Game Theory Based Multi-attribute Negotiation between MA and MSAs

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Abstract. This paper focuses on the multi-attribute negotiation between Manufacture Agent (MA) and Material Supplier Agent (MSA) of supply chain network (SCN). A modified two-stage negotiation protocol is proposed based on the two-stage negotiation protocol proposed in the previous work. The negotiation between MA and MSAs, where the quantity of the order of MA depends on the demand of Consumer Agent (CA), are discussed to decide the final supplier and the final strategies. The strategies of the negotiation are the wholesale price of the product, the quantity of the order, and the lead time. The final solution is solved by finding the Stackelberg equilibrium of MA-Stackelberg game. Numerical case is provided to illustrate the proposed protocol.

Keywords: Multi-agent, supply chain, negotiation, game theory

1 Introduction

A Supply Chain (SC) can be defined as a system consists of suppliers, manufacturers, distributors, retailers, and customers, where materials flow downstream from suppliers to customers and information flows in both directions [1]. Game theory is a powerful tool for analyzing situations in which the decisions of multiple agents affect each agent's payoff [2]. It has become a primary methodology used in supply chain network (SCN) related problems.

Related to the topic, a lot of researches have been done. *Hall et al.* [3] modeled the manufacturer's capacity allocation problem of the SC with a manufacturer and several distributors. [4] considered a profit-maximizing retailer using Stackelberg game and Nash equilibrium. A game theoretic model of a three-stage supply chain consisting of one retailer, one manufacturer and one subcontractor was developed to study ordering, wholesale pricing and lead-time decisions in [5]. *Sinha et al.*[6] analyzed the coordination and competition issues in a two-stage SC distribution system where two vendors compete to sell differentiated products through a common retailer in the same market. *Xia* [7] studied the market competition and pricing strategies for suppliers in a SC with two competitive suppliers and multiple buyers. In the previous research, the single attribute negotiation between one Manufacture Agent (MA) and multiple Material Supplier Agents (MSA) has been discussed [8]. It was assumed that the quantity of the order of MA was fixed. However, in the real market, the quantity of MA is not fixed and it related to the demand of Consumer Agent (CA). Thus, this research extends to the multi-attribute negotiation between one MA and multi-MSA, where the demand is based on the selling price of MA. In particular we focus on the situation of MA monopolies or oligopolies, as it is often the case in the trading business.

This paper is organized as follows: section 2 describes negotiation model and settings used in this research; section 3 describes the modified two-stage negotiation protocol; numerical example and analysis are given in section 4. In conclusion, the contributions and the directions of the future work are commented.

2 Model

All the used notations are shown as follows:

$\alpha_{j'}^{Max}$	maximum percentage of profit of MSA j'		
β^{Max}	maximum percentage profit of MA		
$\delta_{LT,w}^{M}$	concession function of lead time of MA		
$\delta_{Q,w}^{M}$	concession function of quantity of MA		
$\delta_{w,LT}^{M}$	concession function of wholesale price of MA		
$\delta_{w,Q}^{S}$	concession function of wholesale price of SF_j		
$\delta_{w,LT}^{S}$	concession function of wholesale price of SF_j		
γ_j	productivity of SF_j	$\pi_j[k]$	profit of MA at k
$\pi_j^S[k]$	profit of SF_j at k	$\pi_j^{SM}[k]$	profit of SF_j takes strategy of MA at k
AC_j	combined ability of SF_j	PCA_j	maximum price of SF_j
cf	fixed cost per order of MA	PCI_j	minimum price of SF_j
cp_j	unit production cost of SF_j	PMA	maximum price of MA
cs_j	set-up cost per order of SF_j	$ps_j[k]$	selling price of MA at k
cst	shortage cost of MA	$Q_j^S[k]$	quantity of SF_j at k
$D_j[k]$	demand of CA at k	$Q_j^M[k]$	quantity of MA at k
f_Q	function of quantity of SF_j	sv	salvage value of unsold product of MA
h^M	holding cost of MA	TN	deadline of the negotiation
h^S	holding cost of SF_j	TS	time of each negotiation round
$LT_j^M[k]$	lead time of MA at k	$w_j^M[k]$	wholesale price of MA at k
$LT_j^S[k]$	lead time of SF_j at k	$w_j^S[k]$	wholesale price of SF_j at k

This research considers the multi-attribute negotiation between one MA and multiple MSAs, where MA should determine the quantity of his order based on the demand of CA. The negotiations between one MA and multiple MSAs, where the order requested by MA is too large for MSA to complete independently, have been discussed in the previous work[8]. It tried to find another way to resolve this problem which can maintain the integrity of the order. It assumed that the MSAs accepted only the orders which were able to fulfill independently, and tried to combine with the other MSAs as a coalition when the order was out of their abilities. Furthermore, it assumed that the quantity of the order of MA was fixed. However, in the real market, it must be related to the demand of CA. Thus, this research tries to extend the negotiation under fixed demand to the negotiation under the situation where the demand depends on the selling price of MA. Three main attributes considered in this research are the wholesale price of the product,the quantity of the order, and the lead time of the order. MA negotiates with the MSAs to reach agreements on the strategies of the three attributes. We assume that the demand of CA is in an additive form as (1), where $a, b > 0$. It depends on the selling price of MA. $ps_j[k]$ is the selling price of MA of the negotiation between SF_j at k.

$$D_j[k] = a - bps_j[k]. \tag{1}$$

3 Negotiation Protocol

This research proposes a modified two-stage negotiation protocol based on the two-stage negotiation protocol proposed in [8]:

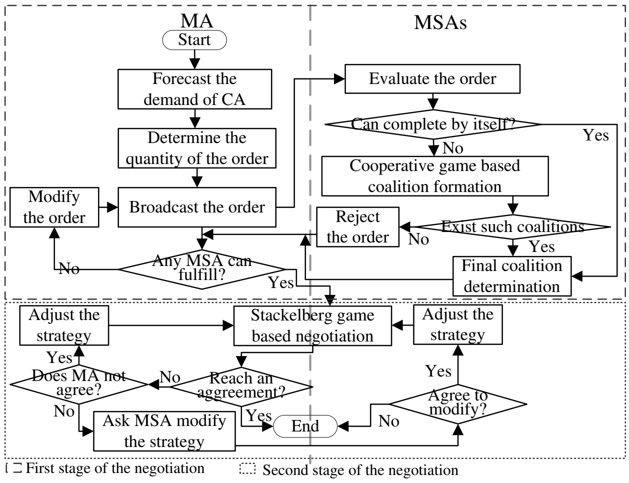

Fig. 1. Flowchart of modified two-stage negotiation protocol

Stage1: Negotiation among MSAs

- *Step 1:* MA forecasts the demand of CA, determines the initial price, quantity and lead time of the order which he wants to place, and then broadcasts the order to all the MSAs.
- *Step 2:* MSAs evaluate the order and check whether the order can be finished by themselves. If they can do it, themselves will be determined as the final coalition SF_j and fed back to MA; if they cannot do it, then they can negotiate with the other MSAs to build a coalition SF_j and feed back to MA. A cooperative game is used for coalition formation.
- *Step 3:* MA checks whether there exists any SF_j can fulfill the order, if there exists, goes to the second stage, if there does not exist, then MA modifies the order and re-broadcasts the order.

Stage2: Negotiation between MA and SF_j

- *Step 4:* MA starts to negotiate with SF_j. MA-Stackelberg game is introduced to find the final solution.
- *Step 5:* MA checks whether an agreement is reached, if the agreement is reached, negotiation ends, if the agreement is not reached, then MA checks whether he agrees with the strategies or not. If he agrees with it, he asks the MSA to modify his strategy, if he does not agree with it, then he adjusts his own strategy and gives a response.

– *Step 6:* MSA checks whether he agrees to modify his strategy or not. If he agrees to modify, he adjusts his own strategy, if he does not agree, then the negotiation ends and fails to find an agreement.

The flowchart of the modified two-stage negotiation is shown in Fig. 1. The processes in the left-hand side of the gray dash line are done by MA, and the processes in the right-hand side are done by MSA. The details of the negotiation protocol will be discussed in the following subsections.

3.1 Negotiation among MSAs

The negotiation among MSAs aims to find partners to establish a coalition when the order of MA is profitable but out of their abilities. It has been discussed in details in [8, 9]. The final coalition SF_j of MSA j will be determined after the negotiation.

3.2 Negotiation between MA and SF_j

MA and SF_j make their decisions sequentially in this part. They negotiate on the wholesale price, the quantity of the order, and the lead time. SF_j wants to increase the wholesale price, quantity and lead time to improve his profit. However, MA wants to decrease the wholesale price to increase his profit. Therefore, the main point of this part is to find a balance between the profits of SF_j and MA.

The negotiation between MA and SF_j can be modeled as a Stackelberg game, where MA is indicated as a leader and SF_j can be seen as a follower. In the formulation of a Stackelberg game, it is usually assumed that players' profit functions are common knowledge [10]. Thus, MA can use this common knowledge to construct SF_j's most profitable responses to all the possible decisions. Then, MA's profit function from corresponding decisions based on all the SF_j's responses can be explored. In the end, MA makes a decision which can maximize his profit. SF_j then has a best response to matching the decision. We assume that both MA and SF_j are rational, thus, they do not want to deviate from this set of decisions, which is called Stackelberg equilibrium in the game theory. The strategies of MA and SF_j are determined in the following sections.

Determination of the strategies of MA From the analysis above, we can see that MA offers his strategies from low to high. The strategies of MA for SF_j at round k are ($Q_j^M[k]$, $w_j^M[k]$, $LT_j^M[k]$), where:

$$w_j^M[k] = w_j^M[k-1] + \frac{PMA - w_j^M[k-1]}{(TN - kTS)/TS} + \delta_{w,LT}^M(LT_j^S[k-1]) \tag{2}$$

$$\delta_{w,LT}^M(LT_j^S[k-1]) = \begin{cases} \theta_z^{wLT}, \text{ if } LT_{ij}^S[k-1] \leq x_1^{LT} \\ \theta_{z-1}^{wLT}, \text{ if } x_{z-1}^{LT} < LT_{ij}^S[k-1] < x_z^{LT} \\ 0, \qquad \text{if } LT_{ij}^S[k-1] \geq x_z^{LT} \end{cases} \tag{3}$$

$$Q_j^M[k] = D_j[k] + \delta_{Q,w}^M(w_j^S[k-1]) \tag{4}$$

$$LT_j^M[k] = LT_j^M[k-1] + \delta_{LT,w}^M(w_j^S[k-1]) \tag{5}$$

$$ps_j[k] = w_j^M[k](1 + \beta^{Max}). \tag{6}$$

At each round of negotiation, MA should determine strategies of all the three attributes. MA determines the value of the first attribute according to his preferential choice and we assume that MA in this research firstly determines his price. MA not only considers the concession which related to the remnant negotiation time at round k (the second item of (2)), but also takes the strategies of SF_j at $k-1$ into account (the third item of (2)). He can increase certain amount of price if SF_j can offer the product in shorter time. The quantity $Q_j^M[k]$ of MA depends on the demand of CA(the first item of (4)) and the increment according to the price of SF_j (the second item of (4)). If SF_j can offer lower price, MA will increase the total amount to buy. The lead time $LT_j^M[k]$ of MA related to the price of SF_j (the second item of (5)). If SF_j can offer lower price, MA will extend the lead time of his order. $\delta_{w,LT}^M$ is a piece-wise function as (3), it means when $LT_j^S[k-1]$ belongs to the range (x_{z-1}^{LT}, x_z^{LT}), MA will give a concession of the price at the value of θ_{z-1}^{wLT}. All the threshold values x_{z-1}^{LT} of each segment and the related mapping values θ_{z-1}^{wLT} are determined by himself. $\delta_{Q,w}^M$ and $\delta_{LT,w}^M$ have the same form as (3).

Determination of the strategies of SF_j Each SF_j tries to determine his strategies to maximize his profit. The higher value of the three attributes, the better. Thus, he replies his strategies from high to low. However, it is better to reach an agreement than does not reach an agreement. Thus, SF_j should also take the strategies of MA into account and then decide his strategies. The strategies of SF_j are defined as follows:

$$w_j^S[k] = w_j^S[k-1] - \frac{w_j^S[k-1] - PCI_j}{(TN - kTS)/TS} - \delta_{w,Q}^S(Q_j^M[k]) - \delta_{w,LT}^S(LT_j^M[k]) \quad (7)$$

$$Q_j^S[k] = f_Q(w_j^S[k]) \quad (8)$$

$$LT_j^S[k] = Q_j^S[k]/\gamma_j \quad (9)$$

$$f_Q(w_j^S[k]) = \begin{cases} \theta_Q^{max}, \text{ if } w_j^S[k] \le PCI_j \\ \theta_Q[k], \text{ if } PCI_j < w_j^S[k] < PCA_j \\ \theta_Q^{min}, \text{ if } w_j^S[k] \ge PCA_j \end{cases} \quad (10)$$

$$\theta_Q[k] = AC_j - \frac{[\frac{cs_j}{w_j^S[0]-cp_i} - AC_j]PCI_j}{PCA_j - PCI_j} + w_j^S[k]\frac{\frac{cs_j}{w_j^S[0]-cp_j} - AC_j}{PCA_j - PCI_j}. \quad (11)$$

SF_j firstly decides his price according to the strategies of MA. (7) means SF_j will give a discount if MA can increase the quantity of his order (the third item of (7)) or extend the lead time (the last item of (7)). The quantity (8) and lead time(9) of SF_j strongly depends on his productivity. Thus, all the three attributes constraint each other. $\delta_{w,Q}^S(Q_j^M[k])$ and $\delta_{w,LT}^S(LT_j^M[k])$ are also piecewise functions as (3) but has opposite tendency (threshold and related mapping values). $\theta_Q[k]$ is used to determine the minimum quantity to ensure the order will be profitable (for each order, minimum setup cost is needed).

Determination of the final equilibrium MA has his own preferences for wholesale price, quantity and lead time and he is looking for the offer that best satisfies these preferences. Both MA and SF_j want to maximize their profits by choosing their preferred strategies. The profits of MA at k, where he agrees with the strategy of SF_j, is defined as:

$$\pi_j^M\ [k] = ps_j[k]D_j[k] + sgn(Q_j^M[k] - D_j[k])(Q_j^M[k] - D_j[k])sv - sgn(D_j[k] - Q_j^M[k])(D_j[k] - Q_j^M[k])cst - w_j^M[k]Q_j^M[k] - \frac{cfD_j[k]}{Q_j^M[k]} - \frac{hQ_j^M[k]}{2} \quad (12)$$

where $sgn(x)$ equals to 1, if $x > 0$, otherwise it equals to 0.

We can see that in this model MA is the leader and he has more decision power. Thus, the objective of the Stackelberg game is to find the equilibrium which can maximize the profit of MA. It can be transformed into finding the agreements on the strategies of the three attributes and so to maximize the profit of MA. However, the strategies must be accepted by SF_j. Thus, the equilibrium of the negotiation between MA and SF_j can be determined as the strategies ($Q_j^M[k]$, $w_j^M[k]$, $LT_j^M[k]$) of MA at k. Moreover, these final strategies must meet the following conditions:

$$\max\ \{\pi_j^M[k]\} \quad (13)$$

$$\textbf{s.t.}\ \ \pi_j^{SM}[k] \geq \pi_j^S[k], \text{if } k < TN\text{-}1 \quad (14)$$

$$\pi_j^{SM}[k] > 0, \text{if } k{=}TN\text{-}1 \quad (15)$$

$$Q_j^M[k] \leq AC_j[k] \quad (16)$$

where (14)-(15) are used for SF_j to evaluate the acceptability of the strategies, and these mean that the order must be profitable for SF_j. (16) means the order must be in ability of SF_j. $\pi_j^{SM}[k]$ is the profit of SF_j adopts the strategies of MA at k (as (2)-(5)), $\pi_j^S[k]$ is the profit of SF_j adopts his own strategies at k (as (7)-(9)). The agreement can be reached only if the profit of taking the strategies of MA is greater than the one of taking his own strategies. Finally, MA decides the final supplier which can maximize his profit based on the equilibrium acquired from (13)-(16).

The characteristics of the proposed protocol are:

- The agreement can be reached as long as (14)-(16) are satisfied, no matter MA cannot reach an agreement with SF_j on the wholesale price or not.
- The agreement may not be reached even MA has reached an agreement with SF_j on the wholesale price.
- The attributes may not be monotone changing.

4 Numerical case and analysis

A numerical case is provided to illustrate the processes of the multi-attribute negotiation and how does the negotiation find the final equilibrium. Assume there is one MA and 5 MSA in SCN and all the used parameters are defined as Table 1.

Firstly, we discuss about the details of the multi-attribute negotiation between one MA and one SF_j. We take the negotiation between MA and coalition

Table 1. Parameter settings and threshold values of the concession functions

MSA (J=5)		**MA**		
$\gamma_j \sim$U(100,300)	$cp_j \sim$U(7,8)	α^{max}=0.5	$h^M = 3$	$cst^M = 5$
β_j^{min}=0.2	β_j^{max}=0.5	α^{min}=0.3	$psIn_i^M \sim$U(13,14)	$cf^M = 100$
h_j^S=3	$cs_j \sim$U(200,300)	$sv^M = 2$	$a \sim$U(1000,2000)	$b \sim$U(0,100)

{12} as an example. The fluctuations of three attributes are shown as Fig. 2. We can see that:

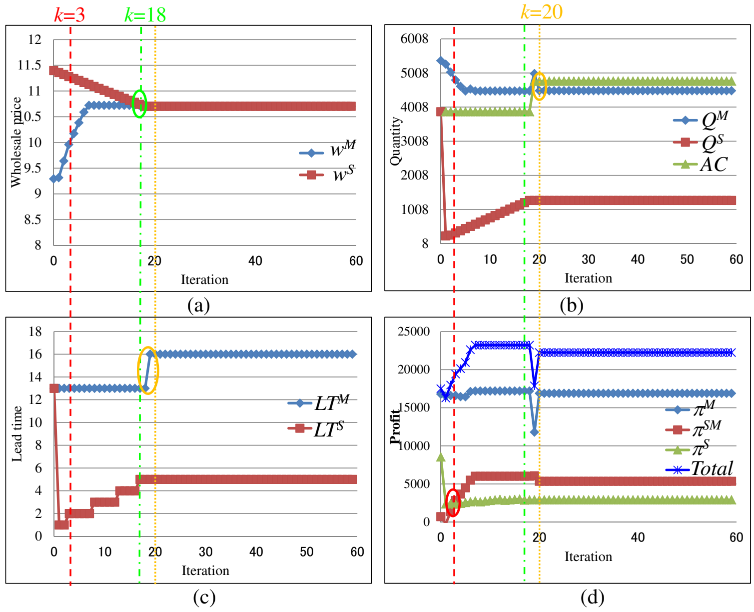

Fig. 2. The fluctuation of the multi-attribute negotiation

- At $k = 3$, the profit of {12} takes the strategies of MA is greater than takes his own strategies (see the area marked by ellipse in Fig. 2(d), where $\pi^{SM}[3] > \pi^S[3]$), that means (14) is satisfied. However, the quantity of MA is also greater than the ability of {12} (see the quantity at k=3 of Fig. 2(b), where $Q^M[3] > AC[3]$), that means (16) is not satisfied. Therefore, the agreement is not reached and the negotiation goes by.
- At $k = 18$, MA reaches an agreement with {12} on the wholesale price (see the area marked by ellipse in Fig. 2(a)) and (14) is satisfied. However, (16) is still not satisfied because $Q^M[18] > AC[18]$ from Fig. 2(b). Therefore, the agreement is not reached and the negotiation goes by.
- At $k = 20$, the wholesale price of MA keeps unchanging, but he makes a concession of his lead time (see the area marked by ellipse in Fig. 2(c)) and then (16) is satisfied (see the area marked by ellipse in Fig. 2(b), where $Q^M[20] < AC[20]$). Therefore, both constraints (14) and (16) are satisfied.

Then we can get final equilibrium between MA and{12} is the strategies of MA at $k = 20$ where the strategies are (10.704, 4489, 16). What we should pay attention to are: 1) The equilibrium not always exists; 2) The order MA may become out of ability of SF_j even it was in ability of the first time.

Similarly, we can get all the equilibriums and then MA decides the final supplier which can maximize his profit. In this case, the final supplier for MA is $\{32\}$ with the final strategies (10.703, 6735, 16) and the profit equals to 25501.295.

5 Conclusion

The multi-attribute negotiation between MA and MSA, where the quantity of the order of MA depends on the demand of CA, was discussed in this paper. A modified two-stage negotiation protocol was proposed. MA-Stackelberg game was introduced to decide the final strategies by finding the Stackelberg equilibrium. A different criterion was proposed for MSA to decide whether accept the order or not. The strategies of MA at k was defined as final equilibrium if the constraints are satisfied. In this research, we only provided a method to find the equilibrium of multi-attribute negotiation. However, it cannot ensure the equilibrium always exists. There still exists the situation where the negotiation cannot find the equilibrium. For future work, we will take the dynamic coalition formation into account to improve the success rate of finding the equilibrium. Moreover, the performance of the proposed protocol will be discussed, including what effects will be caused by changing the parameter, what's the effect of the ability of MSA on the formation of the coalition.

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