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INVERSE CONDUCTIVITY RECOVERY PROBLEM IN A SPHERICAL GEOMETRY FROM EEG DATA: UNIQUENESS, RECONSTRUCTION AND STABILITY RESULTS

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Abstract

Electroencephalography (EEG) is a non invasive imaging technique that measures the effect of the electric activity of active brain regions, called sources, through values of the electric potential furnished by a set of electrodes placed at the surface of the scalp. A fundamental problem there is the inverse problem of source localization which aims at locating the sources of the electric activity using the acquired EEG measurements [2]. The quality of the source estimation depends on the accuracy of the conductivity model used to solve the problem. Among the head tissues, the skull conductivity is the one that influences most the accuracy of EEG source localization [3]. Often, conductivity estimation is performed prior to the source estimation to determine the unknown conductivity using either supplementary EEG measurements or even measurements acquired by other imaging techniques. Such a technique is electrical impedance tomography (EIT) where current is injected through a pair of EEG electrodes while the unknown conductivities can be estimated by the resulting measurements at the rest electrode locations.

We examine the inverse skull conductivity estimation problem, which aims at recovering the electrical conductivity properties of the skull from measurements given at the surface of the head by EEG measurements. Our goal is to show uniqueness and a constructive scheme for the inverse skull conductivity estimation problem using partial boundary EEG data from a single experiment, in the preliminary case of an homogeneous skull conductivity. This is a version of the many inverse conductivity issues still under study nowadays [1].

The head is assumed to be an isotropic piecewise homogeneous medium and we examine a layered spherical head model made of three concentric nested spheres, each of them modelling scalp, skull and brain tissues (from the outermost to the innermost layer). Each of the three layers is supposed to have a constant conductivity. We also assume that the conductivities of the brain and the scalp are known, while the conductivity to be recovered is the one of the intermediate spherical layer (skull).

We solve the above conductivity estimation problem from the available EEG partial boundary data, expanded on the spherical harmonics basis, and transmitted over the spherical interfaces by transfer functions, while we consider that the source term is already estimated (through a number of coefficients of its spherical harmonics expansion). Linear algebra computations then allow us to find polynomials that possess a root which should coincide with the unknown skull conductivity, thus solving the estimation problem. We derive uniqueness properties and a reconstruction algorithm for the skull conductivity. A numerical study shows that the algorithm is able to accurately estimate the skull conductivity, with good robustness properties with respect to various levels of noise. The properties of the inverse conductivity estimation problem are also examined with various source configurations (partially known sources) and EIT measurements.

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