A Disruption Recovery Model in a Production-Inventory System with Demand Uncertainty and Process Reliability

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Abstract. This paper develops a risk management tool for a production-inventory system that involves an imperfect production process and faces production disruption and demand uncertainty. In this paper, the demand uncertainty is represented as fuzzy variable and the imperfectness is expressed as process reliability. To deal with the production scheduling in this environment, a non-linear constrained optimization model has been formulated with an objective of maximizing the graded mean integration value (GMIV) of the total expected profit. The model is applied to solve the production-inventory problem with single as well as multiple disruptions on a real time basis that basically revises the production quantity in each cycle in the recovery time window. We propose a genetic algorithm (GA) based heuristic to solve the model and obtain an optimal recovery plan. A numerical example is presented to explain usefulness of the developed model.

Keywords: Production inventory, series of disruptions, demand uncertainty, process reliability, genetic algorithm.

1 Introduction

Batch production is a well accepted technique in advanced manufacturing and logistics management system. Production lot size is determined to minimize the costs of the system. There are numerous industries, such as pharmaceutical, textile and food, that produce the products using the batch production systems. There are several risks factors in real life problems which should be taken into consideration when production system is analyzed. Production disruptions i.e. raw material shortage, machine breakdown, labor strike, or any other production interruptions are very common scenario in the production systems. Moreover, it is very difficult to find the production process that produces 100% non-defective products. So the production process reliability, can be less than 100%, is also an important factor because of the imperfect production process. In real life situations, product demand cannot be known with uncertainty. In this paper, the process reliability and demand uncertainty are consid-

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ered with production disruption to make the research problem very close to the practical scenario.

Over the last few decades, one of the most widely studied research topics, in operations research and industrial engineering, is the production inventory system. Few examples of such studies in single stage production inventory system include a single-item inventory system with non-stationary demand process [1], determination of lot size and order level for a single-item inventory model with a deterministic time-dependent demand [2], a single-item periodic review stochastic inventory system [3] and a single-item single-stage inventory system with stochastic demand in a periodic review where the system must order either none or at least as much as a minimum order quantity [4].

The above studies, with many others, are conducted under ideal conditions. However, production disruption is a very familiar event in the production environment. Production disruption is defined as any form of interruption that may cause due to shortage of material, machine breakdown and unavailability, or any other form of disturbance. The development of an appropriate recovery policy can help to minimize the loss and maintain the goodwill of the company. Lin and Gong [5] analysed the impact of machine breakdown on EPQ model for deteriorating items in a single stage production system with fixed period of repair time. Widyadana and Wee [6] extended the model of Lin and Gong [5] for deteriorating items with random machine breakdown and stochastic repair time with uniform and exponential distribution. A disruption recovery model for single stage and single item production system is developed by Hishamuddin et al. [7] and the model was formulated for a single disruption, for recovering within a given time window, considering back order as well as lost sales option. Recently, a transportation disruption recovery model in a two-stage production and inventory system was developed by Hishamuddin et al. [8]. In the production and inventory modelling, numerous studies have been performed considering supply disruptions. Parlar and Perry [9] developed inventory models considering supplier availability with deterministic product demand under the continuous review framework. Özekici and Parlar [10] considered back orders to analyse a production-inventory model under random supply disruptions modelled as a Markov chain. Recently, other models of supply disruptions considering deterministic product demand in the inventory models have been studied [11] and [12].

There are some recent studies, where reliability of the imperfect production process has been considered. At first, process reliability is considered by Cheng [13] in a single period inventory system and formulated as unconstrained geometric programming problem. Later, it was extended in [14] by considering fuzzy random demand. Later, process reliability of the imperfect production process was incorporated to determine the optimal product reliability and production rate that achieved the biggest total integrated profit [15], to study unreliable supplier in a single-item stochastic inventory system [11] and to analyze an EPQ model with price and advertising demand pattern under the effect of inflation [16].

Many authors considered only fuzzy characteristics of the variables to tackle uncertainty. Lee and Yao [17] introduced fuzzy senses in the EPQ model considering demand and production per day as fuzzy variables. Later fuzzy product quantity as a triangular fuzzy number [18], order quantity and total demand quantity as triangular fuzzy numbers Yao et al. [19], demand as a fuzzy random variable in a single-period inventory model [20] are considered in developing production inventory modelling. Recently, Islam and Roy [21] developed a modified geometric programming program in an EPQ model under storage space constraint and reliability of the production process considering inventory related costs, storage spaces and others parameters as triangular fuzzy number.

In the previous studies of production inventory modelling, no study considered demand uncertainty and process reliability to develop a disruption recovery model. Also most of the previous studies focused on developing recovery plan only from single disruption. In this paper, a real time disruption recovery model is developed where the production process faces single or multiple disruptions. Other risk factors, process reliability and demand uncertainty, are also incorporated to make the model realistic. Finally, a genetic algorithm based heuristic is proposed to solve the model with single or multiple disruptions on a real time basis.

2 Problem Definitions

In the real life production system, disruptions are very common scenario and it can happen at any time at any point. It needs to develop an optimal plan to recover from those production disruptions. Revision in the production quantities and use of the idle timeslot of the systems are the significant ways to obtain the recovery plan [7]. After each disruption, production quantities in each cycle during the recovery period are revised. We develop a solution approach to obtain the recovery plan that deals with single or series of disruptions on a real time basis.

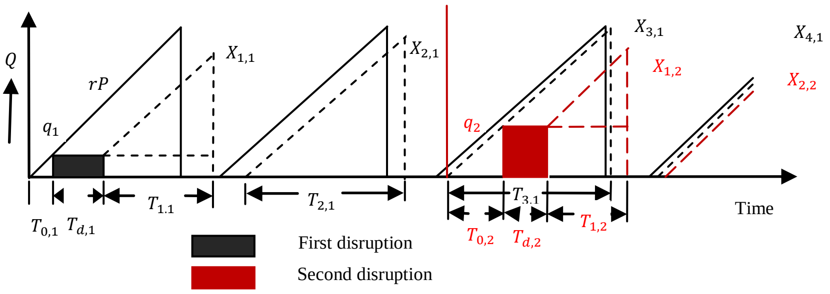

Fig. 1. Disruption recovery plan

The recovery plan after a production disruption is presented in Figure 1. The recovery plan is a new schedule which includes the revised production quantities in each cycle to maximize the total profit in the recovery period. Number of cycles allocated to return to the original production schedule from the disrupted cycle is known as recovery period. The first disrupted production cycle is considered as first cycle (l=1). Now an optimal recovery plan is proposed to revise the production quantities $X_{i,1}$ $(i = 1,2, \dots, M)$ to recover from that disruption which is shown as black dashed line in the Figure 1. Again after the second disruption within the recovery period of previous disruption, production quantities in each cycle $X_{i,2}$ $(i = 1,2, \dots, M)$ are revised by considering the effect of both disruptions which is shown as red dashed line

in Figure 1. It will be continued same way if there is any other disruption. The model is generalized by formulating for the n^{th} disruption.
In this study, we have made a number of assumptions as follows.

i. Production rate is greater than GMIV of the demand rate.
ii. Single item is produced in the system.
iii. All products are inspected and defective products are rejected.
iv. Total cost of interest and depreciation per production cycle $F(A,r)$ is inversely related to set-up cost (A) and is directly related to the process reliability (r) according to following general power function [13]:

$$F(A,r) = aA^{-b}r^{c}$$

Where a, b and c are positive constants chosen to provide the best fit of the estimated cost function.

2.1 Notations used in the Study

The following notations have been used in this study.

S_t	Set-up time for a production cycle
δ	Idle time of a production cycle
$\widetilde{D}$	Fuzzy demand per year
H	Holding cost per unit per year ($ per unit per year)
r	Reliability of the production process – which is known from the historical data of the production system
Q	Production lot size per normal cycle with reliability r
A	Set up cost per cycle ($ per set-up)
P	Production rate (units per year) in a 100% reliable system
u	Production downtime for a cycle (set-up time + idle time) $= S_t + \delta$
M	Number of cycles to recovery after the n^{th} disruption – given from the management
l	New disrupted cycle number from previous disruption
$T_{d,n}$	Disruption period in the n^{th} disruption
q_n	Pre-disruption production quantity in the n^{th} disruption
$T_{0,n}$	Production time for $q_n = \frac{q_n}{rP}$
$X_{i,0}$	Production quantity for a normal cycle i
$X_{i,n}$	Production quantity for cycle i the recovery period after n^{th} disruption– which is the decision variable; $i = 1,2,\ldots,M$
$T_{i,0}$	Productions up time for cycle a normal cycle $i = \frac{X_{i,0}}{rP}$
$T_{i,n}$	Productions up time for cycle i in the recovery period after n^{th} disruption
B	Unit back order cost per unit time ($ per unit per unit time)
L	Unit lost sales cost ($ per unit)
C_P	Per unit production cost ($ per unit)
C_R	Rejection cost per unit ($ per unit)
C_I	Inspection cost as a percentage of production cost
m_1	Mark-up for selling price ($m_1 C_P$) – must be greater than 1

3 Model Formulation

In this section, economic production lot size (Q), equations for different costs and revenues and final objective function are derived for the single stage production inventory system that considers process reliability and demand uncertainty. Economic lot size is calculated to minimize the total annual set-up and holding cost. For a single item production system, with lot-for-lot condition under ideal situation [22] with process reliability, the economic production quantity can be formulated as:

$$Q = \sqrt{\frac{2ArP}{H}} \tag{1}$$

3.1 Costs and Revenues Formulation

Holding, set-up, back order, lost sales, production, rejection, inspection and depreciation costs are identified as the relevant costs. Holding cost is determined as unit holding cost multiplied by total inventory during the recovery period which is equivalent to the area under the curve of Figure 1. Set-up cost is calculated as cost pet set-up multiplied by number of set-up in the recovery period. Back order cost is determined as unit back order cost multiplied by back order units and it's time delay [7]. Lost sales cost is determined as unit lost sales cost multiplied by lost sales units [7]. Unit production cost multiplied by total quantity produced during the recovery period is the total production cost. Rejection cost is determined as unit rejection cost multiplied by total rejected quantities [23]. Inspection cost is considered as a certain percentage of the production cost [23]. Cost of interest and depreciation equation is considered as a general power function [13]. The model is generalized by considering the production quantity in cycle i after the n^{th} disruption as $X_{i,n}$ and the original production quantity as $X_{i,0}$.

$$\text{Holding cost} = \frac{1}{2}H\left[\frac{(q_n)^2}{rP} + 2q_n\left(T_{d,n} + S_t\right) + \frac{2X_{1,n}q_n}{rP} + \sum_{i=1}^{M}\frac{(X_{i,n})^2}{rP}\right] \tag{2}$$

$$\text{Set-up cost} = AM \tag{3}$$

$$\text{Production cost} = C_P P\left(\sum_{i=1}^{M} T_{i,n} + T_{0,n}\right) = \frac{C_P}{r}\left(\sum_{i=1}^{M} X_{i,n} + q_n\right) \tag{4}$$

$$\text{Rejection cost} = C_R(1-r)P\left(\sum_{i=1}^{M} T_{i,n} + T_{0,n}\right) = C_R\left(\frac{1}{r} - 1\right)\left(\sum_{i=1}^{M} X_{i,n} + q_n\right) \tag{5}$$

$$\text{Inspection cost} = \frac{C_I C_P}{r}\left(\sum_{i=1}^{M} X_{i,n} + q_n\right) \tag{6}$$

$$\text{Cost of interest and depreciation} = Ma\,(A_1)^{-b}(r)^c \tag{7}$$

$$\text{Back-order cost} = B\left[(X_{1,n} + q_n)\left[T_{d,n} + \frac{q_n}{rP} + \frac{X_{1,n}}{rP} - \frac{X_{l,n-1}}{rP}\right] + \sum_{i=2}^{M} X_{i,n}\cdot\left[T_{d,n} + (i-1)S_t + \frac{q_n}{rP} + \sum_{j=1}^{i}\frac{X_{j,n}}{rP} - \sum_{j=1}^{i}\frac{X_{l+j-1,n-1}}{rP} - (i-1)u\right]\right] \tag{8}$$

$$\text{Lost sales cost} = L\left(\sum_{i=1}^{M} X_{l+i-1,n-1} - \sum_{i=1}^{M} X_{i,n} - q_n\right) \tag{9}$$

Selling price of the acceptable items, which is revenue, in the recovery period is determined as unit selling price multiplied by the demand in the recovery period [23].

$$\text{Revenues} = m_1 C_P\,\widetilde{D}\,\left[\sum_{i=1}^{M}\frac{X_{i,n}}{rP} + \frac{q_n}{rP} + MS_t\right] \tag{10}$$

3.2 Final Objective Function

Total profit, the objective function, is derived by subtracting all costs from the total revenues. Considering all the equations from (2) to (10), the objective function is obtained as follows.

$$\text{Max } \widetilde{Z} = \text{Total Revenues- Total Costs} \tag{11}$$

4 Fuzzy Parameter

In this paper, we consider product demand as a triangular fuzzy number (TFN) to tackle uncertainty. A TFN $\widetilde{D}$ is specified by a triplet (d_1, d_2, d_3) and is defined by its continuous membership function $\mu_{\widetilde{D}}(x): x \rightarrow [0,1]$ as follows:

$$\mu_{\widetilde{D}}(x) = \begin{cases} L(x) = \left(\frac{x-d_1}{d_2-d_1}\right) & if\ d_1 \leq x \leq d_2 \\ R(x) = \left(\frac{d_3-x}{d_3-d_2}\right) & if\ d_2 \leq x \leq d_3 \\ 0 & otherwise \end{cases} \tag{12}$$

$L(x)$ and $R(x)$ indicates the left and right branch of the TFN $\widetilde{D}$ respectively. An α-cut of $\widetilde{D}$ can be expressed by the following interval [17]:

$$D(\alpha) = [d_1 + (d_2 - d_1)\alpha,\ d_3 - (d_3 - d_2)\alpha],\ \ \alpha \in [0,1]$$

The graded mean integration value (GMIV) of a LR-fuzzy number is introduced by Chen and Hsieh [24]. The graded mean integration representation method is based on the integral value of the graded mean α-level of the LR-fuzzy number for defuzzifing LR-fuzzy numbers. By considering $\widetilde{D}$ is a LR-fuzzy number and according to Chen and Hsieh [24], the GMIV of $\widetilde{D}$ is defined as:

$$G(\widetilde{D}) = \frac{\int_0^1 \left(\frac{\alpha}{2}\right)\{L^{-1}(\alpha)+R^{-1}(\alpha)\}d\alpha}{\int_0^1 \alpha d\alpha} = \int_0^1 \alpha\{L^{-1}(\alpha) + R^{-1}(\alpha)\}\, d\alpha \tag{13}$$

5 Disruption Recovery Model with Fuzzy Demand

In this section, fuzziness of demand is incorporated to the final mathematical model. The GMIV of the expected total profit function is evaluated. Relevant constraints are also developed with the GMIV of expected fuzzy demand. After simplifying the equation (11), the following equation of the total profit is obtained:

$$\widetilde{Z} = C\,\widetilde{D} + Y \tag{14}$$

Now, considering the fuzzy random demand $\widetilde{D}$ with the given set of ta $(\tilde{d}_1, \tilde{p}_1), (\tilde{d}_2, \tilde{p}_2), (\tilde{d}_3, \tilde{p}_3), \ldots\ldots, (\tilde{d}_v, \tilde{p}_v)$, the profit $(\widetilde{Z})$ is also a fuzzy random variable and its expectation is a unique fuzzy number [14] which is,

$$E\widetilde{Z} = C\sum_{k=1}^{v} \tilde{d}_k \tilde{p}_k + Y$$

In this paper, demand data are considered as a triangular fuzzy number (TFN). Demand TFN and associated probabilities are taken as a triplet $\left(\underline{d_k}, d_{k,}\overline{d_k}\right)$ and $\left(\underline{p_k}, p_{k,}\overline{p_k}\right)$ respectively. Where, $k= 1, 2, 3...., v$. Then the fuzzy expected profit function will also be a TFN, $E\widetilde{Z} = \left(\underline{\mathrm{EZ}}, \mathrm{EZ}, \overline{\mathrm{EZ}}\right)$ which is determined as follows:

$$EZ = E[Z(\alpha = 1)] = C\sum_{k=1}^{v} d_k\, p_k \ + Y$$

$$\underline{EZ} = E[Z_L(\alpha = 0)\ ] = C\sum_{k=1}^{v} \underline{d_k}\ \underline{p_k} + Y$$

$$\overline{EZ} = E[Z_R(\alpha = 0)] = C\sum_{k=1}^{v} \overline{d_k}\ \overline{p_k} + Y$$

Here the α-level set of the fuzzy number $E\widetilde{Z}$ are considered as $EZ(\alpha) = E[Z(\alpha)] = \left[E\left(Z_L(\alpha)\right), E\left(Z_R(\alpha)\right)\right]$; $0 \leq \alpha \leq 1$ and different α -cut intervals for the fuzzy number $E\widetilde{Z}$ are obtained for different α between 0 and 1.Taking, α-cut on both sides of equation of $E\widetilde{Z}$.

$$E\tilde{Z}_\alpha = C\sum_{k=1}^{v} \tilde{d}_{k\alpha}\tilde{p}_{k\alpha} + Y$$

The arithmetic interval of fuzzy demand and associated probabilities using an α-cut is determined as follows.

$$\tilde{d}_{k\alpha} = \left[\underline{d_k} + \alpha\left(d_k - \underline{d_k}\right),\ \overline{d_k} - \alpha\left(\overline{d_k} - d_k\right)\right]$$

$$\tilde{p}_{k\alpha} = \left[\underline{p_k} + \alpha\left(p_k - \underline{p_k}\right),\ \overline{p_k} - \alpha(\overline{p_k} - p_k)\right]$$

By using these arithmetic intervals, $E\tilde{Z}_\alpha$ is evaluated as:

$$E\tilde{Z}_\alpha = \left[\left[C\sum_{k=1}^{v}\left[\underline{d_k} + \alpha\left(d_k - \underline{d_k}\right)\right]\left[\underline{p_k} + \alpha\left(p_k - \underline{p_k}\right)\right] + Y\right],\right.$$
$$\left.\left[C\sum_{k=1}^{v}\left[\overline{d_k} - \alpha\left(\overline{d_k} - d_k\right)\right]\left[\overline{p_k} - \alpha(\overline{p_k} - p_k)\right]\ + Y\right]\right]$$

From the representation of graded mean integration methods based on the integral value of the graded mean α-level of the LR-fuzzy number of the total profit, $\mathrm{L}^{-1}(\alpha)$ and $\mathrm{R}^{-1}(\alpha)$ are obtained as follows.

$$\mathrm{L}^{-1}(\alpha) = C\sum_{k=1}^{v}\left[\underline{d_k} + \alpha\left(d_k - \underline{d_k}\right)\right]\left[\underline{p_k} + \alpha\left(p_k - \underline{p_k}\right)\right] + Y$$

$$\mathrm{R}^{-1}(\alpha) = C\sum_{k=1}^{v}\left[\overline{d_k} - \alpha\left(\overline{d_k} - d_k\right)\right]\left[\overline{p_k} - \alpha(\overline{p_k} - p_k)\right] + Y$$

The unique fuzzy number $\mathrm{G}\left(E\widetilde{Z}\ \right)$ is determined by substituting the value of $\mathrm{L}^{-1}(\alpha)$ and $\mathrm{R}^{-1}(\alpha)$ to the equation (13),

$$G\left(E\widetilde{Z}\ \right) = \int_0^1\left[\alpha\ \left[C\sum_{k=1}^{v}\left[\underline{d_k} + \alpha\left(d_k - \underline{d_k}\right)\right]\left[\underline{p_k} + \alpha\left(p_k - \underline{p_k}\right)\right] + Y\right]\right]d\alpha$$
$$+ \int_0^1\left[\alpha\left[C\sum_{k=1}^{v}\left[\overline{d_k} - \alpha\left(\overline{d_k} - d_k\right)\right]\left[\overline{p_k} - \alpha(\overline{p_k} - p_k)\right]\ + Y\right]\right]d\alpha$$

After integrating and simplifying the above equation of $G\left(E\widetilde{Z}\right)$, the GMIV of the total profit function, which is to be maximized, and obtained as:

$$\text{Max } \mathrm{G}\left(E\widetilde{Z}\right) = C(Z_1 + Z_2) + Y \tag{15}$$

Where,

$$Z_1 = \sum_{k=1}^{v} \{\frac{1}{2}\underline{d_k}\,\underline{p_k} + \frac{1}{3}\underline{d_k}\left(p_k - \underline{p_k}\right) + \frac{1}{3}\underline{p_k}\left(d_k - \underline{d_k}\right) + \frac{1}{4}\left(d_k - \underline{d_k}\right)\left(p_k - \underline{p_k}\right)\}$$

$$Z_2 = \sum_{k=1}^{v} \{\frac{1}{2}\overline{d_k}\,\overline{p_k} - \frac{1}{3}\overline{d_k}(\overline{p_k} - p_k) - \frac{1}{3}\overline{p_k}(\overline{d_k} - d_k) + \frac{1}{4}(\overline{d_k} - d_k)(\overline{p_k} - p_k)\}$$

GMIV of the expected fuzzy demand, $\mathrm{G}\left(E\widetilde{D}\right) = Z_1 + Z_2$

Subject to the following constraints:

$$X_{i,0} = Q \tag{16}$$

$$X_{1,n} + q_n \le X_{l,n-1} \tag{17}$$

$$X_{i,n} \le X_{l+i-1,n-1}\ ;\ i = 2,3,4,\ldots\ldots, M \tag{18}$$

$$rP \ge G\left(E\widetilde{D}\right) \tag{19}$$

$$r \le 1 \tag{20}$$

$$\sum_{i=1}^{M} X_{i,n} + q_n \le rP\left(\sum_{i=1}^{M} \frac{X_{l+i-1,n-1}}{G(E\widetilde{D})} - MS_t - T_{d,n}\right) \tag{21}$$

$$\sum_{i=1}^{M} X_{i,n} + q_n \ge \left(\frac{\sum_{i=1}^{M} X_{i,n} + q_n}{rP} + MS_t\right) G(E\widetilde{D}) - \left(\sum_{i=1}^{M} X_{l+i-1,n-1} - \sum_{i=1}^{M} X_{i,n} - q_n\right) \tag{22}$$

$$\frac{X_{1,n} + q_n}{G(E\widetilde{D})} - \frac{X_{2,n}}{rP} - S_t \ge 0 \tag{23}$$

$$\frac{X_{i,n}}{G(E\widetilde{D})} - \frac{X_{i+1,n}}{rP} - S_t \ge 0;\ i = 2, 3, \ldots., M \tag{24}$$

$$T_{d,n} + \frac{q_n}{rP} + \frac{X_{1,n}}{rP} - \frac{X_{l,n-1}}{rP} \ge 0 \tag{25}$$

$$T_{d,n} + (i-1)S_t + \frac{q_n}{rP} + \sum_{j=1}^{i} \frac{X_{j,n}}{rP} - \sum_{j=1}^{i} \frac{X_{l+j-1,n-1}}{rP} - (i-1)u \ge 0;\ i = 2,3,4,\ldots\ldots, M \tag{26}$$

6 Solution Approach

We propose a genetic algorithm based heuristic to solve the model. Genetic algorithm is very popular technique to solve complex non-linear constrained optimization problem. GAs are general purpose optimization algorithms which apply the rules of natural genetics to explore a given search space [25]. The heuristic is designed to make a recovery plan from a single or a series of production disruptions. The proposed heuristic revises the production lot size of each cycle as long as disruptions take place in the system. For a series of disruptions, the heuristic revises the lot size of each cycle by considering the effect of all previous dependent disruptions. The proposed genetic algorithm based heuristic is presented in the Figure 2. The above mentioned heuristic is coded in MATLAB R2012a with the help of its optimization toolbox. In the proposed heuristic, following GA parameters are used to solve the model.

Population size: 100; Population type: Double vector; Crossover fraction: 0.8; Maximum number of generations: 3000; Function tolerance: 1e-6; Non linear constraint tolerance: 1e-6 and other parameters are set as default of the optimization toolbox.

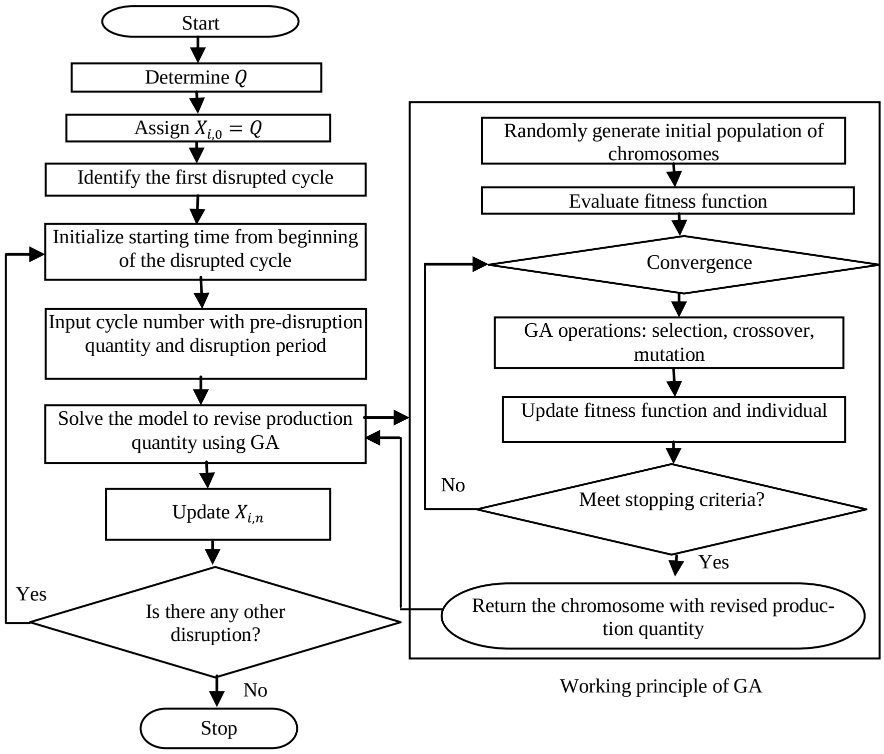

Fig. 2. Flowchart of proposed GA based heuristic

7 Results Analysis

Results have been analysed for both single and multiple disruptions on a real time basis. For single disruption, there is only one random disruption in the system and there is no more disruption within the recovery period. For multiple disruptions, there is a series of disruptions, one after another, on a real time basis in the system.

7.1 Results Analysis for Single Disruption

Following data are considered to analyze the results for single disruption:
$S_t = 0.000057, u = 0.000077,\ A = 60,\ H = 1.4,\ r = 0.92,\ P = 550000,\ M = 5,$ $B = 20,\ L = 25,\ T_{d1} = 0.01,\ q_1 = 500, C_P = 40,\ C_R = 15,\ C_I = 0.01, a = 1000,$ $b = 0.5, c = 0.75, m_1 = 2.5$, and demand data are considered as TFN which is shown in the Table 1.

Table 1. Demand data as TFN and associated probabilities

Demand rate	Probability
(300000, 320000, 340000)	(0.050, 0.055, 0.060)
(350000, 370000, 390000)	(0.144, 0.150, 0.156)
(400000, 420000, 440000)	(0.293, 0.300, 0.307)
(450000, 470000, 490000)	(0.194, 0.202, 0.210)
(500000, 520000, 540000)	(0.104, 0.110, 0.117)
(550000, 570000, 590000)	(0.094, 0.100, 0.106)
(600000, 620000, 640000)	(0.088, 0.093, 0.098)

The problem is solved using the proposed GA based heuristic. The results are obtained from 30 different runs. The best recovery plan after single disruption is shown in Table 2. The production system returns to original schedule from the sixth cycle after the disruption with $X_{6,1} = 6586$, $X_{7,1} = 6586$ and so on. The maximum total profit in the recovery period is obtained as 1381112.5.

Table 2. Best results obtained for the single disruption

Disruption number (n)	Revised production quantity					Total profit
	$X_{1,1}$	$X_{2,1}$	$X_{3,1}$	$X_{4,1}$	$X_{5,1}$	
1	5305	5638	6066	6469	6563	1381112.5

7.2 Results Analysis for a Series of Disruptions

In this case, a series of disruptions on a real time basis is considered, which is shown in Table 3. In this series of disruptions, seven dependent disruptions are considered and each one occurs within the recovery period of the previous disruption. Other data remain same as in section 7.1.

Table 3. Data for the series of disruptions

Disruption number (n)	Disrupted cycle number from previous disruption	Pre-disruption quantity	Disruption period
1	1	1000	0.0045
2	2	650	0.0092
3	3	500	0.0025
4	5	1500	0.0065
5	2	0	0.0110
6	4	800	0.0098
7	3	0	0.0078

The production system with multiple disruptions is also solved using the GA based heuristic on a real time basis. The results are obtained from 30 different runs. Production quantity in each cycle is revised after each disruption considering the effect of entire dependent disruptions to maximize the total profit in the recovery period. The best recovery plan obtained from the heuristic for the series of disruptions is shown in Table 4. The production system returns to original schedule from the sixth cycle after each disruption with $X_{6,n} = 6586$, $X_{7,n} = 6586$ and so on.

Table 4. Best results obtained for the series of disruptions

Disruption number (n)	Revised production quantity					Total profit
	$X_{1,n}$	$X_{2,n}$	$X_{3,n}$	$X_{4,n}$	$X_{5,n}$	
1	5585	6561	6545	6567	6563	1545915.7
2	5272	5671	6111	6553	6572	1404616.5
3	5611	6548	6570	6541	6550	1524257.8
4	4804	6539	6433	6476	6522	1506722.6
5	5271	5647	5994	6336	6381	1324912.7
6	4941	5668	5936	6271	6532	1364154.2
7	5657	5914	6049	6486	6443	1407321.9

8 Conclusions

The objective of this research was to incorporate demand uncertainty and process reliability in developing a disruption recovery model for managing risk in a production inventory system. A single or a series of disruptions on a real time basis was considered to make the model applicable in practical problems. The model was formulated as a non-linear constrained optimization problem and generalized by formulating the model for the n^{th} disruption. A genetic algorithm based heuristic was proposed to solve the model with single or multiple disruptions on a real time basis. This model can be applied in an imperfect production process where the process countenances a single or multiple production disruptions and product demand is uncertain. The model can be extended by considering multiple stages in the production system.

References

1. Graves, S. C.: A single-item inventory model for a nonstationary demand process. Manufacturing & Service Operations Management, 1(1), 50-61, (1999).
2. Dave, U..: A deterministic lot-size inventory model with shortages and a linear trend in demand. Naval Research Logistics, 36(4), 507-514, (2006).
3. Chan, G. H., & Song, Y.: A dynamic analysis of the single-item periodic stochastic inventory system with order capacity. European Journal of Operational Research, 146(3), 529-542, (2003).
4. Kiesmüller, G. P., De Kok, A. G., & Dabia, S.: Single item inventory control under periodic review and a minimum order quantity. International Journal of Production Economics, 133(1), 280-285, (2011).
5. Lin, G. C., & Gong, D. C.: On a production-inventory system of deteriorating items subject to random machine breakdowns with a fixed repair time. Mathematical and Computer Modelling, 43(7), 920-932, (2006).

6. Widyadana, G. A., & Wee, H. M.: Optimal deteriorating items production inventory models with random machine breakdown and stochastic repair time. Applied Mathematical Modelling, 35(7), 3495-3508, (2011).
7. Hishamuddin, H., Sarker, R. A., & Essam, D.: A disruption recovery model for a single stage production-inventory system. European Journal of Operational Research, 222(3), 464-473, (2012).
8. Hishamuddin, H., Sarker, R. A., & Essam, D.: A Recovery Model for a Two–Echelon Serial Supply Chain with Consideration of Transportation Disruption. Computers & Industrial Engineering, 64(2), 552-561, (2013).
9. Parlar, M., & Perry, D.: Inventory models of future supply uncertainty with single and multiple suppliers. Naval Research Logistics, 43(2), 191-210, (1998).
10. Özekici, S., & Parlar, M.: Inventory models with unreliable suppliers in a random environment. Annals of Operations Research, 91, 123-136, (1999).
11. Mohebbi, E., & Hao, D.: An inventory model with non-resuming randomly interruptible lead time. International Journal of Production Economics, 114(2), 755-768, (2008).
12. Qi, L., Shen, Z. J. M., & Snyder, L. V.: A continuous-review inventory model with disruptions at both supplier and retailer. Production and Operations Management, 18(5), 516-532, (2009).
13. Cheng, T. C. E.: An economic production quantity model with flexibility and reliability considerations. European Journal of Operational Research, 39(2), 174-179, (1989).
14. Bag, S., Chakraborty, D., & Roy, A. R.: A production inventory model with fuzzy random demand and with flexibility and reliability considerations. Computers & Industrial Engineering, 56(1), 411-416, (2009).
15. Sana, S. S.: A production–inventory model in an imperfect production process. European Journal of Operational Research, 200(2), 451-464, (2010).
16. Sarkar, B.: An inventory model with reliability in an imperfect production process. Applied Mathematics and Computation, 218(9), 4881-4891, (2012).
17. Lee, H. M., & Yao, J. S.: Economic production quantity for fuzzy demand quantity, and fuzzy production quantity. European Journal of Operational Research, 109(1), 203-211, (1998).
18. Chang, S. C.: Fuzzy production inventory for fuzzy product quantity with triangular fuzzy number. Fuzzy sets and Systems, 107(1), 37-57, (1999).
19. Yao, J. S., Chang, S. C., & Su, J. S.: Fuzzy inventory without backorder for fuzzy order quantity and fuzzy total demand quantity. Computers & Operations Research, 27(10), 935-962, (2000).
20. Dutta, P., Chakraborty, D., & Roy, A. R.: A single-period inventory model with fuzzy random variable demand. Mathematical and Computer Modelling, 41(8), 915-922, (2005).
21. Islam, S., & Roy, T. K.: Fuzzy multi-item economic production quantity model under space constraint: A geometric programming approach. Applied Mathematics and Computation, 184(2), 326-335, (2007).
22. Sarker, R. A., & Khan, L. R.: An optimal batch size for a production system operating under periodic delivery policy. Computers & Industrial engineering, 37(4), 711-730, (1999).
23. Paul, S.K., Azeem, A., Sarker, R., & Essam, D.: Development of a production inventory model with uncertainty and reliability considerations. Optimization & Engineering, Accepted manuscript, (2013). doi: 10.1007/s11081-013-9218-6
24. Chen, S. H., & Hsieh, C. H.: Graded mean integration representation of generalized fuzzy number. Journal of Chinese Fuzzy Systems, 5(2), 1-7, (1999).
25. Homaifar, A., Qi, C. X., & Lai, S. H.: Constrained optimization via genetic algorithms. Simulation, 62(4), 242-253, (1994).