed on subcarrier n, i.e., $\mathcal{U}_n = \{u, v\}$, then

$$\Pi_n = \big\{(u, v), (v, u)\big\}.$$

Let $\boldsymbol{\pi}_n \in \Pi_n$ be the decoding order of the users on subcarrier n. Let $\pi_n(i)$, where $i \in \{1, 2, \ldots, |\mathcal{U}_n|\}$, be its i-th component, which means that user $\pi_n(i)$ first decodes the signals of $\pi_n(1)$ to $\pi_n(i-1)$, subtracts these signals and finally decodes its intended message by treating the signals of the remaining users on subcarrier n as noise. Note that $\boldsymbol{\pi}_n$ is a vector function of the normalized noise power of each multiplexed user on subcarrier n, i.e., $\tilde{\eta}_k^n$ where $k \in \mathcal{U}_n$ [18, Section 6.2] and is defined as follows:

$$\boldsymbol{\pi}_n \triangleq (\pi_n(1), \pi_n(2), \ldots, \pi_n(|\mathcal{U}_n|)),$$

such that the following two criteria are satisfied:

1) The normalized noise power of multiplexed users on subcarrier n are arranged in descending order: $\tilde{\eta}^n_{\pi_n(1)} \geq \tilde{\eta}^n_{\pi_n(2)} \geq \cdots \geq \tilde{\eta}^n_{\pi_n(|\mathcal{U}_n|)}$;
2) When there is a tie, we arrange those users in ascending order of their indices, i.e., if $\tilde{\eta}^n_{\pi_n(i)} = \tilde{\eta}^n_{\pi_n(j)}$, then, $\pi_n(i) < \pi_n(j)$ for $i < j$.

Once the decoding order is determined according to the normalized receiver noise power, the achievable rate of user k on subcarrier n can be obtained as

$$R_k^n \triangleq W_n \log_2\Big(1 + \frac{p_k^n}{\sum_{j=\pi_n^{-1}(k)+1}^{|\mathcal{U}_n|} p^n_{\pi_n(j)} + \tilde{\eta}_k^n}\Big), \quad (1)$$

where $\pi_n^{-1}(k)$ represents the order of user k in $\boldsymbol{\pi}_n$. More precisely, $\pi_n^{-1}(k) = i$ if $\pi_n(i) = k$.

B. Problem Formulation

The objective of this work is to maximize the sum of data rates subject to power constraints and a maximum of multiplexed users per subcarrier. Mathematically, the problem can be formulated as follows:

$$\text{maximize} \quad R_{\text{sum}} \triangleq \sum_{n=1}^{N} \sum_{k=1}^{K} R_k^n,$$

subject to

$$C1: \ \sum_{n=1}^{N} p_k^n \leq \bar{p}_k, \ k \in \mathcal{K} \quad (2)$$

$$C2: \ p_k^n \geq 0, \ k \in \mathcal{K}, \ n \in \mathcal{N} \quad (3)$$

$$C3: \ |\mathcal{U}_n| \leq M, \ n \in \mathcal{N} \quad (4)$$

$$C4: \ p_k^n = 0, \ k \notin \mathcal{U}_n, \ n \in \mathcal{N}. \quad (5)$$

Note that $C1$ and $C2$ represent power constraints for user k. Moreover, $C3$ restricts that the number of multiplexed users on each subcarrier is no more than M. When $M = 1$, the problem reduces to orthogonal multiple access (OMA). In this paper, we consider the case where $M = 2$, which is an important special case for practical systems.

This maximization problem has been proved to be NP-hard [16]. For this reason, a near-optimal polynomial-time solution based on LDA has been proposed [16]. In this work, we design another algorithm based on iterative waterfilling, which is more time efficient at the expense of slight degradation in sum rate.

III. Synchronous Iterative Waterfilling Algorithm

In this section, we introduce the iterative waterfilling algorithm (IWA) which will be applied to our resource allocation algorithm, and we focus on the *synchronous* version (SIWA).

For $k \in \mathcal{K}$, let $\mathcal{N}_k \subseteq \mathcal{N}$ be the set of subcarriers allocated to user k. Let $\boldsymbol{p}_k \triangleq (p_k^n)_{n\in\mathcal{N}_k}$ be an *indexed family* of non-negative real numbers with index set $\mathcal{N}_k$. The set of all feasible powers for user k is denoted by

$$\mathcal{P}_k \triangleq \{\boldsymbol{p}_k : \sum_{n\in\mathcal{N}_k} p_k^n \le \bar{p}_k\}. \tag{6}$$

The set of all feasible powers for all users in the system is $\mathcal{P} \triangleq \prod_{k\in\mathcal{K}} \mathcal{P}_k$.

For $k \in \mathcal{K}$ and $n \in \mathcal{N}_k$, define

$$\tilde{I}_k^n \triangleq \sum_{j=\pi_n^{-1}(k)+1}^{|\mathcal{U}_n|} p_{\pi_n(j)}^n + \tilde{\eta}_k^n, \tag{7}$$

which is the normalized interference plus noise of user k on subcarrier n. Assuming fixed power allocation for other users and constant channel gains, the optimal power allocation for user k is obtained using the following result [19]:

Theorem 1. *$\boldsymbol{p}_k^* \in \mathcal{P}_k$ maximizes $R_k \triangleq \sum_{n\in\mathcal{N}_k} R_k^n$ if and only if there exists a water level, ω, such that*

$$p_k^{*n} = [\omega - \tilde{I}_k^n]^+, \text{ for } n \in \mathcal{N}_k, \tag{8}$$

where

$$[X]^+ = \max\{0, X\}, \tag{9}$$

and

$$\sum_{n\in\mathcal{N}_k} p_k^{*n} = \bar{p}_k. \tag{10}$$

Let $\boldsymbol{p}_{-k} \triangleq (\boldsymbol{p}_1, \dots, \boldsymbol{p}_{k-1}, \boldsymbol{p}_{k+1}, \dots, \boldsymbol{p}_K)$ for $k \in \mathcal{K}$. We define the waterfilling function for user k as

$$f_k(\boldsymbol{p}_{-k}) \triangleq (p_k^{*n})_{n\in\mathcal{N}_k}, \tag{11}$$

where p_k^{*n} is defined in Theorem 1. Furthermore, we define $F : \mathcal{P} \to \mathcal{P}$ as the waterfilling function of the whole system as

$$F(\boldsymbol{p}_1, \boldsymbol{p}_2, \dots, \boldsymbol{p}_K) \triangleq (f_k(\boldsymbol{p}_{-k}))_{k=1}^K. \tag{12}$$

Note that SIWA is an iterative algorithm. For $k \in \mathcal{K}$ and $n \in \mathcal{N}_k$, let $p_k^n(t)$ be the power of user k on subcarrier n at time t, and $\boldsymbol{p}_k(t)$ be the corresponding indexed family at time t. According to (7), we define $\tilde{I}_k^n(t)$ as a function of $\{p_j^n(t) : j \in \mathcal{U}_n \setminus \{k\}\}$. SIWA is then defined as

$$(\boldsymbol{p}_1^{(t+1)}, \boldsymbol{p}_2^{(t+1)}, \dots, \boldsymbol{p}_K^{(t+1)}) = F(\boldsymbol{p}_1^{(t)}, \boldsymbol{p}_2^{(t)}, \dots, \boldsymbol{p}_K^{(t)}), \tag{13}$$

with $\boldsymbol{p}_k^{(0)} = 0$ for $k \in \mathcal{K}$.

Let $\omega_k(t)$ be the water level of user k at time t. Because of (10), we have

$$\sum_{n\in\mathcal{N}_k} [\omega_k(t+1) - \tilde{I}_k^n(t)]^+ = \bar{p}_k. \tag{14}$$

Note that $\omega_k(t+1)$ can be regarded as a function of $\tilde{\boldsymbol{I}}_k(t) \triangleq (\tilde{I}_k^n(t))_{n\in N_k}$, and we denote it by

$$\omega_k(t+1) = g_k(\tilde{\boldsymbol{I}}_k(t)). \tag{15}$$

IV. Subcarrier and Power Allocation

The sum-rate performance of a scheme is principally affected by two factors, namely, subcarrier allocation and power control for multiplexed users. Our proposed DIWA consists of three steps. In the first two steps, we allocate subcarriers to users based on SIWA. Then, based on the subcarrier allocation obtained, we allocate power to the users in the third step using SIWA again.

In this section, we first present our proposed resource allocation algorithm. Subsequently, we analyze its convergence.

A. Double Iterative Waterfilling Algorithm (DIWA)

We state the three steps of DIWA as follows:

1) Relax constraints $C3$ and $C4$, and allow all users to use all N subcarriers simultaneously, i.e., $\mathcal{N}_k = \mathcal{N}$ for $k \in \mathcal{K}$. Apply SIWA for T_1 iterations to obtain each user's power allocation strategy, $\boldsymbol{p}_k$.
2) Assign subcarriers to users based on the power allocation obtained in Step 1. For $n \in \mathcal{N}$,
   - If two or more users have positive power on subcarrier n, allocate subcarrier n to the two users who have the highest and second highest allocated power on subcarrier n, with ties broken arbitrarily;
   - If only one user has positive power on subcarrier n, allocating subcarrier n only to that user;
   - If no one has positive power on subcarrier n, allocate subcarrier n to user k^*, where $k^* \triangleq \operatorname{argmax}_{k\in\mathcal{K}} g_k^n$, with ties broken arbitrarily.

   After this step, $\mathcal{U}_n$ is determined with $|\mathcal{U}_n| \le M = 2$ for all $n \in \mathcal{N}$.
3) Assign power to users based on the subcarrier allocation obtained in Step 2. Apply SIWA repeatedly (with at most T_3 iterations) until the sum rate improvement is smaller than a certain threshold [20], i.e.,

$$\left| \frac{R_{\text{sum}}(t+1) - R_{\text{sum}}(t)}{R_{\text{sum}}(t)} \right| \le \epsilon, \tag{16}$$

   where ϵ is a small constant and $R_{\text{sum}}(t)$ is the sum rate obtained after iteration t.

Note that in Step 1, we impose a maximum number of iterations, T_1, which provides a tradeoff between computation and sum-rate performance. The detailed result is shown in Section V. Additionally, through our simulations, when the link gains are generated according to standard assumptions in cellular systems, Step 1 converges within $T_1 = 5$ iterations in all the 8,000 random instances considered in our simulations. For Step 3, when $|\mathcal{U}_n| \le 2$ for all n, SIWA is guaranteed to converge, which is proved in the next subsection.

B. The Convergence Analysis for Step 3

In this subsection, we will investigate the convergence of SIWA in Step 3 of our proposed method. We consider two waterfilling scenarios for user k. The normalized interference at subcarrier n in the two scenarios are $\tilde{I}_k^n$ and $\tilde{I}_k^{n\prime}$, respectively. After waterfilling, we denote the water levels in the two scenarios by ω_k and ω_k', respectively. With this setting, we have the following lemma:

Lemma 2. *For any $k \in \mathcal{K}$, if $\tilde{I}_k^n \geq \tilde{I}_k^{n\prime}$ for all $n \in \mathcal{N}_k$, then $g_k(\tilde{\boldsymbol{I}}_k) \geq g_k(\tilde{\boldsymbol{I}}_k')$.*

Proof: Let $\omega_k \triangleq g_k(\tilde{\boldsymbol{I}}_k)$ and $\omega_k' \triangleq g_k(\tilde{\boldsymbol{I}}_k')$. By contradiction, assume $\omega_k < \omega_k'$. First, note that

$$\sum_{n\in\mathcal{N}_k}[\omega_k - \tilde{I}_k^n]^+ \leq \sum_{n\in\mathcal{N}_k}[\omega_k' - \tilde{I}_k^n]^+ \leq \sum_{n\in\mathcal{N}_k}[\omega_k' - \tilde{I}_k^{n\prime}]^+, \quad (17)$$

where the first inequality follows from the assumption that $\omega_k < \omega_k'$ and the second inequality follows from the condition that $\tilde{I}_k^n \geq \tilde{I}_k^{n\prime}$. According to (14), both sides are equal to $\bar{p}_k$, which implies, in particular, equality holds in the first inequality. This is possible only if

$$[\omega_k - \tilde{I}_k^n]^+ = [\omega_k' - \tilde{I}_k^n]^+ = 0 \quad (18)$$

for all $n \in \mathcal{N}_k$. As a result, $\bar{p}_k = 0$, which violates our assumption in the system model.

□

Theorem 3. *Given $|\mathcal{U}_n| \leq 2$ for all $n \in \mathcal{N}$, SIWA always converges.*

Proof: Since $|\mathcal{U}_n| \leq 2$, there are at most two multiplexed users in subcarrier n. For each subcarrier $n \in \mathcal{N}_k$, user k may suffer from intra-band interference if subcarrier n is also assigned to another user and that user has a smaller normalized noise power than user k. We denote this subset of subcarriers by $\mathcal{T}_k$, and its complement by $\mathcal{S}_k$, i.e., $\mathcal{S}_k = \mathcal{N}_k \setminus \mathcal{T}_k$. For $n \in \mathcal{T}_k$, we define $-k_n$ as the index of the user who shares subcarrier n with user k.

Since each user has a total power constraint, $\tilde{I}_k^n(t)$ is bounded from above for all k and n. Therefore, according to (14), $\omega_k(t)$ is also bounded from above for all $k \in \mathcal{K}$. The convergence of SIWA in Step 3 is established if

$$\boldsymbol{\omega}(t) \triangleq (\omega_1(t), \omega_2(t), \ldots, \omega_K(t)), \quad (19)$$

is monotone increasing, i.e., for any $t \geq 1$,

$$\boldsymbol{\omega}(t+1) \succeq \boldsymbol{\omega}(t), \quad (20)$$

which we now prove by induction.

Basis: Since $\boldsymbol{p}_k^{(0)} = 0$ for all k, we have $\tilde{I}_k^n(0) = \tilde{\eta}_k^n$ for all k and n. It is obvious that $\tilde{I}_k^n(1) \geq \tilde{I}_k^n(0)$. Lemma 2 and (15) imply $\boldsymbol{\omega}(2) \succeq \boldsymbol{\omega}(1)$.

Inductive step: Suppose (20) holds for $t = L$, i.e.,

$$\boldsymbol{\omega}(L+1) \succeq \boldsymbol{\omega}(L). \quad (21)$$

First, consider $n \in \mathcal{S}_k$. By the definition of $\mathcal{S}_k$, $\tilde{I}_k^n(t) = \tilde{\eta}_k^n$ for all t, which implies

$$\tilde{I}_k^n(L+1) = \tilde{I}_k^n(L), \quad \text{for } n \in \mathcal{S}_k. \quad (22)$$

Next, consider $n \in \mathcal{T}_k$. According to (7), for any t, we have

$$\tilde{I}_k^n(t) = p_{-k_n}^n(t) + \tilde{\eta}_k^n, \quad \text{for } n \in \mathcal{T}_k. \quad (23)$$

By the definition of $\mathcal{T}_k$, user $-k_n$ experiences no intra-band interference in subcarrier n. The waterfilling method dictates that

$$p_{-k_n}^n(t) = [\omega_{-k_n}(t) - \tilde{\eta}_{-k_n}^n]^+. \quad (24)$$

Substituting it back to (23), we obtain

$$\tilde{I}_k^n(t) = [\omega_{-k_n}(t) - \tilde{\eta}_{-k_n}^n]^+ + \tilde{\eta}_k^n, \quad \text{for } n \in \mathcal{T}_k. \quad (25)$$

which, together with the inductive hypothesis in (21), implies

$$\tilde{I}_k^n(L+1) \geq \tilde{I}_k^n(L), \quad \text{for } n \in \mathcal{T}_k. \quad (26)$$

Invoking Lemma 2 with (22) and (26) and using (15), we obtain $\boldsymbol{\omega}(L+2) \succeq \boldsymbol{\omega}(L+1)$, which completes the proof.

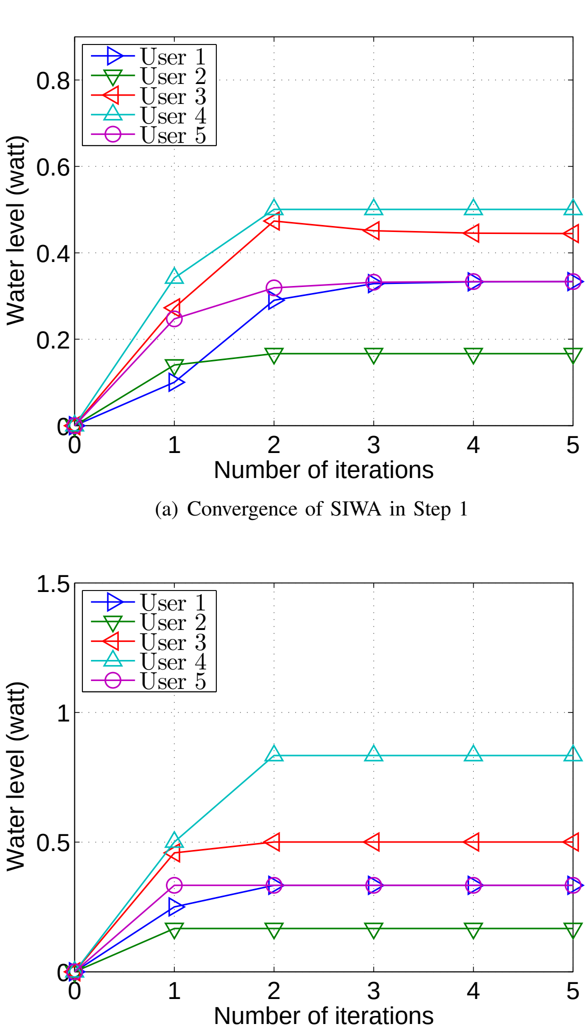
□

TABLE I
SIMULATION PARAMETERS

Parameters	Value
Cell radius	250 m
Minimum distance from user to BS	35 m
Path loss	$128.1 + 37.6\log_{10} d$ dB, d is in km
Shadowing	Log-normal, standard deviation 8 dB
Fading	Rayleigh fading with variance 1
Users distribution scheme	Randomly uniform distribution
Noise power spectral density	-174 dBm/Hz
Overall system bandwidth, W	5 MHz
Subcarrier number, N	5
Number of users, K	3 to 10
Throughput calculation	Shannon's capacity formula
Decay factor of FTPC	0.4
Number of power levels of LDA	11, ([0 W, 0.5 W], step by 0.05 W)
Power constraint for each user	0.5 W
T_1 in Step 1 of our method	1, 5
T_3 in Step 3 of our method	100
ϵ in Step 3 of our method	10^{-4}
Parameter M	1 (OMA), 2 (NOMA)

V. SIMULATION RESULTS

This section evaluates the performance of our subcarrier and power allocation algorithm. The radius R of the cell is set to 250 meters. Within the cell, there is one BS located at the center and K users uniformly distributed inside it. The system bandwidth W is assumed to be 5 MHz and $W_n = W/N$ for $n \in \mathcal{N}$, where $N = 5$. The noise power spectral density is assumed to be -174 dBm/Hz. In the propagation model, we consider the distance-dependent path loss, shadow fading and small-scale fading based on [21]. The path loss component is given by $128.1 + 37.6\log_{10} d$, in which d is the distance between the transmitter and the receiver in km. Lognormal shadowing has the standard deviation of 8 dB. For small-scale fading, each user experiences independent Rayleigh fading with variance 1.



(a) Convergence of SIWA in Step 1

(b) Convergence of SIWA in Step 3

Fig. 1. The convergence of SIWA, $K = 5$.

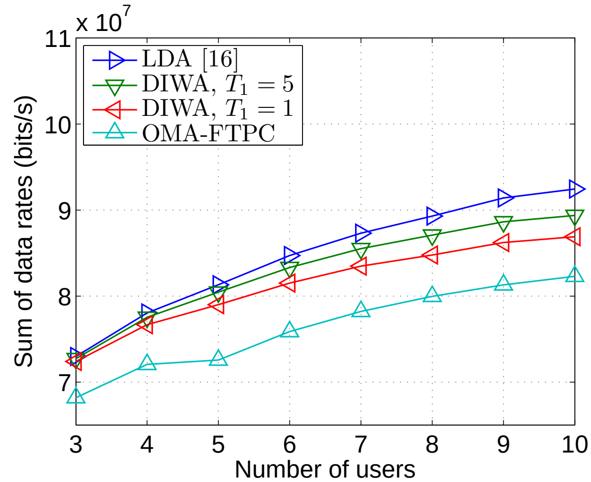


Fig. 2. Sum of data rates versus different number of users.

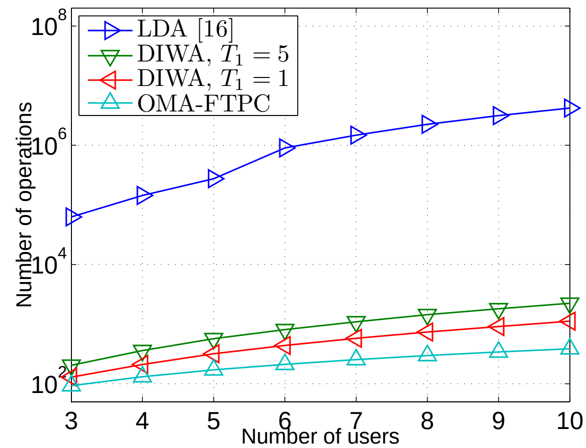


Fig. 3. Number of operations versus different number of users.

We compare the performance of our proposed DIWA with LDA [16] and orthogonal multiple access (OMA) with fractional transmit power control (FTPC), which is denoted by OMA-FTPC [1], [4], [5], [15]. The number of power levels of LDA is assumed to be 11 and the decay factor of FTPC is assumed to be 0.4; we will have the same settings. For our proposed resource allocation algorithm, in Step 1, the parameter T_1 is set to 1 or 5. Furthermore, in Step 3, we assume $T_3 = 100$ and $\epsilon = 10^{-4}$. The simulation parameters are summarized in Table I.

In the following, we consider three important system-level performance metrics: the convergence time of SIWA, the sum of data rates and the number of operations spent.

A. Convergence Time of SIWA

Fig. 1 shows the convergence of SIWA, where the number of users is equal to 5 (i.e., $K = 5$). We use the water level of each user during iterations to show this performance. Fig. 1(a) and Fig. 1(b) illustrate the convergence of SIWA in Step 1 and Step 3 of our proposed resource allocation algorithm, respectively. The x-axis indicates the number of iterations, while the y-axis depicts the water level of each user. It is observable that the SIWA in both Step 1 and Step 3 takes only a few iterations to converge. Additionally, it is worth pointing out that the water level of each user in Step 1 is not monotonous since the monotonicity is not guaranteed for $M > 2$. However, as expected that the water level of each user in Step 3 is monotonically increasing.

B. Sum of Data Rates

Fig. 2 shows the sum of data rates of the proposed DIWA, the aforementioned LDA and OMA-FTPC with different number of users. Each data point is obtained by averaging over 1,000 random instances. Clearly, the sum of data rates of each method will increase with the increasing number of users, K. As expected, DIWA with $T_1 = 5$ has higher system performance than that with $T_1 = 1$. Additionally, it is worth pointing out that DIWA with $T_1 = 5$ could achieve comparable sum rates to that of LDA, see for example, when $K = 10$, the proposed algorithm with $T_1 = 5$ only has a performance loss of 3.3% compared with LDA. Besides, OMA-FTPC has the worst system performance among all.

C. Number of Operations

Fig. 3 shows the number of operations required by different resource allocation algorithms. For each algorithm, we count the number of additions, multiplications, and comparisons used, which reflects the time complexity, as an estimation. Obviously, the number of operations for each algorithm will increase with the increasing of user number. In addition, we can see that OMA-FTPC requires the fewest operations. The proposed DIWA, with $T_1 = 1$ or $T_1 = 5$, requires slightly more operations than OMA-FTPC. However, both of them are much more time efficient than LDA especially when the number of users is high. For example, when $K = 10$, the number of operations required by DIWA with $T_1 = 1$ is less than 0.1% of that required by LDA.

VI. Conclusion

In this paper, we investigate the subcarrier and power allocation problem in single-cell MC-NOMA system. A heuristic, namely DIWA, is proposed to solve the sum of data rates maximization problem. For practical reasons, the number of users that can be multiplexed on a subcarrier is usually limited to a certain number. Such a constraint causes the optimization problem hard to solve. To circumvent this difficulty, we first relax this contraint and use SIWA to obtain an initial power allocation, which provides clue on the comparative advantage of the users in using a certain subcarrier. Based on this information, we assign subcarriers, respecting the previously relaxed constraint. Lastly, with the subcarrier allocation obtained in the previous step, SIWA is applied again to allocate powers. Analytical result guarantees that this last step always converges. By simulation, we show that our proposed resource allocation strategy could achieve comparable data rates performance to LDA but is much more time efficient. Future work includes solving the subcarrier and power allocation problem for multi-cell MC-NOMA systems and other scenarios.

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