Conformal hexahedral mesh coarsening by agglomeration

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1 Introduction

Reservoir simulation involves to compute dynamic flow of different phases in a porous medium. The initial state of the reservoir is usually precomputed via geo-statistics methods, extrapolating measures of the terrain. so, the input of reservoir simulation is given as a fine mesh containing heterogeneous data. However, due to the complexity of the system of partial differential equations (PDE) governing the flow dynamic, the use of this fine mesh is impracticable. Nowadays, industrial software uses upscaled model [1] a coarse mesh is generated homogenizing fine properties from the fine mesh.

In this paper, we will describe an agglomeration strategy, to dynamically coarsen the fine mesh without loss of the fine properties. The adaptivity may be driven by physics and/or geometric estimators. Ideally, the coarsening should be applied locally in low gradient regions, whereas high gradient regions keep the fine mesh.

After recalling connectivity of hexahedral meshes, we detail the spatial twist components,[3],[5],[4]. These components form an arrangement of mesh connectivity. They also form meshes of sub dimension, denoted paths and leaves. Moreover set of leaves and paths are linked each other forming a graph object. The coarsening algorithm we use begin by a coloring of the mesh to extract its leave and path structure of the mesh. This structure gives coarse hexahedral decompositions of the mesh that easily identify leaves agglomeration that preserve the conformity of the hexahedral mesh. We illustrate the use of this algorithm on a simulation platform, [2], on a several meshes including an example of mesh used for reservoir simulations.

2 Extracting hexahedral structure

2.1 Cubes and connectivity

Hexahedron belongs to the more extended class of polytopes, the k-cubes with k denotes the dimension. The sequence of k-cubes is built from a single node, by a recursive transformation named extrusion. It follows an interesting property : all polytopes of co-dimension l, induced in a n-cube is a (k-l)-cube;

Another property of n-cubes concerns symmetry. The boundary graph, formed by the sets of (k-1)-cubes and (k-2)-cubes is a complete-n-partite graph, with each independent set having exactly two (k-1)-cubes. Then we denote $\underset{k-1}{\sim}$, the symmetry through a k-cube. Thus for each independent set of (k-1)-cubes, $\{x, y\}$, it follows $x \underset{k-1}{\sim} y$.

Now we consider $\mathcal{M}$ as an hexahedral mesh, with $\mathcal{B}_3$, $\mathcal{B}_2$, $\mathcal{B}_1$ and $\mathcal{B}_0$, form sets of hexahedrons, quads, edges and nodes. Local binary relations over hexaedrons, $\underset{2}{\sim}$, over faces, $\underset{1}{\sim}$, and over edges $\underset{0}{\sim}$ can be gathered into sets. Considering these binary relation as symmetric, reflexive and transitive, then each defines equivalence classes partitioning the sets of faces, edges or nodes.

Relations $\underset{1}{\sim}$ and $\underset{2}{\sim}$ are used to define components of the specific components of a hexaedral mesh, as detailed in [3]. $[x]_1 = \{y \in \mathcal{B}_1 | x \underset{1}{\sim} y\}$ is the class of edges whose x belongs; and $[x]_2 = \{y \in \mathcal{B}_2 | x \underset{2}{\sim} y\}$ is the class of faces whose x belongs.

2.2 Paths an leaves

Equivalence relations are used to decompose dual graph into paths of the mesh. Since the mesh has a conformal connectivity, each internal quads has exactly two neighboring hexahedrons and defines an edge of the dual graph. In contrast boundary quads has only one neighboring hexahedrons. It follows that $[x]_2$ forms a finite or sequence of internal faces linked by relation $\underset{2}{\sim}$ in a hexahedron.

As result the sequence of internal faces define a path included in the dual graph of the mesh. If no there is no boundary faces in the sequence, the path is closed. Otherwise, two boundary faces belongs to $[x]_2$ and the path is opened. Paths may be self intersected, but do not occur in our study case.

The classes $[x]_2$ are parsed by component, in the meantime paths are defined as list of hexahedrons. In case of opened paths, boundary faces are stored as extra information. When components have been parsed, a set of paths, $\mathcal{P}$, decomposes the dual graph of the mesh and such any hexahedron belongs to three paths.

Same way, the dual graph of the boundary mesh into boundary paths. But $\underset{1}{\sim}$ is used only for boundary elements. As result $[x]_1$ forms a finite sequence of

boundary faces linked by relation $\underset{1}{\sim}$ in boundary quads. Each boundary path is closed and defineed as a circular list of boundary faces. The set of boundary path decomposes the dual graph of the boundary mesh such any boundary faces belongs to two boundary paths.

Leaves are subgraph form with faces incident to class of internal edges. It involves that each path belong to two leaves and each hexaedron belongs to three leaves. So subgraph is defined as a union of paths such intersection of leaves is a path.

To make the union, open paths are linked to their extra boundary via the boundary faces. Then the open paths are gathered in a leave bounded by a single boundary path. However if it appears that two independent boundary paths are linked to a same open path, they are match to form a leaves bounded by two boundary paths. When all open path belongs to two leaves. We may add closed path. Finally, if it lefts closed paths connected to a single leave, closed leaved are formed.

2.3 Coloring

Leaves and paths are defined such that for all paths p in $\mathcal{P}$, it exists exactly two leaves, l_1 and l_2 in $\mathcal{L}$ and $l_1 \cap l_2 = p$. Basically two intersecting leaves cannot be agglomerated. Partitioning the set of leaves is useful to identify a maximum of leaves that can be agglomerated. So we define the graph of intersection of leaves, $\mathcal{G}$, whose vertices are leaves and of edges are defined by their intersections. Then, $\mathcal{L}$ is partitioned into subsets of non-intersecting leaves, by graph coloring.

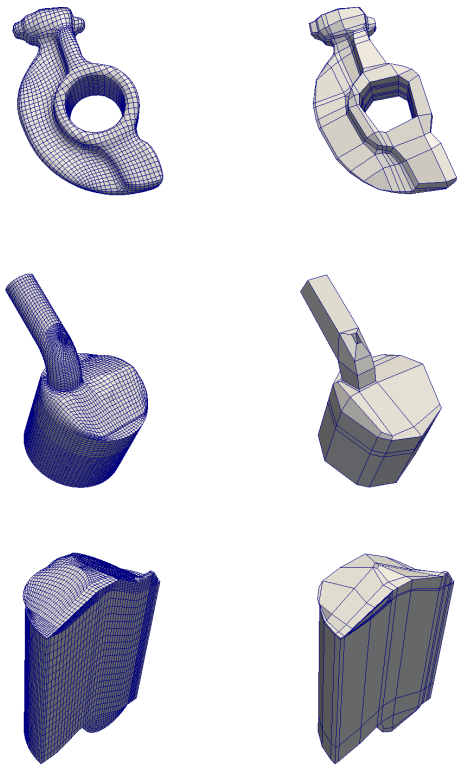
Optimal graph coloring is a NP-complete problem as soon as $\chi(\mathcal{G}) > 2$. Nevertheless there are heuristic algorithms, as the greedy coloring, that color a graph in polynomial time. Greedy coloring, is an iterative procedure considering vertices in specific order and and mapping it to the smallest color not used by its neighbours.

We test classic ordering, BFS and decreasing degrees. As we know that $\chi(\mathcal{G}) \geq 3$, if greedy coloring involves only 3 colors, such coloring is optimal. An immediate consequence is that it determines that the mesh is globally structured. However, even for structured meshes the greedy algorithm may be non-optimal. It appears in general when domain has concave shape. Nevertheless the supplementary colors appears only with singularities on boundary mesh.

3 Coarsening

3.1 Conformal agglomerations

Our coarsening algorithm is based on atomic agglomerations such two neighboring hexaedrons form a coarser hexaedron by removing their common faces.



self-intersect	closed p.	2-boundary l.	closed l.	colors	fine mesh	coarsened
no	no	yes	no	3	7874	322
no	yes	yes	no	5	55789	78
no	yes	yes	yes	6	35869	270

Fig. 1. Coarsest hexahedral decomposition, perfomed by successive conformal leave agglomeration. Table presents few properties of paths, leaves and coloring. and number of hexaedrons.

The opposite quads to the one removed are preserved whereas others are agglomerate by two, forming coarser quads. And new coarse edges are also formed with two fine edge.

However to preserve a conformal hexahedral mesh, atomic agglomeration must be applied on two matching leaves. This means that each hexahedron has a unique neighbour in the other leave. It involves that two matching leaves intersect the same leaves. For this reason they must have been colored identically.

3.2 Dynamic agglomeration

Algorithms iterate on the the color to search agglomeration in all directions. To identify matching leaves of a given color, algorithm enumerate faces of the mesh. Faces shared by hexadrons of distinct leaves but same color are count. In the meantime algorithm can evaluate local coarsening criteria. The all the conformal agglomeration are identified and local criteria is reduced by leaves. The chosen leave agglomerations involves a set of atomic agglomeration. Mesh is modified, but coarse element update links with the fine element.

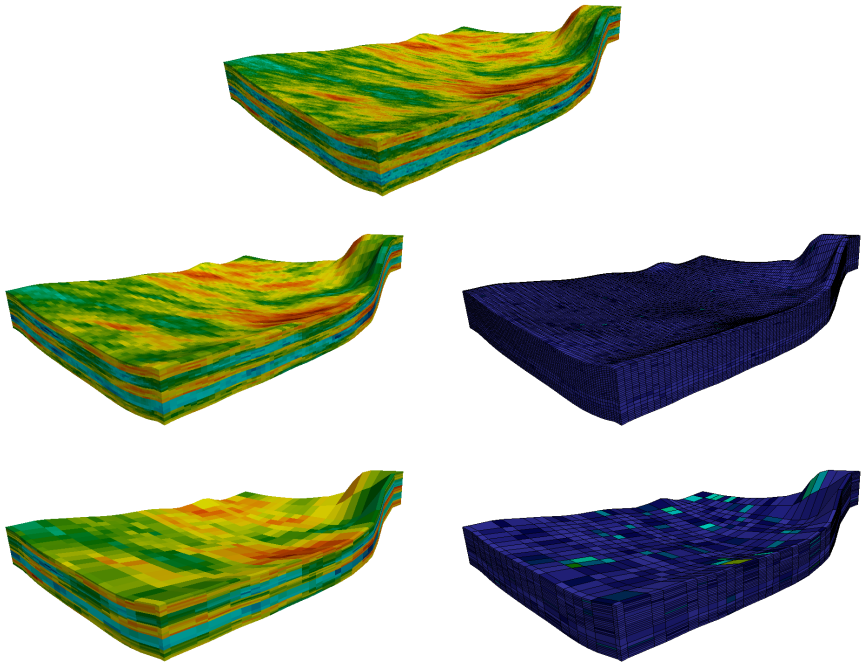


Fig. 2. Porosity defined on a coarse mesh and standard deviation between porosity defined with fine cell porosity (number of cells : $\approx 10^6, \approx 10^5$ and 10^4)

4 Conclusion and perspectives

In petroleum engineering reservoir simulations use upscaled models where a coarse mesh is generated from a very fine one gathering all the petro-physical parameters of the terrain. In this note, we have described a coarsening algorithm for hexaedral meshes that allows by a quick reference to this parent

fine mesh to recover easily these informations. This algorithm is based on the notions of paths and leaves. After having generated leaves decomposing the mesh, a coloring algorithm is used to identify non-intersecting leaves. Then agglomeration of leaves can be performed. This algorithm have been applied to several examples of meshes including meshes used for reservoir simulations. We plan to extend our work into two complementary directions. The first one will be to study the control of the agglomeration of leaves and hexahedron by the use of local error estimates coming from the solution of the governing PDE. Another work direction will be to extend the algorithm to hex-dominant meshes (CPG meshes) that contains degenerated hexaedron by allowing the algorithm to generate semi-conforming meshes.

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