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# Minimal Noise Variance Decoder for Uncoordinated Multiple Access in VLC

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**Abstract**—In a visible light communications system (VLC), light sources are responsible for both illumination, communications and positioning. These light sources inevitably interfere each others at the receiver. To retain the appealing advantage that VLC systems can reuse existing lighting infrastructure, using an extra network to control or synchronize the light sources should be avoided. This paper proposes an uncoordinated multiple access scheme for VLC systems with positioning capability. The proposed scheme does not require a central unit to coordinate the transmission of the transmitters. Transmitters can be asynchronous with one another and with the receiver. Each transmitter is allocated a unique codeword with  $L$  chips for a system with up to  $\frac{L-1}{2}$  transmitters where  $L$  is prime. Due to the linear growth in complexity with respect to number of transmitters, our proposed scheme is feasible for systems with large numbers of transmitters. Our novel decoder can minimize the effect of additive Gaussian noise at the receiver side. Simulation results show that the proposed decoder outperforms zero-forcing decoder.

## I. INTRODUCTION

Visible light communications (VLC) has recently gained a lot attention as a promising future technology to overcome the scarcity of the radio frequency spectrum. Together with having several appealing features, VLC is usable for illumination, communications and positioning. It uses visible light for transmission generated by light emitting diodes (LEDs) as transmitters and detected by photodiodes (PDs) as receivers.

For broadcasting information using VLC, many different schemes are proposed in literature such as expurgated pulse position modulation [1], adaptive modulation based on power control [2] and optical code division multiple access (OCDMA) [3]. All these techniques require a central unit to coordinate the transmission of the LEDs.

Visible light positioning systems can achieve accurate positioning. [4, 5] present two positioning systems based on the received light intensity, where the receiver needs to distinguish the light intensity received from each LED. To achieve that in [4, 5], time division multiple access (TDMA) was used, which also requires a central unit to coordinate the transmission of the LEDs.

To reduce system complexity and cost, it is necessary to develop uncoordinated multiple access (UMA) schemes that work without the need for a central unit nor the extra

infrastructure used for controlling the LEDs. Such schemes can simply be installed by replacing the lights by LED transmitters. Thus, the installation cost is reduced as well. However, implementing UMA schemes needs to solve the following challenges. 1) Due to the possibility of using the receiver at arbitrarily locations with arbitrarily orientations, the transmitted signal suffers random channel gain that is unknown to the receiver. 2) LEDs may be controlled by different switches so that they may be switched on at different times. As a results, transmitters may be asynchronous with one another and with the receiver. 3) There is no communication link among the transmitters.

Recently, some UMA schemes usable only for positioning are proposed [6, 7]. In [7], a unique sinusoidal-based code of length  $2N$  is allocated to each transmitter for a system with  $N$  transmitters. The receiver eliminates multiple access interference (MAI) entirely by applying FFT of length  $2N$ . Moreover, its scheme delay, i.e., the minimum received signal duration needed by the receiver to estimate the average received powers from the transmitters, grows linearly with number of transmitters. However, it does not support communications. An UMA scheme usable for communication and positioning is proposed in [8, 9]. In this scheme, each transmitter is assigned a unique code, however the length of the code grows exponentially with the number of transmitters. As a result, the scheme delay grows exponentially with the number of transmitters. In [10], asynchronous OCDMA is proposed for communications. To obtain a probability of error below  $10^{-3}$  for a system with 20 transmitters, the code length  $L \geq 150$  chips is needed so that the system throughput  $\leq 0.133$  bits/chip. Moreover, if  $k$  bits are wanted to be transmitted in each symbol, then  $2^k$  different codes are allocated to each transmitter that increases the system complexity.

In this paper, we modify the sinusoidal-based codes proposed in [7] and generalize them to further support information broadcast. We also propose a new decoder for our uncoordinated multiple access scheme with sinusoidal-based codes (UMA-SC) and compare it with zero-forcing decoder (ZFD). The proposed decoder not only eliminates MAI totally but also minimizes the noise variance. Our proposed scheme supports communications and positioning and does not need a central unit. Each transmitter is assigned a unique code with

$L$  chips for a system with up to  $\frac{L-1}{2}$  transmitters where  $L$  is prime. Transmitters can transmit more than one bit per symbol without the need for extra codes, which reduces the system complexity.

This paper is organised as follows. Section II presents the overview of the proposed system. The proposed scheme is detailed in Section III. Simulation results are given in Section IV, and the paper is concluded in Section V.

*Notation:*  $S(\mathbf{x}, \mu)$  means the cyclic shift of the vector  $\mathbf{x}$  to the left by  $\mu$  steps and  $\bar{S}(\mathbf{x}, \mu, m)$  means the subset of the vector  $S(\mathbf{x}, \mu)$  from the index 0 to  $m-1$ . For example, if  $\mathbf{x} = (x_0, x_1, x_2)$ , then  $S(\mathbf{x}, 1) = (x_1, x_2, x_0)$  and  $\bar{S}(\mathbf{x}, 1, 2) = (x_1, x_2)$ . All the estimates of variables denoted by the symbols of the original variables together with a hat. For example, the estimate of  $P_i$  is denoted by  $\hat{P}_i$ .

## II. SYSTEM MODEL AND PROBLEM FORMULATION

### A. System Model

In our proposed system, each transmitter has a single LED. We assume that  $N$  LEDs are installed in a space. The receiver is a mobile device equipped with a PD, and it may appear at random location. The receiver's position can be estimated by any positioning algorithm based on the received light intensity. Furthermore, we assume that each LED broadcasts different data information, and this information includes the location information of each LED that is required by the positioning algorithm. The message can also depend on the system application. We assume that there is no central unit to coordinate the transmission of the LEDs, and the  $N$  LEDs may be switched on and start transmission at different times. All LEDs transmit simultaneously and they interfere the others.

### B. Channel Model

Suppose the distance between LED  $i$  in a transmitter and a PD in a receiver is  $d_i$  for  $1 \leq i \leq N$ . If  $\phi_i$  is the irradiance angle with respect to the LED  $i$ 's normal and  $\psi$  is the incidence angle with respect to the PD's normal, then the channel gain of a LoS optical wireless channel [11] between Transmitter  $i$  and a PD is given by

$$h_i = \frac{(l+1)A}{2\pi d_i^2} \cos^l(\phi_i) T(\psi) g(\psi) \cos^M(\psi), \quad (1)$$

where the parameters are explained as follows. The Lambertian parameters of the LED and PD are given by  $l = \frac{-\log 2}{\log(\cos(\phi_{1/2}))}$  and  $M = \frac{-\log 2}{\log(\cos(\psi_{1/2}))}$ , where  $\phi_{1/2}$  is the half-power angle of irradiance of an LED and  $\psi_{1/2}$  is the half-power angle of incidence of a PD. The effective area of the PD at the receiver is given by  $A$ . The filter gain and concentrator gain are represented by  $T(\psi)$  and  $g(\psi)$ , respectively. When the average transmitting optical flux of LED  $i$  is  $\Phi_i$  (in lumens), the average received optical power<sup>1</sup> of the PD is given by  $P_i = \Phi_i h_i$  (in lux · m<sup>2</sup>). In our system, we do not use filter and concentrator at the receiver (i.e.,  $T(\psi) = g(\psi) = 1$ ).

<sup>1</sup>Here, we use the photometric unit lux · m<sup>2</sup> for power. Physical units lux · m<sup>2</sup> and Watts are interchangeable and the constant for conversion depends on the device.

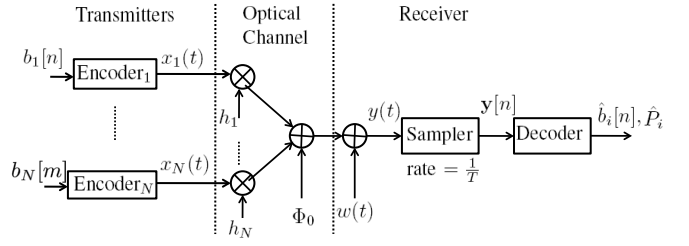


Fig. 1: A channel model of an VLC system for broadcasting  $N$  data streams by  $N$  transmitters which are asynchronous.

In this setting, we assume without loss of generality that the optical power incident on the PD from LED  $i$  is given by

$$P_i = \frac{\Phi_i(l+1)A}{2\pi d_i^2} \cos^l(\phi_i) \cos^M(\psi). \quad (2)$$

An UMA scheme using visible light is depicted in Fig. 1. Assume that the receiver starts receiving signals from  $N$  LEDs at time  $t = 0$ . Define  $b_i[k]$  as the  $k$ -th message sent by LED  $i$  and  $x_i(t)$  as the transmitted signal from LED  $i$  at time  $t$ . Suppose LED  $i$  started to transmit at  $t = -\tau_i$ . Since LEDs may begin transmission at different times,  $\tau_i$  may not be the same in general<sup>2</sup>. The superposition of signals from all LEDs is received together with the background light intensity  $\Phi_0$  at the receiver. Here, we assume that  $h_i$  and  $\Phi_0$  are constant over a short period of time. This is justified by that it is common to achieve transmission rate over  $10^6$  symbols per second in VLC [12]. If the receiver's displacement and the change in the background light intensity are negligible within  $10^{-3}$  seconds,  $h_i$  and  $\Phi_0$  can be seen as invariant for more than  $10^3$  symbols. Without loss of generality, assume that  $R_p$ , the responsivity of the PD, is equal to 1. So the received signal is modelled as

$$y(t) = \sum_{i=1}^N x_i(t) h_i + \Phi_0 + w(t), \quad (3)$$

where  $w(t)$  is the thermal noise which is a real value additive white Gaussian noise (AWGN) with zero mean and variance  $\sigma_w^2$ . At the receiver, the signal  $y(t)$  is sampled at a finite rate  $\frac{1}{T}$ .

### C. Problem Formulation

Since the LEDs transmit simultaneously,  $y(t)$  is the sum of the received powers from different LEDs. Furthermore,  $\tau_i$  is unknown to the receiver as the system is asynchronous. The receiver also has no information about the channel gains  $h_i$  because the receiver may appear at random location with arbitrary orientation. Therefore, we need to cleverly design  $x_i(t)$  such that for all  $i$ , the receiver

- 1) can decode the broadcast information  $b_i[n]$  and
- 2) can estimate the average received power  $P_i = \Phi_i h_i$  for LED  $i$ .

Then positioning algorithms such as [5] can be applied.

<sup>2</sup>  $\tau_i$  can also be used to capture propagation delay.

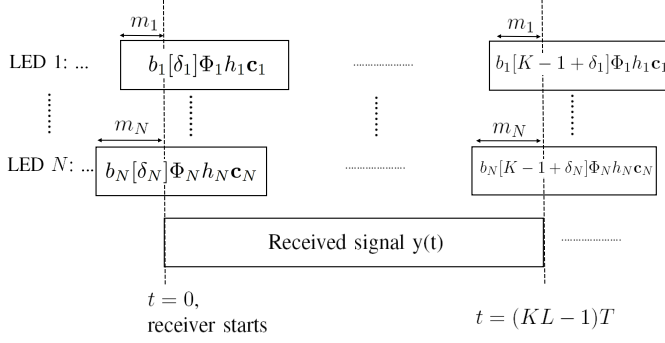


Fig. 2: The time tags among the transmitters and the receiver.

### III. PROPOSED MULTIPLE ACCESS SCHEME

In this section, the transmitter and receiver designs in our proposed system will be described. Our proposed transmitter design in Section III-A generalizes the design in [7] where only positioning but no data communications is supported. A completely different receiver design will be shown in Section III-B.

#### A. Transmitter Design

**Definition 1.** Consider a prime number  $L$  with  $L > 2N$ . For  $i = 1, 2, \dots, N$ , a codeword  $\mathbf{c}_i$  with  $L$  chips is assigned to LED  $i$  where the  $j$ -th chip of  $\mathbf{c}_i$  is defined as

$$\mathbf{c}_i[j] = \cos\left(\frac{2\pi i j}{L}\right) + 1, \quad (4)$$

so that  $0 \leq \mathbf{c}_i[j] \leq 2$  and  $\frac{1}{L} \sum_{j=0}^{L-1} \mathbf{c}_i[j] = 1$ . Assume that the chip duration is  $T$ .

Suppose  $b_i[k]$ , the  $k$ -th message sent by Transmitter  $i$ , is uniformly distributed in  $\{b', 2b', \dots, Mb'\}$  where  $b' = \frac{2}{M+1}$  so that the expected value of  $b_i[k]$  is always 1. The instantaneous optical flux generated from Transmitter  $i$  is defined as

$$x_i(t) = \sum_{k=0}^{\infty} \sum_{j=0}^{L-1} \Phi_i b_i[k] \mathbf{c}_i[j] \text{rect}(t - T(j - kL) + \tau_i), \quad (5)$$

where  $\Phi_i$  is a constant for LED  $i$  to control its optical flux per chip and

$$\text{rect}(t) = \begin{cases} 1 & 0 \leq t < T, \\ 0 & \text{otherwise.} \end{cases} \quad (6)$$

Therefore,  $M$ -PAM is used in each transmitter.

#### B. Receiver Design

The receiver receives the superposition of signals transmitted from the LEDs within its field of view (FOV). Suppose LED  $i$  has transmitted  $\delta_i$  messages before the receiver starts detecting signals. Due to the asynchronous transmission, there exists time lag  $m_i$  between LED  $i$  and the receiver, where  $m_i = \lfloor \frac{\tau_i}{T} \rfloor \bmod L$ . The system parameters are illustrated in Fig. 2. In the following, we present how the receiver first estimates  $m_i$  and then decodes the received message.

1) *Estimating time lag  $m_i$* : This is done by performing cross correlation between the received signals and a shifted version of  $\mathbf{c}_i$  as follows. Construct  $G(\mathbf{c}_i, \mu, K)$  by concatenating  $K$  copies of  $S(\mathbf{c}_i, \mu)$ . Here  $\mu$  will be determined below and the value of  $K$  will be discussed in Section IV-A. Let

$$B = [y(0) \ y(T) \ y(2T) \ \dots \ y((KL-1)T)]. \quad (7)$$

The estimate of  $m_i$ , denoted by  $\hat{m}_i$ , is the  $\mu$  which yields the maximum correlation between  $B$  and  $G(\mathbf{c}_i, \mu, K)$ , i.e.,

$$\hat{m}_i = \arg \max_{0 \leq \mu \leq L-1} (B \cdot G(\mathbf{c}_i, \mu, K)), \quad (8)$$

where  $\cdot$  denotes the dot product between two vectors. After the receiver estimates  $\hat{m}_i$  for all  $i$ , it can start decoding the messages  $b_i[k]$  as follows.

2) *Minimal Noise variance Decoder (MND)*: Recall that the received signal  $y(t)$  is sampled at a rate  $\frac{1}{T}$ . To recover  $h_i v_i[n]$  for  $1 \leq i \leq N$  and  $n \geq 0$ , we construct a vector

$$\mathbf{y}[n] = [y(nLT) \ y(nLT+T) \ \dots \ y(nLT+(\alpha-1)T)], \quad (9)$$

where  $\alpha = (2\gamma+1)L+1$  with a positive integer  $\gamma$  to be determined in Section IV-A. Due to (3) and (5),

$$\mathbf{y}[n] = \boldsymbol{\xi}[n] \mathbf{C} + \mathbf{w}, \quad (10)$$

where

$$\boldsymbol{\xi}[n] = \{h_1 v_1[n] \ \dots \ h_1 v_1[n+2\gamma+1] \ \dots \ h_N v_N[n] \ \dots \ h_N v_N[n+2\gamma+1] \ \Phi_0\} \quad (11)$$

is a row vector of length  $(2\gamma+2)N+1$  with  $v_i[n] = b_i[n+\delta_i]\Phi_i$  so that  $\boldsymbol{\xi}[n]$  contains  $(2\gamma+2)N$  unknown messages together with the undetermined background light intensity  $\Phi_0$ . Here,  $\mathbf{w}$  is a noise vector of length  $\alpha$  (sampling of  $w(t)$ ) and  $\mathbf{C}$  is a  $((2\gamma+2)N+1) \times \alpha$  matrix

$$\mathbf{C} = [\tilde{\mathbf{C}}_1^\top \ \tilde{\mathbf{C}}_2^\top \ \dots \ \tilde{\mathbf{C}}_N^\top \ \mathbf{1}_{\alpha \times 1}^\top]^\top, \quad (12)$$

where  $\mathbf{M}^\top$  is the transpose of matrix  $\mathbf{M}$ ,  $\mathbf{1}_{\alpha \times 1}$  is a column vector of ones with length  $\alpha$  and for  $1 \leq i \leq N$

$$\tilde{\mathbf{C}}_i = \begin{bmatrix} \bar{S}(\mathbf{c}_i, \hat{m}_i, L - \hat{m}_i) & 0 & 0 \cdots 0 & 0 & 0 \\ 0 & \mathbf{c}_i & 0 \cdots 0 & 0 & 0 \\ \vdots & & \ddots & & \vdots \\ 0 & 0 & 0 & \mathbf{c}_i & 0 \\ 0 & 0 & 0 & 0 & \bar{S}(\mathbf{c}_i, 0, \hat{m}_i + 1) \end{bmatrix}. \quad (13)$$

For  $1 \leq i \leq N$ , the optimal  $\boldsymbol{\beta}_i^*$  is obtained by solving the optimization problem

$$\text{minimize} \quad \sum_{j=0}^{\alpha-1} \beta_i[j]^2, \quad (14)$$

subject to

$$\mathbf{1}_{1 \times (2\gamma+2)} \cdot \tilde{\mathbf{C}}_i \cdot \boldsymbol{\beta}_i = 1, \quad (15)$$

$$\tilde{\mathbf{C}}_j \cdot \boldsymbol{\beta}_i = \mathbf{0}_{(2\gamma+2) \times 1}, \quad \forall j \neq i \quad (16)$$

$$\mathbf{1}_{1 \times \alpha} \cdot \boldsymbol{\beta}_i = 0, \quad (17)$$

variables:  $\boldsymbol{\beta}_i$ ,

where  $\mathbf{0}_{(2\gamma+2)\times 1}$  is a column vector of 0's with length  $2\gamma+2$ . Due to (15)–(17), our decoder gives

$$\mathbf{y}[n]\boldsymbol{\beta}_i^* = h_i \sum_{j=n}^{n+2\gamma+1} \lambda_j v_i[j] + \mathbf{w} \cdot \boldsymbol{\beta}_i^*, \quad (18)$$

where  $\lambda_j$  equals the multiplication of the  $j$ -th row of  $\tilde{\mathbf{C}}_i$  and  $\boldsymbol{\beta}_i^*$ . Due to (15),  $\sum_{j=n}^{n+2\gamma+1} \lambda_j = 1$ . Thus the variance contributed by the additive Gaussian noise becomes

$$\text{var}(\mathbf{w} \cdot \boldsymbol{\beta}_i^*) = \sigma_w^2 \sum_{j=0}^{\alpha-1} \boldsymbol{\beta}_i^*[j]^2. \quad (19)$$

So the proposed decoder not only eliminates MAI perfectly and removes the effect from background light, but also minimizes the noise variance in (19) due to (16), (17) and (14), respectively.

The receiver builds a set  $\mathcal{S}_i = \{\mathbf{y}[n]\boldsymbol{\beta}_i^*\}$  for different values of  $n$ . When the set  $\mathcal{S}_i$  is sufficiently large, the average of the elements in  $\mathcal{S}_i$  is equal to the expected value of the right side of (18), i.e.,  $h_i\Phi_i$ . Therefore,  $h_i$  can be estimated. So positioning can be done by applying algorithm such as [5].

3) *Comparison with Zero Forcing Decoder:* Due to (10), it seems that ZFD can be used to decode the transmitted messages. However, we are going to see that ZFD is worse than our proposed receiver in minimizing (19).

In ZFD, the receiver estimates  $\boldsymbol{\xi}[n]$ , which has  $(2\gamma+2)N+1$  messages, as follows

$$\hat{\boldsymbol{\xi}}[n] = \mathbf{y}[n]\mathbf{C}^\dagger = \boldsymbol{\xi}[n] + \mathbf{w}\mathbf{C}^\dagger, \quad (20)$$

where  $\hat{\boldsymbol{\xi}}[n]$  is the estimate of  $\boldsymbol{\xi}[n]$  and  $\mathbf{C}^\dagger = (\mathbf{C}^\top\mathbf{C})^{-1}\mathbf{C}^\top$  is the pseudo-inverse of the matrix  $\mathbf{C}$  [13]. Due to (20), ZFD eliminates MAI totally.

Now, consider the noise variance if ZFD is used. Define  $\mathbf{c}_i^\dagger$  as the  $i$ -th column of the matrix  $\mathbf{C}^\dagger$ . The noise in the  $i$ -th element in  $\hat{\boldsymbol{\xi}}[n]$  due to the additive Gaussian noise becomes

$$\sigma_w^2 \sum_{j=0}^{\alpha-1} \mathbf{c}_i^\dagger[j]^2, \quad (21)$$

which is very similar to (19). The receiver builds a set  $\mathcal{S}'_i = \{\mathbf{y}[n]\mathbf{c}_i^\dagger\}$  for different values of  $n$ , where

$$i^* = \underset{(i-1)(2\gamma+2) \leq j < i(2\gamma+2)}{\arg \min} \sum_{j=0}^{\alpha-1} \mathbf{c}_i^\dagger[j]^2. \quad (22)$$

So the one with the smallest noise variance belong to LED  $i$  is chosen. In contrast, MND finds an optimal way to combine the estimates of  $h_i v_i[n]$  in order to minimize (19). The following example shows how the noise variance changes with  $i$  and  $m_i$ .

**Example 1.** Consider a system with  $N = 2$ ,  $L = 5$ ,  $\gamma = 2$ ,  $\sigma_w^2 = 1$ ,  $m_1 = 1$  and  $m_2 = 2$ . If ZFD is used, the noise variances of LEDs 1 and 2 in (21) for all  $i$  are [198.1451 1.5515 0.2202 0.2168 0.2168 0.2169] and [197.5615 0.7271 0.2181 0.2168 0.2168 0.2170], respectively. Then ZFD selects the minimum noise variances

TABLE I: Simulation Parameters.

| Transmitters configuration                          | Values                                   |
|---|--|
| Number of LEDs $N$                                  | 4  |
| LEDs positions                                      | (1,1,3), (1,2,3), (2,1,3), (2,2,3)       |
| Codeword length $L$                                 | 11                                       |
| Average transmitting optical flux per chip $\Phi_i$ | 500 lm                                   |
| LED half power-angle $\phi_{1/2}$                   | $\frac{\pi}{3}$ rad                      |
| Background light intensity $\Phi_0$                 | $0.2 \mu\text{A}$                        |
| $\tau_i$  | uniform between [0, 1]                   |
| Receiver configuration                              | Values                                   |
| Receiver position                                   | (1.5,1.5,1)                              |
| Number of PD  | 1  |
| $K$   | 100                                      |
| $\gamma$  | 100                                      |
| PD's Lambertian parameter $M$                       | 1.4                                      |
| PD's FOV  | $\frac{\pi}{2}$ rad                      |
| PD's responsivity $R_p$                             | 22 nA/lux                                |
| Receiver's area $A$                                 | 15 mm <sup>2</sup>                       |
| $\sigma_{\text{noise}}^2$                           | $2.0856 \times 10^{-6} [\text{V}^2]$ [5] |

belong to LED 1 and LED 2 which are 0.2168 for both LEDs. If MND is used, the noise variances are 0.049 and 0.048 for LEDs 1 and 2, respectively.

4) *Decoding Data:* To compare the two decoders, suppose  $\{\hat{h}_i\}$  is obtained by either MND or ZFD. We use (20) to get an estimate of  $\boldsymbol{\xi}[n]$ . Together with the known  $\Phi_i$  and  $\{\hat{h}_i\}$ , the data  $b_i[n]$  can be estimated.

## IV. SIMULATION RESULTS

### A. Simulation Setup

Monte Carlo simulation is used to evaluate the performance of our proposed MND in terms of mean square error (MSE) of the estimated channel gains  $\hat{h}_i$  and bit error rate (BER) of the decoded data vs system throughput. The performance of ZFD is also shown to compare with MND. We assume that each LED transmits the same average optical flux per chip (i.e.,  $\Phi_i$  is a constant for all  $i$ ). The receiver is equipped with a PD. LEDs start at random times and then continue transmitting. The parameters of the simulation setup and LEDs and receiver positions are listed in Table I.

### B. System Performance versus System Throughput

Here, we evaluate the system performance through MSE of the estimated channel gains  $\hat{h}_i$  and BER of the decoded data vs system throughput. Assume that the chip duration  $T$  is equal to the inverse of the modulation frequency of the LEDs so that the throughput measured in bits/chip is equivalent to the throughput measured in bits/sec/Hz. To obtain different system throughput, we fix number of LEDs  $N = 4$  and Codeword length  $L = 11$  and vary the modulation scheme M-PAM, i.e.,  $M = \{2, 4, 8, 16, 32, 64\}$ . Let  $\eta = \log_2 M$  so that the system throughput =  $\frac{\eta N}{L}$ .

1) *MSE of estimated channel gains  $\hat{h}_i$ :* We define the MSE of the estimated channel gains  $\hat{h}_i$  as follows:

$$\text{MSE} = \frac{1}{N} \sum_{i=1}^N \left| \frac{h_i - \hat{h}_i}{h_i} \right|^2. \quad (23)$$

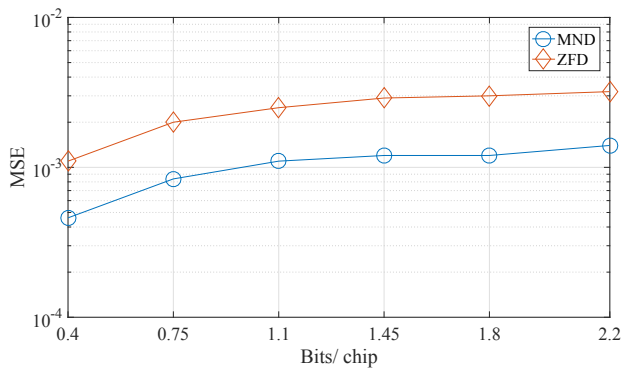


Fig. 3: MSE of estimated channel gains  $\hat{h}_i$  versus system throughput.

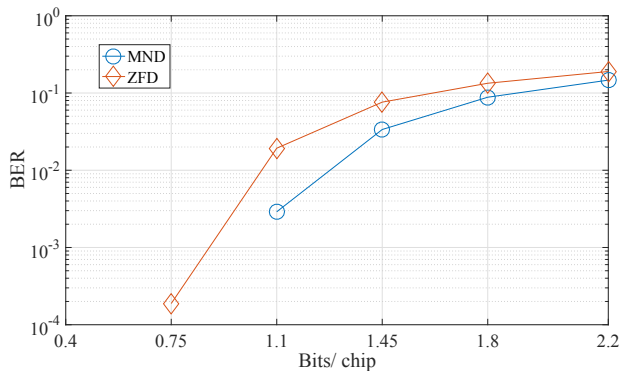


Fig. 4: BER versus system throughput.

Figure 3 shows the MSEs of both decoders versus the system throughput. The proposed MND outperforms ZFD because it minimizes the effect of Gaussian noise in (18) due to (14). Figure 3 also reveals that the MSEs of both decoders improve with reducing the modulation order  $M$  because smaller  $M$  gives smaller variance in data, i.e., the summation in (18).

2) *BER of the decoded data:* Figure 4 illustrates the BERs of both decoders versus the system throughput. It is shown that MND outperforms ZFD because MND minimizes the noise impact in (18) and gives smaller MSE in the estimation of  $h_i$ . The BER of MND for 0.75 Bits/chip is not shown because even  $4.4 \times 10^7$  bits had been transmitted in the simulations, all the bits were correctly received. The BERs of both decoders improve with reducing the modulation order  $M$  due to the following two reasons. 1) Improving MSE and therefore more accurate thresholds for decoding are obtained. 2) Increasing the Euclidean distances among transmitted symbols.

## V. CONCLUSION

This paper has proposed a multiple access scheme for visible light communications with positioning capability. The proposed scheme reduces the system cost and complexity by avoiding using central units and extra infrastructure to coordinate the transmitters. We have compared the performance

of our proposed scheme with zero-forcing decoder by both analysis and simulation. We have argued why our scheme always has the smaller effect due to the additive Gaussian noise. Simulation results showed that the proposed decoder outperforms zero-forcing decoder in terms of MSE, BER and system throughput.

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