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Moon today has no global magnetic field, nevertheless there are evidences showing that it used to have one in the past [1]. Paleomagnetic scientists try to understand the reasons of the creation and the evolution of this ancient magnetic field, by studying the remanent magnetism of surface Moon rocks.

In our study, we consider dipole recovery issues from sparse magnetic data, with the use of best quadratic rational approximation techniques, together with geometrical and algebraic properties of the poles of the approximants.

The data acquisition process generates the main "cylindrical" geometry of our study. A special magnetometer has been constructed by scientists at CEREGE* which they call "lunometer". Lunometer provides measurements of the components of the magnetic field B, on 3 circular sections $i = \{1, 2, 3\}$.

The underlying magnetic phenomenon is modeled by Maxwell equations in the magnetostatic and macroscopic framework [2]. We preliminary assume that the sample contains a unique pointwise dipolar unknown magnetic source located at X_d, with moment M_d. The expression of the magnetic field at $X \neq X_d$ (where μ_0 is the permeability of the free space) has the form:

$$-\frac{4\pi}{\mu_0}B(X) = \frac{|X - X_d|^2 M_d - 3\left[M_d \cdot (X - X_d)\right](X - X_d)}{|X - X_d|^5} \tag{1}$$

From measurements of the magnetic field B at sensors positions, we want to recover the moment M_d and the location X_d of the dipole. Our strategy is to decompose this inverse problem into two subproblems and search firstly for the location X_d of the dipole (which is a nonlinear problem) and secondly for its moment M_d (which becomes a linear problem when the location is known).

More precisely, the idea is to use the available data of a component of B denoted as B_i. Then, B_i^2 can be seen as the trace on the circle of a rational function of the complex variable in the corresponding plane [3]. This function has one pole inside the disk and one outside. From the values of this function, we use the best quadratic rational algorithm called RARL2‡ which provides us with an approximation of the pole inside. Vieta's formula allows us to estimate the pole outside, hence to fully recover the denominator of the rational function. We establish relations between the denominator and the unknown actual dipole position X_d.

Observe that a variety of factors can affect the moment recovery process. Indeed, the data is subject to measurement errors and the condition number of the problem depends on the actual dipole location/orientation. Geometrical methods could work as supplementary to the algebraic methods. During the talk, these techniques will be explained and numerical results will be discussed.

References

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[3] L. Baratchart, J. Leblond and J. P. Marmorat, *Inverse source problem in 3D ball from best meromorphic approximation on 2D slices*, Electronic Transactions on Numerical Analysis, 25, pp. 41-53, 2006.

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‡`https://project.inria.fr/rarl2/`

**`http://maglune.cerege.fr/`